Complementarity at GAMS Development

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How Important is CP?

- In 1989, no CP possible in GAMS
- In 1999, CP comprises over 10% of solver sales
- Active modeling & application area
- Many different parties involved
Talk Outline

• The MCP model type
  – Modeling syntax
  – Solution algorithms

• MPSGE

• New Directions
  – reformulation tools
  – MPEC models
Why Use GAMS for CP?

• Benefits for the modeler
  – Powerful set-based indexing, large scale
  – Symbolic differentiation
  – Self-documenting model: portable, flexible, extendable, readable
  – Solver independence
  – Access to model library
Why Use GAMS for CP?

• Benefits for the algorithm developer
  – Readily available test problems: GAMSLIB, MCPLIB
  – Immediate access to market, user community
  – Environment allows algorithm comparison, enforces rigor
MCP: Definition

Given $F : \mathbb{R}^n \rightarrow \mathbb{R}^n$, $-\infty \leq \ell \leq u \leq +\infty$,

find $z \in \mathbb{R}^n$ s.t.

either $z_i = \ell_i$ and $F_i(z) \geq 0$

or $z_i = u_i$ and $F_i(z) \leq 0$

or $\ell_i < z_i < u_i$ and $F_i(z) = 0$. 
Extending GAMS for CP

• CP requires:
  – (nonlinear) functions - ✓
  – (bounded) variables - ✓
  – complementary pairing - ?

• MCP model type introduces pairing F.z
• Allows reuse of functions
• Builds on user expertise in NLP
A Walrasian Equilibrium Example

Find a price $p \in \mathbb{R}^m$ and an activity level $y \in \mathbb{R}^n$ such that

$$S(p, y) := b - d(p) + Ay \geq 0, \quad p \geq 0, \quad \perp$$

$$L(p) := -A^T p \geq 0, \quad y \geq 0, \quad \perp$$

An equivalent MCP:

$$F(p, y) := \begin{bmatrix} S(p, y) \\ L(p) \end{bmatrix}, \quad B := \mathbb{R}_+^m \times \mathbb{R}_+^n,$$
The GAMS Model

parameters b(I), s(I), A(I,J);

positive variables p(I), y(J);
equations S(I), L(J);

S(I) .. b(I) + sum{J, A(I,J)*y(J)}
    - s(I)*sum{K, b(K)*p(K)} / p(I)
    =g= 0;
L(J) .. - sum{I, p(I)*A(I,J)}
    =g= 0;

model walras / S.p, L.y /;

solve walras using mcp;
CPLIB  (Rutherford)

• An interface layer between GAMS & solver
  – Provides function F and its Jacobian, box B, initial iterate, solution reporting
  – Allows integration of MPSGE
• Fortran code, supports C solvers as well
• Cornerstone of CP solvers at GAMS
MPSGE: Intro

• A language for GE models
  – Economists say a lot with a few words
  – MPSGE “speaks economics”

• This language foreign to GAMS
  – Preprocessor reads MPSGE code
  – GAMS code and MPSGE code integrated
MPSGE: Advantages

- Shorthand that reduces errors, tedium
- Efficient constraint representation
- Structures and manages complexity
- Consequence: increased solver demand
  - Forces robustness, speed
  - Drives sales of MCP solvers
Intl. Impact Assessment Model

- 80-country GE model
- Evaluate effects of environmental policy
- User input of key parameters
- Displayed data and graphics allows convenient, rapid comparison b/t policies
MCP Solvers: MILES, PATH

• (Josephy) Newton-based algorithms
  – Local quadratic convergence
  – Require exact Jacobians
  – Line/pathsearch techiques increase robustness

• Use sparse linear algebra (LUSOL)
  – Scale to large problems, efficient pivoting
  – Employ dynamic memory allocation
Why use PATH?

- Crashing
  - Quickly finds near-optimal basis
  - Major speed boost on large models
- Proximal point term to handle rank-deficiency
- Robustness: restarts, pathsearch, merit functions
- Diagnostics
A Simple NLP

Let $c_i, i = 1..1000$ be in $[0, 2]$, $C = \sum_i c_i$.

minimize \[ \sum_i (c_i - x_i)^2 \]
subject to \[ .4C = \sum_i x_i \]
\[ x \leq .85 \]

NLP solution: 673 superbasic vars, 1 basic, 326 nonbasic.

First order (KKT) conditions for the above: a simple complementarity problem.

\[ 2(x - c) + e\lambda \leq 0 \quad \perp \quad x \leq .85 \]
\[ .4C - \sum_i x_i = 0 \quad \perp \quad \lambda \quad \text{free} \]

MCP solution: 674 basic vars, 326 nonbasic
NLP vs. KKT

- Problems with NLP solvers
  - expect “few” superbasics
  - slow convergence: add superbasics singly
  - memory usage quite high

- Advantages of KKT system
  - Uses second order information
  - no superbasics, exact basis identification
NLP2MCP (Ferris, Horn)

- Output KKT conditions for an NLP
  - Requires taking derivatives symbolically
  - Automates error-prone process
- Makes interesting projects possible
  - Integrate optimization and equilibrium concepts
  - Large NLP models may solve better
  - Build large models out of components
MPEC model type

• Superset of NLP and MCP
  – Application may come from either direction
  – Syntax reflects this

• Some models and solver links exist
  – Bundle code
  – SolvOpt

• Better solvers required
MCP2NLP  (Drud)

- May want to reformulate CP as NLP
  - Improve robustness of arsenal
  - Diagnostics for unsolvable models
  - Starting point for MPEC models
- Initial results:
  - Can improve robustness, speed
  - Comprehensive test requires more automation
Summary

- MCP/MPSGE models no longer “exotic”
  - Large base of demanding users
  - System delivers dependable results
- Active work on new applications in CP
  - MPEC model type
  - reformulation tools: NLP2MCP, MCP2NLP