Including Risk – Part 1 Non Adaptive

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Including Non Adaptive Risk

Many face pervasive uncertainty.

Prices, weather, labor, and other factors induce large changes in revenue, yields, working rates and resources available.

When incorporating risk into the program models there are three big issues

1. What is the nature of risk?
   a. What parameters of the model are uncertain? and
   b. How do we describe their distribution?

2. When during the model time horizon are risk outcomes revealed?

Are there times when the model should reflect that the producer has received information about uncertain events and will make adaptive decisions?

3. How do we model the decision makers behavioral reaction to risk? Is expected profit maximization not to the proper objective but rather some degree of aversion to the variation caused by risk?

Including Non Adaptive Risk

Our risk treatment will be limited and somewhat specialized because of time constraints (see newbook.pdf chapter 14 and probab.pdf for more extensive treatment)

The treatment will be limited in several principal ways

1. Will specialize in risk in objective function coefficients. Will not discuss how to form such probability distributions except through a few casual remarks (see probab.pdf for more extensive treatment)

2. We will only cover the expected value variance formulation of model objective function alterations to risk (this is the two main one used in the literature -- see newbook.pdf chapter 14 for more extensive treatment)

3. We will treat non adaptive behavior here. In a later section adaptive behavior using discrete stochastic programming or stochastic programming with recourse

Including Non Adaptive Risk
Why model risk

Why not just solve for all values of risky parameters

Curses of dimensionality and certainty

Dimensionality Number of possible plans
(3 possible values for 5 parameters $3^5 = 243$)

Certainty Each plan would be certain of data so we would have
243 different things we could do –
What would we do?

General Risk Modeling Aim

Generate a plan which is Robust in the face of the Uncertainty

Not best performer necessarily in any setting, but a good performer across many or most of the uncertainty spectrum

Including Non Adaptive Risk

Risk entry into a programming model

Maximize CX
Subject to AX # b
X $ 0

Objective function returns - C

- Variability in prices
- Variability in production quantities
- Variability in costs
- Variability in market sales

Resource usages - A

- Variability in raw input quality
- Variability in working conditions
- Variability in intermediate product yields
- Variability in product requirements

Resource endowments - b

- Variability in demand firm faces
- Variability in resources available
- Variability in working conditions

Including Non Adaptive Risk

Forms of assumed reaction to risk

Non-Recourse or non adaptive decision making

Decisions made now consequences felt later

No additional decisions made between now and when consequences felt

Example – Buy stock now make no decisions for one year

Recourse or adaptive decision-making

Decisions made now consequences arise over time

Later time during model additional decisions made.

In this later decision period

Decision maker knows what happened between first decision and now.

Decision makers cannot revise prior actions but can adjust current decisions ie current decisions can be employed to make adjustments in the face of realized events -- phenomena called irreversibility and recourse

Example – Buy stock now, review decisions quarterly possibly selling and buying other stocks

Including Non Adaptive Risk
Decision Maker reaction to risk

Expected Value Maximization

$$\begin{array}{lccc} \max & \bar{c}X & & \\ s.t. & \bar{A}X & \leq & \bar{b} \\ & X & \geq & o \end{array}$$

Conservative - Fat or thin coefficients

$$\begin{array}{lccc} \max & \tilde{c}X & & \\ s.t. & \tilde{A}X & \leq & \tilde{b} \\ & X & \geq & o \end{array}$$

where $\tilde{c} = \bar{c} - \text{Risk Discount}$

$\tilde{a} = \bar{a} + \text{Risk Discount}$

$\tilde{b} = \tilde{b} - \text{Risk Discount}$

E- V

Maximize E(income) - RAP * Variance(income)

Expected utility

Maximize Sum(p,Probability(p)*U[Wealth(p)])

S.T. Wealth(p)=InitWealth + Income(p) for all p

Income(p)=C(p)*X for all p

Safety First based

Maximize Sum(p,Probability(p)*Income(p))

S.T. Income(p)=C(p)*X for all p

Income(p)$safety for all p

Including Non Adaptive Risk
First Risk Model

Markowitz mean-variance portfolio choice formulation

Given Problem
max sum(invest, moneyinvest(invest)*avgreturn(invest))
s.t
sum(invest,moneyinvest(invest)*price(invest))# funds

Markowitz observed not all money is invested in the highest valued stock

Inconsistent with LP formulation
Why? Not a basic solution

Markowitz posed the hypothesis that average returns and the variance of returns were important

Including Non Adaptive Risk

E-V Model – Statistical Background

Given a linear objective function

$$Z = c_1 X_1 + c_2 X_2$$

Where X_1, X_2 are decision variables
c_1 , c_2 are uncertain parameters

Then Z is distributed with mean and variance

$$\overline{Z} = \overline{c}_1 X_1 + \overline{c}_2 X_2$$

$$\sigma_Z^2 = s_{11} X_1^2 + s_{22} X_2^2 + s_{a2} X_1 X_2$$

Defining terms

s_{ii} is the variance of objective function coefficient of X_i, which is calculated using $s_{ik} = 3_k (c_{ik} - \bar{c}_i)^2/N$ where c_{ik} is the k^{th} observation on the objective value of X_i and N the number of observations.

s_{ij} for $i \neq j$ is the covariance of the objective function coefficients between X_i and X_j, calculated by the formula $s_{ij} = 3_k (c_{ik} - \bar{c}_i)(c_{jk} - \bar{c}_j)/N$. Note $s_{ij} = s_{ji}$.

$\bar{c}_i$ is the mean value of the objective function coefficient associated with X_i, calculated by $\bar{c}_i = 3_k c_{ik}/N$. (Assuming an equally likely probability of occurrence.)

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E-V Model – Statistical Background

In matrix terms the mean and variance of Z are

$$\overline{Z} = \overline{c}\,X$$

$$\sigma_Z^2 = X' S Z$$

where in the two by two case

$$\overline{Z} = E = [\bar{c}_1 \; \bar{c}_2]\begin{bmatrix} X_1 \\ X_2 \end{bmatrix}$$

$$\sigma_Z^2 = [X_1 \; X_2]\begin{bmatrix} s_{11} & s_{12} \\ s_{21} & s_{22} \end{bmatrix}\begin{bmatrix} X_1 \\ X_2 \end{bmatrix}$$

Markowitz Formulation

$$\begin{aligned} &Min \quad \sigma_Z^2 \\ &s.t. \quad E \quad = \quad K \end{aligned}$$

Freund Formulation

$$Min \quad E - \phi\,\sigma_Z^2$$

or Min E - RAP * Variance

Why Use Freund – reuse of RAP and Transferability

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E-V Model Commonly Used Formulation

Min E - RAP * Variance

$$
\begin{aligned}
Max \quad & \sum_j \bar{c}_j X_j \quad - \phi \sum_j \sum_k s_{jk} X_j X_k & \\
s.t. \quad & \sum_j X_j & \leq funds \\
& X_j & \geq 0 \; for\, all\; j
\end{aligned}
$$

Where

E = expected value of risky c times choice of x

Var = sum of var time x squared minus twice covariance times x's

N = risk aversion parameter

Including Non Adaptive Risk
EV Example Data

Data for E-V Example -- Returns by Stock and Event

	----Stock Returns by Stock and Event----			
	Stock1	Stock2	Stock3	Stock4
Event1	7	6	8	5
Event2	8	4	16	6
Event3	4	8	14	6
Event4	5	9	-2	7
Event5	6	7	13	6
Event6	3	10	11	5
Event7	2	12	-2	6
Event8	5	4	18	6
Event9	4	7	12	5
Event10	3	9	-5	6
	Stock1	Stock2	Stock3	Stock4
Price	22	30	28	26

Mean Returns and Variance Parameters for Stock Example

	Stock1	Stock2	Stock3	Stock4
Mean Returns	4.70	7.60	8.30	5.80
	Variance-Covariance Matrix			
	Stock1	Stock2	Stock3	Stock4
Stock1	3.21	-3.52	6.99	0.04
Stock2	-3.52	5.84	-13.68	0.12
Stock3	6.99	-13.68	61.81	-1.64
Stock4	0.04	0.12	-1.64	0.36

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EV Model Example

Max E - N F^2 = E - RAP * Variance

$$
\begin{array}{lll}
Max & \sum_j \bar{c}_j X_j \quad - \phi \sum_j \sum_k s_{jk} X_j X_k & \\
s.t. & \sum_j X_j & \leq funds \\
 & X_j & \geq 0 \; for \; all \; j
\end{array}
$$

or for the example

$$
Max \; [4.70 \;\; 7.60 \;\; 8.30 \;\; 5.80] \begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \end{bmatrix} - \phi \; [X_1 \;\; X_2 \;\; X_3 \;\; X_4] \begin{bmatrix} +3.21 & -3.52 & +6.99 & +0.04 \\ -3.52 & +5.84 & -13.68 & +0.12 \\ +6.99 & -13.68 & +61.81 & -1.64 \\ +0.04 & +0.12 & -1.64 & +0.36 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \end{bmatrix}
$$

$$
s.t. \;\; 22X_1 + \;\; 30X_2 + \;\; 28X_3 + 26X_4 \; \leq \; 500
$$

or

$$
Max \;\; 4.70X_1 + \;\; 7.60X_2 + \;\; 8.30X_3 + \;\; 5.80X_4
$$

$$
-\varphi \begin{bmatrix} +3.21X_1^2 & -3.52X_1X_2 & +6.99X_1X_3 & +0.04X_1X_4 \\ -3.52X_1X_2 & +5.84X_2^2 & -13.68X_2X_3 & +0.12X_2X \\ +6.99X_1X_3 & -13.68X_2X_3 & +61.81X_3^2 & -1.64X_3X_4 \\ +0.04X_1X_4 & +0.12X_2X_4 & -1.64X_3X_4 & +0.36X_4^2 \end{bmatrix}
$$

$$
s.t. \quad 22X_1 + \;\; 30X_2 + \;\; 28X_3 + 26X_4 \; \leq \; 500_4
$$

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GAMS Formulation (EVPORTFO.GMS)

```
 SETS       STOCKS  POTENTIAL INVESTMENTS / BUYSTOCK1*BUYSTOCK4 /
            EVENTS  EQUALLY LIKELY RETURN STATES OF NATURE /EVENT1*EVENT10 / ;
 ALIAS (STOCKS,STOCK);
 PARAMETERS      PRICES(STOCKS) PURCHASE PRICES OF THE STOCKS
                             / BUYSTOCK1   22
                               BUYSTOCK2   30
                               BUYSTOCK3   28
                               BUYSTOCK4   26 / ;
 SCALAR      FUNDS     TOTAL INVESTABLE FUNDS / 500 / ;
 TABLE RETURNS(EVENTS,STOCKS) RETURNS BY STATE OF NATURE EVENT
           BUYSTOCK1    BUYSTOCK2    BUYSTOCK3    BUYSTOCK4
   EVENT1       7            6            8            5
   EVENT2       8            4           16            6
   EVENT3       4            8           14            6
   EVENT4       5            9           -2            7
   EVENT5       6            7           13            6
   EVENT6       3           10           11            5
   EVENT7       2           12           -2            6
   EVENT8       5            4           18            6
   EVENT9       4            7           12            5
   EVENT10      3            9           -5            6
 PARAMETERS    MEAN  (STOCKS)       MEAN RETURNS TO X(STOCKS)
               COVAR(STOCK,STOCKS) VARIANCE COVARIANCE MATRIX;
 MEAN(STOCKS)  = SUM(EVENTS , RETURNS(EVENTS,STOCKS) / CARD(EVENTS) );
 COVAR(STOCK,STOCKS)=SUM(EVENTS,(RETURNS(EVENTS,STOCKS) - MEAN(STOCKS))
                              *(RETURNS(EVENTS,STOCK)- MEAN(STOCK)))/CARD(EVENTS);
 SCALAR RAP   RISK AVERSION PARAMETER / 0.0 / ;
 POSITIVE VARIABLES    INVEST(STOCKS)  MONEY INVESTED IN EACH STOCK
 VARIABLE              OBJ             NUMBER TO BE MAXIMIZED ;
 EQUATIONS             OBJJ            OBJECTIVE FUNCTION
                       INVESTAV        INVESTMENT FUNDS AVAILABLE            ;
 OBJJ.. OBJ =E=   SUM(STOCKS, MEAN(STOCKS) * INVEST(STOCKS))
                - RAP*(SUM(STOCK, SUM(STOCKS,
                       INVEST(STOCK)* COVAR(STOCK,STOCKS) * INVEST(STOCKS))));
 INVESTAV..     SUM(STOCKS, PRICES(STOCKS) * INVEST(STOCKS)) =L= FUNDS ;
 MODEL EVPORTFOL /ALL/ ;
 SOLVE EVPORTFOL USING NLP MAXIMIZING OBJ ;
 SCALAR VAR  THE VARIANCE ;
 VAR = SUM(STOCK, SUM(STOCKS,INVEST.L(STOCK)*COVAR(STOCK,STOCKS)*INVEST.L(STOCKS)))
 SET RAPS   RISK AVERSION PARAMETERS /R0*R25/
 PARAMETER RISKAVER(RAPS) RISK AVERSION COEFICIENT BY RISK AVERSION PARAMETER/
R0  0.00000,R1  0.00025,R2  0.00050,R3  0.00075,R4  0.00100,R5  0.00150,R6  0.00200
R7  0.00300,R8  0.00500,R9  0.01000,R10 0.01100,R11 0.01250,R12 0.01500,R13 0.02500
R14 0.05000,R15 0.10000,R16 0.30000,R17 0.50000,R18 1.00000,R19 2.50000,R20 5.00000
R21 10.0000,R22  15.   ,R23 20.    ,R24  40.   ,  R25  80./
PARAMETER OUTPUT(*,RAPS) RESULTS FROM MODEL RUNS WITH VARYING RAP
LOOP (RAPS,RAP=RISKAVER(RAPS);
  SOLVE EVPORTFOL USING NLP MAXIMIZING OBJ ;
  VAR = SUM(STOCK, SUM(STOCKS,
                  INVEST.L(STOCK)* COVAR(STOCK,STOCKS) * INVEST.L(STOCKS))) ;
  OUTPUT("RAP",RAPS)=RAP;  OUTPUT(STOCKS,RAPS)=INVEST.L(STOCKS);
  OUTPUT("OBJ",RAPS)=OBJ.L;
  OUTPUT("MEAN",RAPS)=SUM(STOCKS, MEAN(STOCKS) * INVEST.L(STOCKS));
  OUTPUT("VAR",RAPS) = VAR;OUTPUT("STD",RAPS)=SQRT(VAR);
  OUTPUT("SHADPRICE",RAPS)=INVESTAV.M; OUTPUT("IDLE",RAPS)=FUNDS-INVESTAV.L  );
```

Including Non Adaptive Risk

EV Example and Solution

(EVPORTFO.GMS)

$$Max\ [4.70\ \ 7.60\ \ 8.30\ \ 5.80]\begin{bmatrix}X_1\\X_2\\X_3\\X_4\end{bmatrix} - \phi\ [X_1\ \ X_2\ \ X_3\ \ X_4]\begin{bmatrix}+3.21 & -3.52 & +6.99 & +0.04\\ -3.52 & +5.84 & -13.68 & +0.12\\ +6.99 & -13.68 & +61.81 & -1.64\\ +0.04 & +0.12 & -1.64 & +0.36\end{bmatrix}\begin{bmatrix}X_1\\X_2\\X_3\\X_4\end{bmatrix}$$

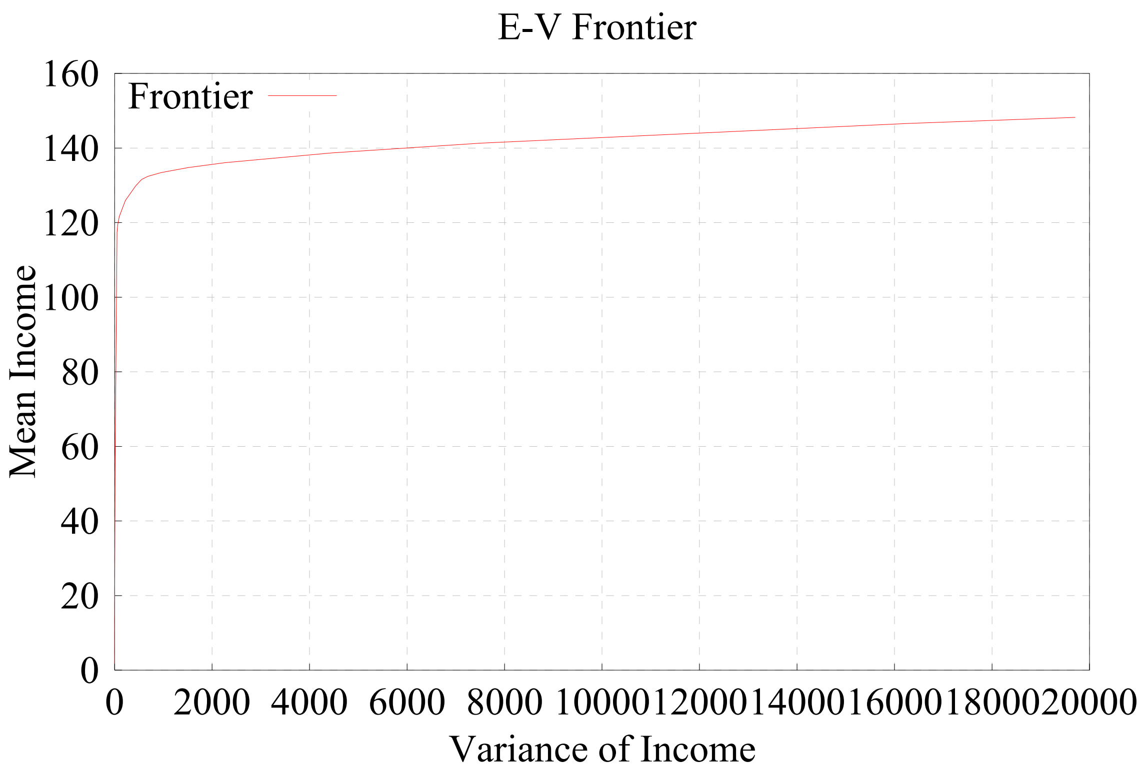
$$s.t.\ \ 22X_1 + 30X_2 + 28X_3 + 26X_4 \le 500$$

E-V Example Solutions for Alternative Risk Aversion Parameters

RAP	0	0.00025	0.0005	0.00075	0.001
BUYSTOCK2			1.263	5.324	7.355
BUYSTOCK3	17.857	17.857	16.504	12.152	9.977
OBJ	148.214	143.287	138.444	135.688	134.245
MEAN	148.214	148.214	146.581	141.331	138.705
VAR	19709.821	19709.821	16274.764	7523.441	4460.478
STD	140.392	140.392	127.573	86.738	66.787
SHADPRICE	0.296	0.277	0.261	0.260	0.260
RAP	0.011	0.012	0.015	0.025	0.050
BUYSTOCK1			1.273	4.372	4.405
BUYSTOCK2	12.893	12.960	12.420	11.070	8.188
BUYSTOCK3	4.043	3.972	3.550	2.561	1.753
BUYSTOCK4					4.168
OBJ	125.441	124.614	123.380	120.375	116.805
MEAN	131.545	131.459	129.839	125.939	121.656
VAR	554.929	547.587	430.560	222.576	97.026
STD	23.557	23.401	20.750	14.919	9.850
SHADPRICE	0.239	0.236	0.234	0.230	0.224
RAP	0.100	0.300	0.500	1.000	2.500
BUYSTOCK1	4.105	3.905	3.865	3.835	1.777
BUYSTOCK2	6.488	5.354	5.128	4.958	2.289
BUYSTOCK3	1.340	1.064	1.009	0.968	0.446
BUYSTOCK4	6.829	8.602	8.957	9.223	4.296
OBJ	113.118	102.254	92.010	66.674	27.185
MEAN	119.327	117.774	117.463	117.230	54.370
VAR	62.086	51.734	50.905	50.556	10.874
STD	7.879	7.193	7.135	7.110	3.298
SHADPRICE	0.214	0.173	0.133	0.032	0
IDLE FUNDS					268.044

Including Non Adaptive Risk

EV Example and Frontier (EVPORTFO.GMS)



RAP	0	0.00025	0.0005	0.00075	0.001
BUYSTOCK2			1.263	5.324	7.355
BUYSTOCK3	17.857	17.857	16.504	12.152	9.977
OBJ	148.214	143.287	138.444	135.688	134.245
MEAN	148.214	148.214	146.581	141.331	138.705
VAR	19709.821	19709.821	16274.764	7523.441	4460.478
STD	140.392	140.392	127.573	86.738	66.787
SHADPRICE	0.296	0.277	0.261	0.260	0.260

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MEAN	131.545	131.459	129.839	125.939	121.656
VAR	554.929	547.587	430.560	222.576	97.026
STD	23.557	23.401	20.750	14.919	9.850
SHADPRICE	0.239	0.236	0.234	0.230	0.224

Including Non Adaptive Risk

Alternative E-V Model

$$
\begin{array}{llllll}
Max & & & +E & -\phi(\sum_k p_k (S_k - E)^2)^\alpha & \\
s.t. & \sum_j r_j X_j & & & & \le funds \\
& -\sum_j c_{kj} X_j & +S_k & & & = 0 \; for \; all \; k \\
& & -\sum_k p_k S_k & +E & & = 0 \\
& X_j & & & & \ge 0 \; for \; all \; j \\
& & S_k & E & & unrestricted \; for \; all \; k
\end{array}
$$

where

j identifies the stock possibilities;
k identifies the states of nature;
X_j is amount of stock j bought;
r_j is the cost of buying X_j;
c_{kj} is the uncertain yield of stock j realized under state of nature k when the buying X_j;
S_k is the income from stocks under state of nature k;
p_k is the probability of state of nature k;
E is average income;
M is a risk aversion parameter
" is 1 for EV models or ½ for E STD models

Including Non Adaptive Risk

Alternative E-V Model

$$
\begin{array}{llllll}
Max & & & +E & -\phi(\sum_k p_k (S_k - E)^2)^{\alpha} & \\
s.t. & \sum_j r_j X_j & & & & \leq funds \\
 & -\sum_j c_{kj} X_j & +S_k & & & = 0 \ for\ all\ k \\
 & & -\sum_k p_k S_k & +E & & = 0 \\
 & X_j & & & & \geq 0 \ for\ all\ j \\
 & & S_k & E & & unrestricted\ \ for\ all\ k
\end{array}
$$

Obj Depicts maximization of expected income minus the risk aversion parameter times the variance of income

First constraint limits funds available

Second equation sums income by state of nature

Third weights income by state of nature by probability forming expected income

Last two requires investment variables to be nonnegative but allows income to be positive or negative.

Including Non Adaptive Risk

Alternative E-V Model (EVPORTF2.GMS)

$$
\begin{array}{llllllll}
Max & & & & & & E-\sum_k 1/10*(I_k-E)^2 & \\
s.t. & 22X_1 & +30X_2 & +28X_3 & +26X_4 & & & \leq 500 \\
 & 7X_1 & +6X_2 & +8X_3 & +5X_4 & -I_1 & & =0 \\
 & 8X_1 & +4X_2 & -16X_3 & +6X_4 & -I_2 & & =0 \\
 & 4X_1 & +8X_2 & +14X_3 & +6X_4 & -I_3 & & =0 \\
 & 5X_1 & +9X_2 & -2X_3 & +7X_4 & -I_4 & & =0 \\
 & 6X_1 & +7X_2 & +13X_3 & +6X_4 & -I_5 & & =0 \\
 & 3X_1 & +10X_2 & +11X_3 & +5X_4 & -I_6 & & =0 \\
 & 2X_1 & +12X_2 & -2X_3 & +6X_4 & -I_7 & & =0 \\
 & 6X_1 & +4X_2 & +18X_3 & +6X_4 & -I_8 & & =0 \\
 & 4X_1 & +7X_2 & +12X_3 & +5X_4 & -I_9 & & =0 \\
 & 3X_1 & +9X_2 & -5X_3 & +6X_4 & -I_{10} & & =0 \\
 & & & & & \sum_k I_k/10 & -E & =0
\end{array}
$$

```
           |                                     R
           |                                     H
           |                                     S     P     N
           |                                           O     E     R
           | I I I I                       a     C     S     G     O
           | N N N N                       v     O     I A   A A   W
           | V V V V   r e v - r e v - r e g     E     T I   T I   C
           | E E E E O                     r     F     I J     J   N
           | S S S S B                     e     F     V ,   V ,   T
           | T T T T J                     v     S     E S   E S   S
           |                             1
           | 1 2 3 4 1 1 2 3 4 5 6 7 8 9 0 1
            --------------------------------
     OBJJ 1|         C                     3   = 0       1     1     2
  INVESTAV 1| E E E E                           < F       4     0     4
    r     1| D D D D   3                       = 0       4     1     5
    e     2| D D E D     3                     = 0       4     1     5
    v     3| D D E D       3                   = 0       4     1     5
    e     4| D D 4 D         3                 = 0       3     2     5
    n     5| D D E D           3               = 0       4     1     5
    u     6| D E E D             3             = 0       4     1     5
    e     7| D E 4 D               3           = 0       3     2     5
    -     8| D D E D                 3         = 0       4     1     5
    r     9| D D E D                   3       = 0       4     1     5
    e    10| D D 4 D                     3     = 0       3     2     5
avrevenue 1|           A A A A A A A A A A 3   = 0     1 0     1   1 1
```

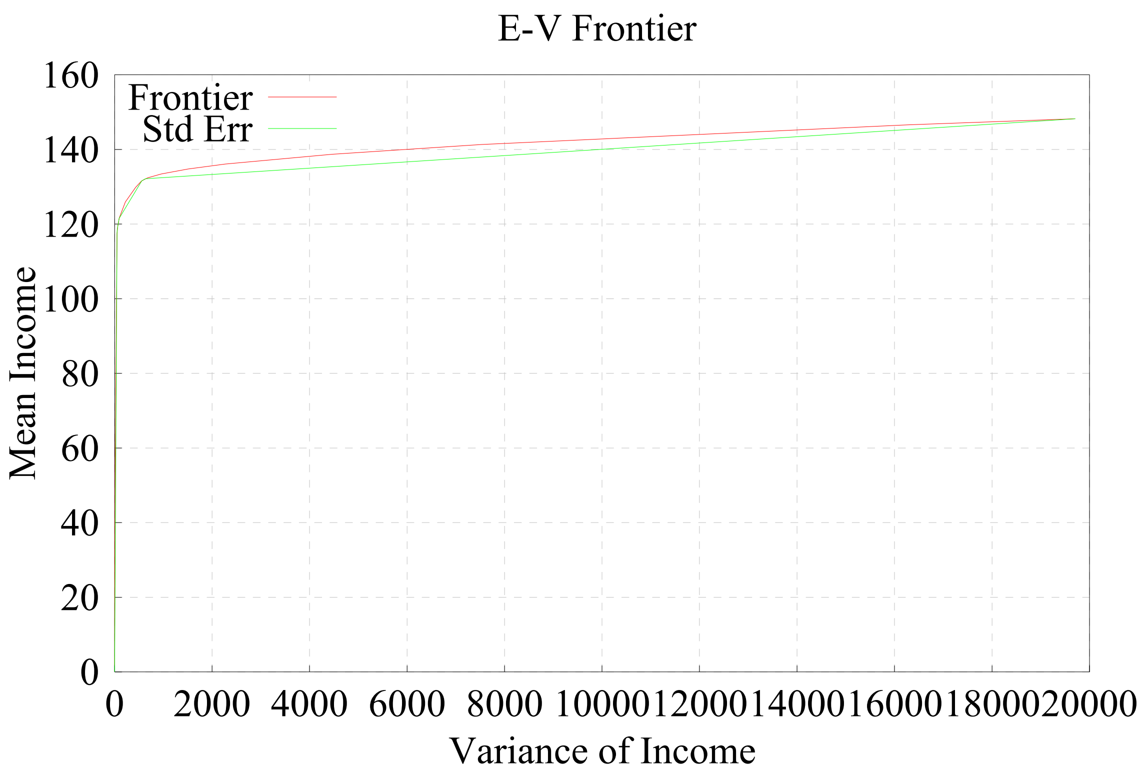
Including Non Adaptive Risk

Alternative E-V Model (EVPORTF2.GMS)

```
SETS STOCKS  POTENTIAL INVESTMENTS / BUYSTOCK1*BUYSTOCK4 /
     EVENTS  EQUALLY LIKELY STATES OF NATURE /EVENT1*EVENT10 / ;
...
 SCALAR RAP   RISK AVERSION PARAMETER / 0.0 /
        alpha term if 1 is var if 0.5 is std err /1/
        n     sample size;
        n=card(events);
 POSITIVE VARIABLES    INVEST(STOCKS)  MONEY INVESTED IN EACH STOCK
 VARIABLE              OBJ             NUMBER TO BE MAXIMIZED
                       rev(events)     revenue by event
                       avgrev          average revenue;
 EQUATIONS             OBJJ            OBJECTIVE FUNCTION
                       INVESTAV        INVESTMENT FUNDS AVAILABLE
                       revenue(events) accounts revenue by event
                       avrevenue       accounts average revenue  ;
 OBJJ..
 OBJ =E= avgrev  - RAP*
(SUM(events,1/n*(rev(events)-avgrev)*(rev(events)-avgrev)))**alpha;
INVESTAV.. SUM(STOCKS, PRICES(STOCKS) * INVEST(STOCKS)) =L= FUNDS ;
 revenue(events).. sum(stocks,RETURNS(EVENTS,STOCKS)*invest(STOCKS))=e=rev(events);
 avrevenue.. sum(events,1/n*rev(events))=e=avgrev;
 MODEL EVPORTFOL /ALL/ ;
 SOLVE EVPORTFOL USING NLP MAXIMIZING OBJ ;
 SCALAR VAR  THE VARIANCE ;
        VAR = (SUM(events,
                          1/n*(rev.l(events)-avgrev.l)*(rev.l(events)-avgrev.l))) ;
 SET RAPS   RISK AVERSION PARAMETERS /R0*R25/
 PARAMETER RISKAVER(RAPS) RISK AVERSION COEFICIENT BY RISK AVERSION /
R0  0.00000,R1  0.00025,R2  0.00050,R3  0.00075,R4  0.00100,R5  0.00150,R6  0.00200
R7  0.00300,R8  0.00500,R9  0.01000,R10 0.01100,R11 0.01250,R12 0.01500,R13 0.02500
R14 0.05000,R15 0.10000,R16 0.30000,R17 0.50000,R18 1.00000,R19 2.50000,R20 5.00000
R21 10.0000,R22  15.   ,R23 20.    ,R24  40.   ,  R25  80./
 PARAMETER OUTPUT(*,RAPS) RESULTS FROM MODEL RUNS WITH VARYING RAP
 LOOP (RAPS,RAP=RISKAVER(RAPS);
           SOLVE EVPORTFOL USING NLP MAXIMIZING OBJ ;
var=(SUM(events,1/n*(rev.l(events)-avgrev.l)*(rev.l(events)-avgrev.l))) ;
 OUTPUT("RAP",RAPS)=RAP;   OUTPUT(STOCKS,RAPS)=INVEST.L(STOCKS);
 OUTPUT("OBJ",RAPS)=OBJ.L; OUTPUT("MEAN",RAPS)=avgrev.l;
 OUTPUT("VAR",RAPS) = VAR;
           OUTPUT("STD",RAPS)=SQRT(VAR);
           OUTPUT("SHADPRICE",RAPS)=INVESTAV.M;
           OUTPUT("IDLE",RAPS)=FUNDS-INVESTAV.L
             );
parameter graphit (*,raps,*);
graphit("Frontier",raps,"Mean")=OUTPUT("MEAN",RAPS);
graphit("frontier",raps,"Var")=OUTPUT("std",RAPS)**2;
 alpha=0.5;
 LOOP (RAPS,RAP=RISKAVER(RAPS);
           SOLVE EVPORTFOL USING NLP MAXIMIZING OBJ ;
           var=(SUM(events,1/n*(rev.l(events)-avgrev.l)*(rev.l(events)-avgrev.l))) ;
  OUTPUT("RAP",RAPS)=RAP; OUTPUT(STOCKS,RAPS)=INVEST.L(STOCKS);
  OUTPUT("OBJ",RAPS)=OBJ.L;  OUTPUT("MEAN",RAPS)=avgrev.l;
  OUTPUT("VAR",RAPS) = VAR;  OUTPUT("STD",RAPS)=SQRT(VAR);
  OUTPUT("SHADPRICE",RAPS)=INVESTAV.M; OUTPUT("IDLE",RAPS)=FUNDS-INVESTAV.L    );
 DISPLAY OUTPUT;
```

Including Non Adaptive Risk

Alternative E-V Model (EVPORTF2.GMS)



EV solutions are identical with earlier, E Standard Error is close

	R0	R1	R2	R3	R4	R5
BUYSTOCK2			1.263	5.324	7.355	9.386
BUYSTOCK3	17.857	17.857	16.504	12.152	9.977	7.801
RAP		2.500000E-4	5.000000E-4	7.500000E-4	0.001	0.001
OBJ	148.214	143.287	138.444	135.688	134.245	132.671
MEAN	148.214	148.214	146.581	141.331	138.705	136.080
VAR	19709.821	19709.821	16274.764	7523.441	4460.478	2272.647
STD	140.392	140.392	127.573	86.738	66.787	47.672
SHADPRICE	0.296	0.277	0.261	0.260	0.260	0.259

+	R6	R7	R8	R9	R10	R11
BUYSTOCK2	10.401	11.416	12.229	12.838	12.893	12.960
BUYSTOCK3	6.713	5.625	4.755	4.102	4.043	3.972
RAP	0.002	0.003	0.005	0.010	0.011	0.012
OBJ	131.753	130.575	129.005	125.999	125.441	124.614
MEAN	134.767	133.454	132.404	131.617	131.545	131.459
VAR	1506.907	959.949	679.907	561.764	554.929	547.587
STD	38.819	30.983	26.075	23.702	23.557	23.401
SHADPRICE	0.257	0.255	0.251	0.241	0.239	0.236

Including Non Adaptive Risk
Dissecting the GAMS formulation
Graphing

```
parameter graphit (*,raps,*);
graphit("Frontier",raps,"Mean")=OUTPUT("MEAN",RAPS);
graphit("frontier",raps,"Var")=OUTPUT("std",RAPS)**2;
*$include gnu_opt.gms
* titles
...
more solves
...
graphit("Std Err",raps,"Mean")=OUTPUT("MEAN",RAPS);
graphit("Std Err",raps,"Var")=OUTPUT("std",RAPS)**2;
$setglobal gp_title  "E-V Frontier "
$setglobal gp_xlabel "Variance of Income"
$setglobal gp_ylabel "Mean Income"
$batinclude gnupltxy graphit mean var
```

This is done using a GNUPLOT interface originally developed by Rutherford but modified to gnupltxy as documented on the Web page agecon.tamu.edu/faculty/mccarl

Including Non Adaptive Risk
Modeling Support from GAMSCHK
Nonlinear Models

```
##  rev('EVENT1')
  SOLUTION VALUE                                      114.697
  EQN                                    Aij          Ui         Aij*Ui
 OBJJ                           ***-0.70261E-01  1.0000     -0.70261E-01
 revenue('EVENT1')                   -1.0000     -0.17026     0.17026
 avrevenue                            0.10000     -1.0000     -0.10000
 TRUE REDUCED COST                                            0.00000E+00

----## EQU OBJJ

 ##  OBJJ
  VAR                                    Aij          Xj         Aij*Xj
 OBJ                                   1.0000      111.81      111.81
 rev('EVENT1')                   ***-0.70261E-01 114.70      -8.0587
 rev('EVENT2')                   ***-0.47985E-02 121.06     -0.58089
 rev('EVENT3')                   *** 0.11205     132.41      14.836
 rev('EVENT4')                   *** 0.10966E-03 121.53      0.13327E-01
 rev('EVENT5')                   *** 0.10129     131.37      13.306
 rev('EVENT6')                   *** 0.13558     134.70      18.263
 rev('EVENT7')                   *** 0.70002E-01 128.33      8.9831
 rev('EVENT8')                   ***-0.10467     111.35     -11.655
 rev('EVENT9')                   ***-0.51262E-01 116.54     -5.9742
 rev('EVENT10')                  ***-0.18804     103.25     -19.416
 avgrev                          *** -1.0000     121.52     -121.52
    =E=                                                       =E=
 RHS COEFF                                                  0.00000E+00
```

*** marks nonl;inear terms

Starting point and accuracy is an issue

Including Firm Level Risk
Finding a Risk Aversion Parameter

E-MF

is one tailed confidence interval

when M=1 under normality confidence interval is 84%
M=1.96 interval is 97.5%

Also E-V relation

$$\begin{array}{llll} Max & cX - \psi\sigma^2(X) & & \\ s.t. & AX & \leq b & \\ & X & \geq 0 & \end{array} \quad versus \quad \begin{array}{llll} Max & cX - \theta\sigma(X) & & \\ s.t. & AX & \leq b \\ & X & \geq 0 \end{array}$$

$$c - 2\psi\sigma(X)\frac{\partial\sigma(X)}{\partial X} - \lambda A = 0 \qquad c - \theta\frac{\partial\sigma(X)}{\partial X} - \lambda A = 0$$

which equates when

$$\psi = \frac{\theta}{2\sigma(X)}$$

Given we generally find θ in a range between 0 and 5 this implies

$$0 \leq \psi \leq \frac{5}{2\sigma(X)}$$

Forming Probability Distributions

Probability distributions state the relative frequency of occurrence of a set of mutually exclusive events.

Finding Probability Distributions Based on Objective Data

Desirable Characteristics

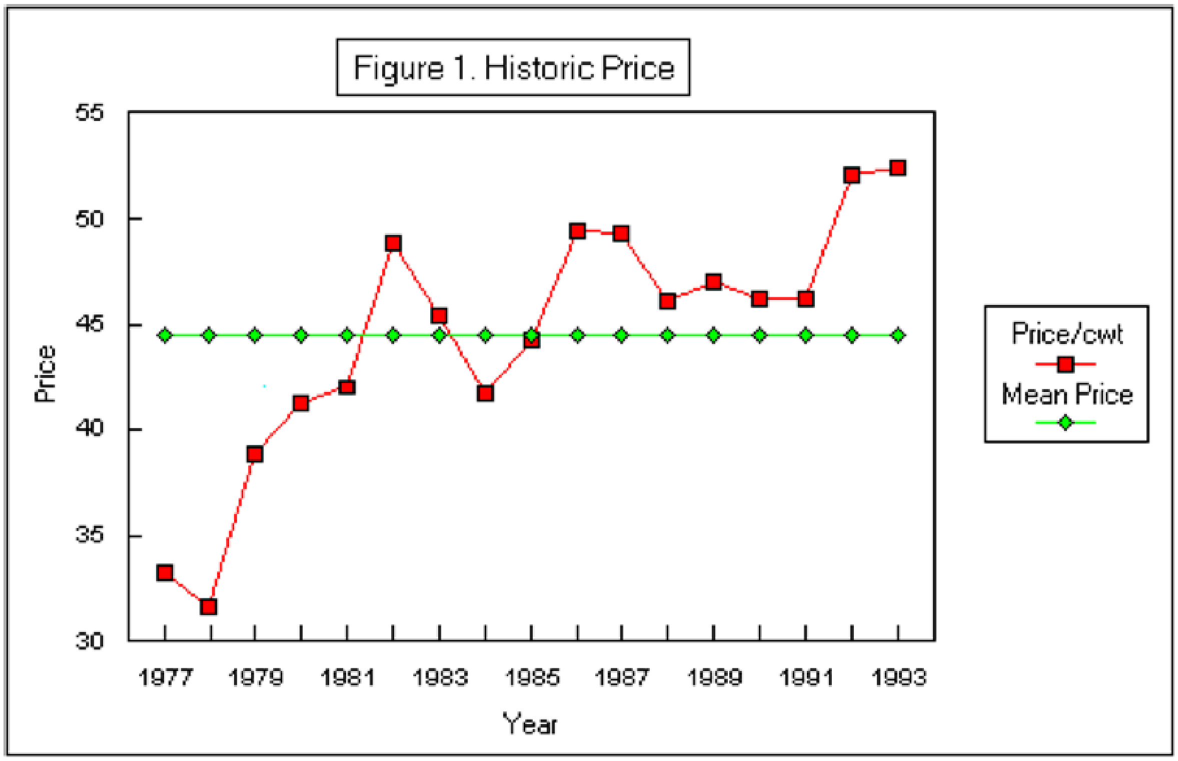
1) each of the states of nature must be mutually exclusive;

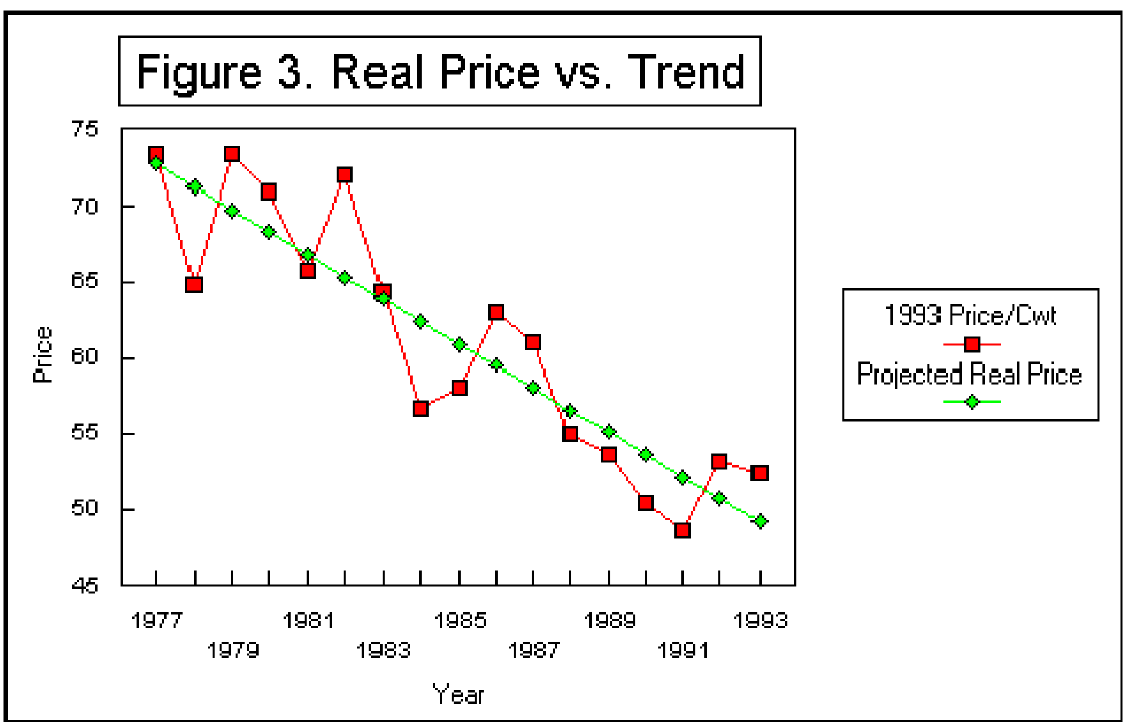
2) probability of occurrence of each of the states of nature must be an unbiased measure of the current probability of that state of nature occurring;

3) the sum of the probabilities across the states of nature must equal one

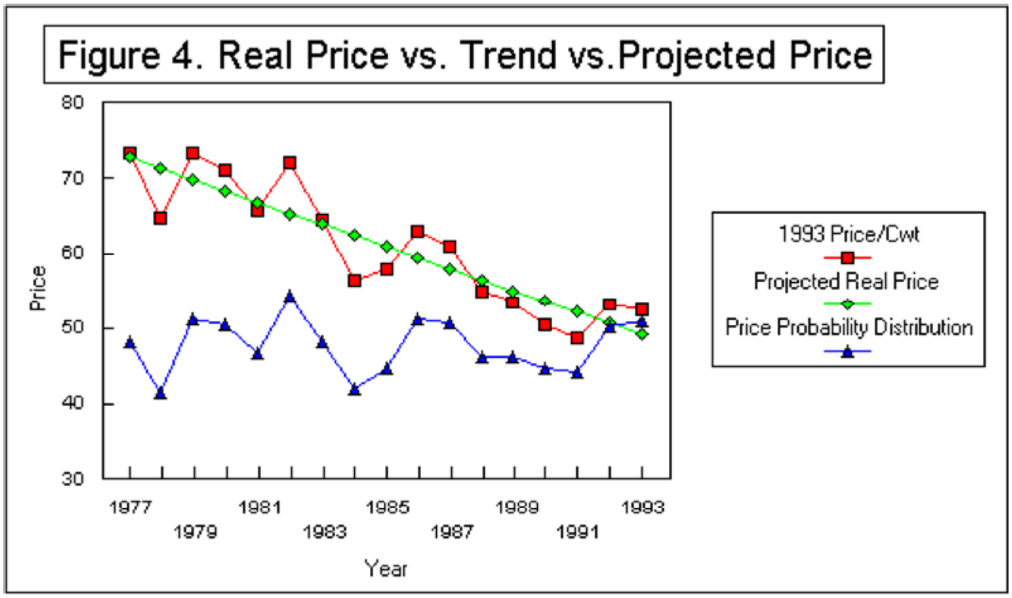
Second property is the most troubling in when using objective, historical data. trends, events

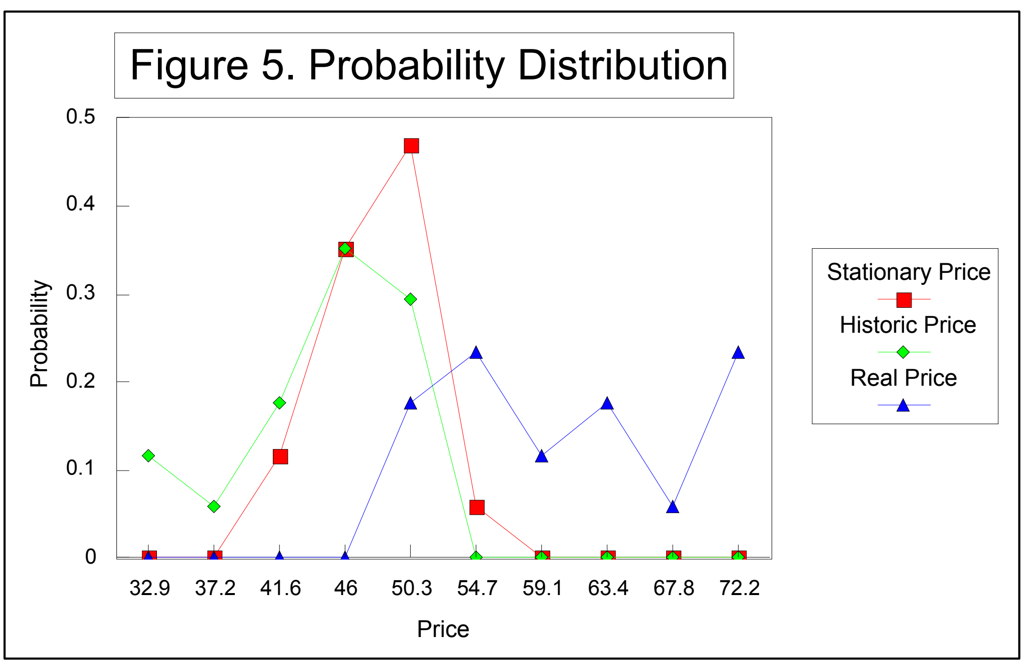
Including Firm Level Risk





Including Firm Level Risk





General Lessons Learned on Objective Probabilities

Use objective data – trends and other systematic effects can bias

Use a procedure like regression to develop values expected

One may find residual terms are hetereoskedastic