Stochastic Optimization: Recent Enhancements in Algebraic Modeling Systems

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GAMS at a Glance

**General Algebraic Modeling System:**

Design Principles:
- Balanced mix of declarative and procedural elements
- Open architecture and interfaces to other systems
- Different layers with separation of:
  - model and data
  - model and solution methods
  - model and operating system
  - model and interface
An introducing Example

• The Newsboy (Newsvendor) Problem:
  – A newsboy purchases newspapers for a price $C$
  – He is faced with uncertain demand $D$
  – He has to satisfy his customers demand or has to pay a penalty $Q>C$ per newspaper

• Decisions to make:
  – How much newspaper should he buy “here and now” (without knowing the outcome of the uncertain demand)?
    $\Rightarrow$ First-stage decision
  – How much customers have to be dropped after the outcome becomes known?
    $\Rightarrow$ Second-stage or recourse decision
  – Recourse decisions can be seen as
    • penalties for bad first-stage decisions
    • variables to keep the problem feasible
Some Stochastic Programming Classes

Source: G. Mitra
Stochastic Programming Claims and ‘Facts’

- Lots of application areas (Finance, Energy, Telecommunication)
- Mature field (Dantzig ’55)
- Variety of SP problem classes with specialized solution algorithms (e.g. Bender’s Decomposition)

- Compared to deterministic mathematical programming (MP) small fraction
  - Only ~0.1% of NEOS submission to SP solvers
- Few commercially supported solvers for SP
- Various frustrations with industrial SP projects
Stochastic Programming Solvers

- LP solver
  - Interior point methods seem to be better than simplex
- Other ready to use solver (e.g. NEOS):
  - DECIS, Infanger (2-stage)
  - FortSP, OptiRisk
  - OSL/SE, IBM (discontinued)
  - Academic codes:
    - MSLiP (Gassmann)
    - BNBS (Altenstedt)
    - DDSIP (Schultz et. al) (2-stage MIP)
    - ...
Stochastic MPS

- Make it easy to convert existing deterministic LP into SLP
- Add information about dynamic and stochastic structure.
  - Core file (deterministic problem)
  - Time file (map core file dynamic structure into stages)
  - Stoch file (information about the random variables)
- SMPS format is extremely flexible
- Difficult for a human to manage
Other Tools and Frameworks

- Scenario (tree) management
  - Reduction
    - ScenRed, (Römisch et. al.)
  - Generation
    - From random variables to scenarios
    - SAMPL, SMPL, AIMMS, Mosel tools for tree generation
    - ScenRed2

- Framework
  - SLP-IOR (Mayer/Kall)
  - SPInE (Stochastic Programming Integrated Environment), OptiRisk
    - Special purpose scenario generators
    - Connection to SAMPL and SMPL
    - FortSP SP solver
AML and Stochastic Programming (SP)

• Algebraic Modeling Languages/Systems good way to represent optimization problems
  – Algebra is a universal language
  – Hassle free use of optimization solvers
  – Simple connection to data sources (DB, Spreadsheets, …) and analytic engines (GIS, Charting, …)
• Large number of (deterministic) models in production
  – Opportunity for seamless introduction of new technology like Global Optimization, Stochastic Programming, …
  – AML potential framework for SP
Example Model: Gas Price Model

gas1.gms (deterministic model)
n-Stage Stochastic Programs

• Construct Scenario Tree:
  – Start with today’s price and use a (discrete) distribution
  – Realizations: up, down

• Stochastic Linear Program (block structure)
  – Nested Bender’s Decomposition (OSLSE, FortSP, AIMMS)
  – Here and Now (HN)
  – In practice Deterministic Equivalent with Barrier method

\[
Z_{HN} = \min_{x_1} \left\{ c_1 x_1 + E_{S_2} \left[ \min_{x_2} c_2 x_2 + E_{S_3} \left[ \min_{x_3} c_3 x_3 + \ldots + E_{S_7} \left[ \min_{x_7} c_7 x_7 \right] \right] \right] \right\}
\]

subject to:

\[
\begin{align*}
A_{11} x_1 & = b_1 \\
A_{21} x_1 + A_{22} x_2 & = b_2 \\
A_{31} x_1 + A_{32} x_2 + A_{33} x_3 & = b_3 \\
& \vdots \\
A_{71} x_1 + A_{72} x_2 + A_{73} x_3 + \ldots + A_{77} x_7 & = b_7 \\
\ell & \leq x_i \leq u_i
\end{align*}
\]

\[
\implies \text{gas2.gms} \\
(n\text{-stage SP, distribution})
\]
ScenRed (Römisch et. al., HU Berlin)

- Find good approximation of original scenario tree of significant smaller size.
- Available since 2002
- Integrated in GAMS system
- No extra cost

`gas2.gms` (n-stage SP, distribution) plus ScenRed
Scenario based Stochastic Programs

- Random variables with distributions versus independent scenarios:
  - Wait-and-see (WS)
    - solve scenarios independently (grid computing)
  - Expected value problem (EV)
    - Calculate EV of random variables and solve
  - Expectation of EV problem (EEV)
    - Implement policy of EV problem and evaluate all scenarios

\[ \text{gas3.gms (Scenario based: WS, EEV)} \]
Value of Stochastic Solution/Visualization

- WS ≥ EEV (maximization!)
- Visualize results!
  - e.g. fan plot (Tom Rutherford, ETH Zürich)

\[ \text{gas3.gms (Scenario based: fan plot)} \]
2-Stage Stochastic Programs

- SP Solver DECIS (Gerd Infanger, Stanford, USA)
  - Stores only one instance of the problem and generates scenario sub-problems as needed
  - Solution Strategies
    - Deterministic Equivalent (all scenarios)
    - Sampling: Crude Monte Carlo/Importance sampling

→ gas4.gms (2-stage, DECIS)
Tree Generation: ScenRed2

- Construct a true scenario tree from independent scenarios:

- Reconstruct underlying distribution from a set of scenarios
- Here-and-now (HN): $WS \geq HN \geq EEV$
- Value of stochastic solution $VSS = HN - EEV$
- Expected value of perfect information $EVPI = WS - HN$

$\Rightarrow$ gas5.gms (n-stage, ScenRed2)
The Rich World of Stochastic Programming

- n-stage stochastic linear programming (SLP) just one option
- SP models from application areas exist (finance)

- Economic modeling
  - mixed complementarity problems
  - scenario trees with few branches
Conclusion

• Stochastic Programming still challenging and developing field
  – GUPOR: *Uncertainty: An OR Frontier* (Greenberg, 2006)
• Lack of solution technology limits the dissemination of SP
• There is more to SP than n-stage SLP
• Representation of results
• Collection of comprehensive & reproducible examples could help to *spread the word*
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