Grid Computing in Finance using an Algebraic Modeling System

Franz Nelißen
FNelissen@gams.com
GAMS Software GmbH
www.gams.de

OR 2007
Saarbrücken, Germany, September 5-7, 2007
Agenda

- Mathematical Optimization in Finance
- Grid Computing
- Sun’s Network.com
GAMS Development / GAMS Software

- **Roots:** Research project World Bank 1976
- Pioneer in **Algebraic Modeling Systems** used for economic modeling
- Went **commercial** in 1987
- **Offices** in Washington, D.C and Cologne

- Professional **software tool provider**, not a consulting company
- Operating in a **segmented niche market**
- Broad **academic & commercial** user base and network

**General Algebraic Modeling System**
Agenda

- Mathematical Optimization in Finance
- Grid Computing
- Sun’s Network.com
Mathematical Optimization in Finance

Very active research field with significant contributions and important practical applications

Some of the reasons:
- Continual stream of challenging problems with obvious impact of uncertainty
- High availability of data
- Validation potential – benchmarking
- Very competitive and liquid markets

Many instruments, tools and strategies
Portfolio Optimization Models

- Seminal Developments: Mean-Variance Portfolio Optimization
- Scenario Optimization
- Stochastic Programming
The Mean-Variance Model

**Markowitz (1952), Nobel prize 1990**

**Given**

- Some investments $x_i$ with historical data
  - Rewards = Expected returns of investments: $\mu_i$ (Mean of historical returns)
  - Risk: Variance of investments $Q_{i,j}$

**Goal**

Balance risk $r$ of portfolio against expected returns of portfolio

Minimize variance $\nu$ of portfolio for a given target return $r$

**Algebra**

- Variance of Portfolio: $\text{Min} \sum_{i=1}^{I} \sum_{j=1}^{J} x_i Q_{i,j} x_j$
- Target return: $\sum_{i=1}^{I} \mu_i x_i \geq r$
- Budget constraint: $\sum_{i=1}^{I} x_i = 1$
- No short sales: $x_i \geq 0$
Efficient Frontier and Portfolios

Return of portfolio (%)

Variance of portfolio

Share of portfolio (%)

Solution points
Business Rules

- Institutional or legal requirements: Describe the way the institution is operating
- Additional constraints, which have to be satisfied
  - Not defined by modeling experts
  - Independent of risk model
  - Basel II
Simple Business Rules

Do not change the model type:

- Short selling
- Risk free borrowing
- Upper or lower bounds on certain instruments
More Complex Business Rules

Require introduction of integer (binary) variables:

- **Cardinality Constraint**: Restrict number of investments $y_i$ in portfolio

- **Threshold Constraint**: Investments $x_i$ can only be purchased at certain minimum $l_{i,i}$ or maximum $u_{u,i}$

- more trading restrictions …
Scenario Optimization Models

Scenarios capture complex interactions between multiple risk factors

- Different methods for risk measurement:
  - Mean Absolute Deviation Models
  - Index Tracking Models
  - Expected Utility Models
  - VAR Models (linear Version: CVAR)

- Models are solved over all scenarios

Modeling Issues:
- Linear Models, but business rules may introduce binary variables
- Lots of independent model runs, which can be handled in parallel
Stochastic Programming (SP)

Stochastic Programming models allow **Sequence of Decisions:**
- **Scenarios:** Complete set of possible discrete realizations of the uncertain parameters with probabilities
- **Stages:** Decisions points. First stage decisions now, second stage decision (depending of the outcome of the first stage decision) after a certain period and so on
- **Recourse:** Decision variables can adapt to the different outcomes of the random parameters at each stage
More Complex Scenario Trees

**Figure 1: US dollar short rate scenarios**

Original load scenario tree
Challenges

**Deterministic equivalent:** Includes all scenarios and stages
- Size of model explodes
  - Generation difficult
  - Solution may not be possible
  - Interpretation and validation of results
- Less applications than one may expect

**But:** Number of uncertain parameters is small:
- Efficient representation of the uncertain data within the Algebraic Modeling System?
- Scenarios may only differ slightly
- Problems are structured → Specialized Algorithmen available (Decomposition)?
Agenda

- Mathematical Optimization in Finance
- Grid Computing
- Sun’s Network.com
What is Grid Computing?

A pool of connected computers managed and available as a common computing resource

- Effective sharing of CPU power
- Massive parallel task execution
- Scheduler handles management tasks
- E.g. Condor, Sun Grid Engine, Globus
- Can be rented or owned in common
- Licensing & security issues
Advantages of Grid Computing

• Solve a certain number of scenarios faster, e.g:
  – sequential: 50 hours
  – parallel (200 CPUs): ~15 minutes

• Get better results by running more scenarios*:

<table>
<thead>
<tr>
<th>#SIM</th>
<th>VaR error</th>
<th>CVaR error</th>
</tr>
</thead>
<tbody>
<tr>
<td>1000</td>
<td>5.42%</td>
<td>6.74%</td>
</tr>
<tr>
<td>20,000</td>
<td>1.21%</td>
<td>1.49%</td>
</tr>
</tbody>
</table>

**GAMS & Grid Computing**

- **Scalable:**
  - support of massive grids, **but also**
  - multi-cpu / multiple cores desktop machines
  - “1 CPU - Grid”

- Platform **independent**

- Only **minor changes** to model required

- **Separation** of model and solution method
  → Model stays **maintainable**
Simple Serial Solve Loop

Loop (p(pp),

ret.fx = rmin +(rmax-rmin) /
(card(pp)+1)*ord(pp) ;

Solve minvar min var using miqcp;

xres(i,p) = x.l(i);
report(p,i,'inc') = xi.l(i);
report(p,i,'dec') = xd.l(i)
);

How do we get to parallel and distributed computing?
GRID Specific Enhancements

1. Submission of jobs

2. “Grid Middleware”
   - Distribution of jobs
   - Job execution

3. Collection of solutions

4. Processing of results
Results for 4096 MIPS on Condor Grid

- Submission started Jan 11, 16:00
- All jobs submitted by Jan 11, 23:00
- All jobs returned by Jan 12, 12:40
  - 20 hours wall time, 5000 CPU hours
  - Peak number of CPU’s: 500
Agenda

- Mathematical Optimization in Finance
- Grid Computing
- Sun’s Network.com
Network.com operated by Sun:
• On-demand grid computing service
• Pay as you go utility: All-inclusive price of 1 USD per CPU-hour (Hardware)
• A few hundred CPU’s (AMD Opteron, 2 CPU SMP, 2 *4 GB RAM) running Solaris 10
• Similar Services:
  – Amazon: “Elastic Compute Cloud (EC2)”
  – ZeroC: IceGrid
Using Network.com

On-demand Applications – Five Easy Steps

1. Select an Application
2. Upload
3. Create Job
4. Run Job
5. Download Results

» Return to overview   » View all on-demand applications   » Request new application
Using Network.com

<table>
<thead>
<tr>
<th>Name</th>
<th>Size (MB)</th>
<th>Type - Status</th>
<th>Source File</th>
<th>Description</th>
<th>Owner</th>
</tr>
</thead>
<tbody>
<tr>
<td>sphot</td>
<td>0</td>
<td>Application</td>
<td>sphot.zip</td>
<td>2D Monte Carlo Simulation</td>
<td>Sun Grid</td>
</tr>
<tr>
<td>pov-ray 3.6</td>
<td>2</td>
<td>Application</td>
<td>povray.zip</td>
<td>Open source ray tracer</td>
<td>Sun Grid</td>
</tr>
<tr>
<td>Hello World</td>
<td>3</td>
<td>Data</td>
<td>helloworld_complete.zip</td>
<td>Make a movie with POY-ray</td>
<td>Sun Grid</td>
</tr>
<tr>
<td>GAMS</td>
<td>45</td>
<td>Application - Approved</td>
<td>gams.zip</td>
<td>GAMS Distribution 22.5</td>
<td>Franz Nelissen</td>
</tr>
<tr>
<td>GAMS-PVM</td>
<td>45</td>
<td>Application</td>
<td>gams.zip</td>
<td>GAMS 22.5 -PVM enabled</td>
<td>Franz Nelissen</td>
</tr>
<tr>
<td>coin</td>
<td>0</td>
<td>Data</td>
<td>coin.zip</td>
<td>coin tests with coin solvers</td>
<td>Franz Nelissen</td>
</tr>
<tr>
<td>final</td>
<td>0</td>
<td>Data</td>
<td>final.zip</td>
<td></td>
<td>Franz Nelissen</td>
</tr>
<tr>
<td>batch</td>
<td>0</td>
<td>Data</td>
<td>batch.zip</td>
<td></td>
<td>Franz Nelissen</td>
</tr>
<tr>
<td>GAMS 22.5b</td>
<td>45</td>
<td>Application - Locked</td>
<td>gams225.zip</td>
<td>GAMS Distribution 22.5b</td>
<td>Franz Nelissen</td>
</tr>
</tbody>
</table>

Parallel Execution of Job Arrays

Job Scheduling Host

Submission Loop

Job 1
- Model 001
- Model ...
- Model nnn

Output Job 1
- Model 001
- ...
- Model nnn

Node 001

Model 001

Node ...

Model ...

Model nnn
Using the GAMS GRID Facilities

Job Scheduling Host

Node 001

GAMS

Solve Loop

Model 1, Instance 001

Model 1, Instance …

Model 1, Instance nnn

Node 002

Node …

Node nnn

Output Job 1

Model 001

Job 1

Model 001
Parallel Solver Threads (SMP)

- Restricted to two parallel Threads
- Not available with all Solvers
Further Developments

- Better Interfaces to different Grid Architectures
- Parallel Submission of Jobs from within GAMS
- Better Job Control
- ...
Conclusions and Summary

- Finance is a success story for OR applications
- Rich set of different risk models available
- Incorporating business rules essential
- Stochastic programming still challenging

- Grid Computing offers lots of promising developments
- Algebraic Modeling Languages fully support parallel environments

- Sun’s Network.com interesting commercial approach
- Currently 250 free CPU hours and free access to GAMS (COIN) at network.com: http://www.gams.com/sungrid
Thank you!
... Questions?
# More Theory and Templates

## Theory
- **Practical Financial Optimization** (forthcoming) by S. Zenios
- **A Library of Financial Optimization Models** (forthcoming) by A. Consiglio, S. Nielsen, H. Vladimirou and S. Zenios
- **Financial Optimization** by S. Zenios (ed.)

## Templates available online
- **GAMS Model Library:**
  http://www.gams.com/modlib/libhtml/subindx.htm
- **Course Notes „Financial Optimization“:**
  http://www.gams.com/docs/contributed/financial/
Contacting GAMS

Europe:
GAMS Software GmbH
Eupener Str. 135-137
50933 Cologne
Germany
Phone: +49 221 949 9170
Fax: +49 221 949 9171
http://www.gams.de

USA:
GAMS Development Corp.
1217 Potomac Street, NW
Washington, DC 20007
USA
Phone: +1 202 342 0180
Fax: +1 202 342 0181
http://www.gams.com