GAMS
Model Development –
Using CHP as an example

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Agenda

- GAMS – Basic Syntax
- Excursus: GDX
- Building a Model: CHP Generation Plant
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- GAMS – Basic Syntax
- Excursus: GDX
- Building a Model: CHP Generation Plant
GAMS Syntax: Declaration

• Sets

\begin{align*}
\text{Sets} & \\
\text{i} & \text{canning plants} & / \text{seattle, san-diego} / \\
h & \text{hours} & / 1*24 / \\
\text{work}(h) & \text{hours of work} & / 9*12, 14*17 /;
\end{align*}

• Parameters

\begin{align*}
\text{Parameters} & \\
\text{a}(i) & \text{capacity of plant i in cases} & / \\
& \text{seattle} & 350 \\
& \text{san-diego} & 600 /;
\end{align*}

\begin{table}[h]
\centering
\begin{tabular}{lccc}
\text{d}(i,j) & \text{distance in thousands of miles} \\
\hline
& \text{new-york} & \text{chicago} & \text{topeka} \\
\text{seattle} & 2.5 & 1.7 & 1.8 \\
\text{san-diego} & 2.5 & 1.8 & 1.4 \\
\end{tabular}
\end{table}

\begin{align*}
\text{Parameter} & \\
\text{pay}(i,h) & \text{Payment per city and hour in $} & / \\
& \text{seattle.9*12} & 9 \\
& \text{seattle.14*17} & 11 /;
\end{align*}

• Scalars

\begin{align*}
\text{Scalar} & \ f \ \text{freight in dollars per case per thousand miles} & /90/ ;
\end{align*}
GAMS Syntax: Data Assignment using Sets

• General

Parameter \( c(i,j) \) transport cost in thousands of dollars per case;
\[
c('seattle','chicago') = f * d('seattle','chicago') / 1000;
\]
\[
c(i,j) = f * d(i,j) / 1000;
\]

• Sum

Parameter \( daypay(i) \) Payment for a complete workday in $;
\[
daypay(i) = \text{sum}(h$work(h),pay(i,h));
\]
\[
daypay(i) = \text{sum}(work(h),pay(i,h));
\]
\[
daypay(i) = \text{sum}(work,pay(i,work));
\]

• Product

scalar \( prodcap \) Product of all capacities;
\[
prodcap = \text{prod}(i,a(i));
\]

• Minimum/Maximum

Scalar \( maxdem \) Maximum of all demands;
\[
maxdem = \text{smax}(j,b(j));
\]
Scalar \( mindist \) Minimum of all distances;
\[
mindist = \text{smin}((i,j),d(i,j));
\]
GAMS Syntax: Defined Elements of a Set

- `Ord()` and `Card()`

```plaintext
Set lasth(h) Last hour of the day;
lasth(h) = (ord(h) = card(h));
lasth(h)$ (ord(h) = card(h)) = yes;
```

- `Sameas()`

```plaintext
Scalar demXny Demand in all markets except for New-York;
demXny = sum(j$(not sameas(j,'new-york')), b(j));
```
GAMS Syntax: Variables

- Free ($-\infty$ to $\infty$)
  
  Variables  
  
  Total transportation costs in thousands of dollars ;

- Positive (0 to $\infty$)
  
  Positive Variable  
  
  Shipment quantities in cases;

- Negative ($-\infty$ to 0)
  
  Negative Variable  
  
  Resource consumption;

- Integer (0, 1, 2, ...)
  
  Integer Variable  
  
  Output;

- Binary (0 or 1)
  
  Binary Variable  
  
  Decision whether to produce or not;
GAMS Syntax: Variables

• Semi continuous (0 or above certain value)

```
Semicont Variable SHIP(i,j) Ship at least 100 tons;
SHIP.lo(i,j) = 100;
```

• Semi integer (0 or integer above certain value)

```
Semiint Variable OUTP(i) Produce at least 12 units;
OUTP.lo(i) = 12;
```

• Special Ordered Sets Type 1 (Only one member in a set of variables can have nonzero value)

```
SOS1 Variable PRODUCE(i) Produce at one location only;
```

• Special Ordered Sets Type 2 (Only two adjacent members in a set of variables can have nonzero value)

```
SOS2 Variable WORKSCED(h) Schedule work so that 2 hours in series are assigned;
```
GAMS Syntax: Equations

• Definition

\[
\text{Equations} \\
\text{cost} \quad \text{define objective function} \\
\text{supply}(i) \quad \text{observe supply limit at plant } i \\
\text{demand}(j) \quad \text{satisfy demand at market } j \\
\]

• Declaration

\[
\text{cost} \quad . . \quad Z = \text{sum}(i, j, c(i, j)*X(i, j)); \\
\text{supply}(i) \quad . . \quad \text{sum}(j, X(i, j)) = \text{a}(i); \\
\]
GAMS Syntax: Model Definition

• Model
  
  ```gams
  Model transport /all/;
  ```

• Solver selection
  
  ```gams
  option lp=coincbc;
  ```

• GAMS options
  
  ```gams
  Option reslim = 60;
  Option iterlim = 100;
  ```

• Solver options
  
  ```gams
  $onecho > cplex.opt
  lpmethod = 4
  $offecho
  ```

• Solve
  
  ```gams
  Solve transport using lp minimizing Z;
  ```
GAMS Syntax: Procedural Elements

- **For**

  ```gams
  scalar scen;
  for(scen=1 to 10 by 0.5,
      f    = 10*scen;
      c(i,j) = f * d(i,j) / 1000;
    Solve transport using lp minimizing Z;
    Display Z.l;);
  ```

- **While**

  ```gams
  Scalar scen /1/;
  while(scen<=10,
      f    = 10*scen;
      c(i,j) = f * d(i,j) / 1000;
    Solve transport using lp minimizing Z;
    scen = scen + 0.5;
  );
  ```

- **Loop**

  ```gams
  loop(h,
      if(work(h),
        pay(i,h) = 0.6*pay(i,h);
      else
        pay(i,h) = 1.5*pay(i,h);
      );
  );
  ```

- **If... else...**
# Mathematical Functions

<table>
<thead>
<tr>
<th>Function</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>erf(x)</td>
<td>Integral of the standard normal distribution from $-\infty$ to $x$</td>
</tr>
<tr>
<td>exp(x)</td>
<td>Exponential, $e^x$</td>
</tr>
<tr>
<td>log(x)</td>
<td>Natural logarithm, $\log_e x$</td>
</tr>
<tr>
<td>log10(x)</td>
<td>Common logarithm, $\log_{10} x$</td>
</tr>
<tr>
<td>normal(x,y)</td>
<td>Random number normally distributed with mean $x$ and standard deviation $y$</td>
</tr>
<tr>
<td>uniform(x,y)</td>
<td>Random number with uniform distribution between $x$ and $y$</td>
</tr>
<tr>
<td>abs(x)</td>
<td>Absolute Value of $x$, i.e. $</td>
</tr>
<tr>
<td>ceil(x)</td>
<td>Ceiling of $x$. Smallest integer $\geq x$</td>
</tr>
<tr>
<td>floor(x)</td>
<td>Floor of $x$. Largest integer $\leq x$</td>
</tr>
<tr>
<td>mapval(x)</td>
<td>Mapping function. Assigns unique numbers to special values.</td>
</tr>
<tr>
<td>max(x,y,...)</td>
<td>Largest value among all arguments.</td>
</tr>
<tr>
<td>min(x,y,...)</td>
<td>Smallest value among all arguments</td>
</tr>
<tr>
<td>mod(x,y)</td>
<td>Remainder. $x - y \times \text{trunc}(x/y)$</td>
</tr>
<tr>
<td>power(x,y)</td>
<td>Integer power. $x^y$, where $y$ must be an integer</td>
</tr>
<tr>
<td>round(x)</td>
<td>round $x$ to the nearest integer</td>
</tr>
<tr>
<td>round(x,y)</td>
<td>Rounds $x$ to $y$ decimal places right (+) or left (-) to the decimal point</td>
</tr>
<tr>
<td>sign(x)</td>
<td>Returns 1 if $x &gt; 0$, -1 if $x &lt; 0$, and 0 if $x = 0$</td>
</tr>
<tr>
<td>sqr(x)</td>
<td>Square of $x$. $x^2$</td>
</tr>
<tr>
<td>sqrt(x)</td>
<td>Square root of $x$. $\sqrt{x}$</td>
</tr>
<tr>
<td>trunc(x)</td>
<td>$\text{sign}(x) \times \text{floor}(\text{abs}(x))$</td>
</tr>
<tr>
<td>arctan(x)</td>
<td>$\tan^{-1} x$. Result in radians</td>
</tr>
<tr>
<td>cos(x)</td>
<td>$\cos x$; $x$ in radians</td>
</tr>
<tr>
<td>sin(x)</td>
<td>$\sin x$; $x$ in radians</td>
</tr>
</tbody>
</table>

*not exhaustive
Compile Time vs. Execution Time

• Compile time arguments…
  – start with $  
  – are executed when compiling a GAMS file  
  – are e.g. $if, $set, $goto, $exit, $call …

• Execution time arguments…
  – are executed during the execution of the compiled GAMS file  
  – are e.g. if, execute, solve, loop, …

**NOTE:** When reading a model from top to bottom, we can see an execution time command before a compile time command, but the latter will be executed first.
Agenda

GAMS – Basic Syntax

Excursus: GDX

Building a Model: CHP Generation Plant
Gams Data eXchange

Binary Data Exchange

- Fast exchange of data
- Syntactical check on data before model starts
- Data Exchange at any stage (Compile and Run-time)
- Platform Independent
- Direct Excel connectivity
- General API
- Scenario Management Support
- Full Support of Batch Runs

GDX Tools

- GDX Viewer
- GDXRank
- GDXMerge
- GDXDiff
- GDXAPI
- GDX
- IDE
- GDXxrw
- GMS
Using GDXXRW to read from Excel

Parameter \(d(i,j)\) distance in thousands of miles;

\$
call GDXXRW dist.xls par=d rng=dist!A1 rdim=1 cdim=1
\$
\$if errorlevel 1 $abort "Problem with file dist.xls!"

\$gdxin dist
\$load d
Using GDXXRW to write to Excel

```gams
execute_unload 'ship' x;
execute 'GDXXRW ship.gdx var=x rng=ship!A1 rdim=1 cdim=1';
```
Agenda

- GAMS – Basic Syntax
- Excursus: GDX
- Building a Model: CHP Generation Plant
Combined Heat and Power (CHP) Plant

- Produces heat and electricity in combination
- Certain demand of heat and electricity has to be satisfied
- Electricity can be traded at energy exchange
- Excess heat cannot be released, the demanded amount has to be generated exactly
- Cogeneration is subsidized by government
Process Model
Process Model

- **External Inputs/Output**
- **Intermediate Commodities**

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
<th>G</th>
<th>H</th>
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Processes with minimum utilization level

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## Demand Electricity/Heat

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</table>
Modeling Task

- Find cost minimal solution
  - Satisfy demand
  - Buy or make electricity
  - Subsidize cogeneration
  - Technical feasible schedule of plants
  - Investment decisions (new/upgraded power plants)
  - Economical aspects (e.g. shared ownership of plants)
  - ...

Minimize [Fuelcosts]  
  + [Costs/returns from electricity trading]  
  – [Bonus for cogeneration]

s.t.  [Matter input] = [Matter output]

[Generated electricity] + [Purchased electricity]  
= [Demand for electricity]

[Generated heat] = [Demand for heat]
Exercise 1: Add Steam Generator (SG)

- At maximum utilization:
  - Output: Steam 11 kg/s
  - Input: Coal 35.24 MWh
  - Electricity 0.5 MWh

- At minimum utilization:
  - Output: Steam 5.5 kg/s
  - Input: Coal 17.62 MWh
  - Electricity 0.25 MWh

- Coal costs: 12.23 $/MWh

- Reduces Output of HTB: 25 MWh → 18 MWh
Exercise 2: Add Heat Bypass (HB)

- Consumes up to 53.2 MWh wasteheat
- Cools it down at costs of 4 $ per MWh
- No relevant output
Exercise 3: Add Heat Storage Tank (HST)

- Maximum capacity of 50 MW heat
- At most 15 MW per hour input
- At most 12 MW per hour output
- “Pump” heat into tank costs 0.05 $ per MW
- 2% of stored heat gets lost per h
Heat Storage Tank

Before:

\[ \text{dem}_{\text{Heat}}(h) = \text{GEN}_{\text{Heat}}(h) \]

After:

\[ \text{dem}_{\text{Heat}}(h) = \text{GEN}_{\text{Heat}}(h) + \text{HOUT}(h) - \text{HIN}(h) \]
\[ \text{HLVL}(h) = \text{HLVL}(h-1) \cdot 0.98 - \text{HOUT}(h) + \text{HIN}(h) \]
\[ \text{HLVL}(h) \leq 50 \]
\[ \text{HIN}(h) \leq 15 \]
\[ \text{HOUT}(h) \leq 12 \]
Exercise 4: Limiting Number of GT Starts

- Startup costs:
  - GT: 500 $  
  - SG: 1000$
- GT may be turned on not more than 8 times during modeled time frame

\[ \text{ONOFF}(h, p) = 1 \land \text{ONOFF}(h-1, p) = 0 \Rightarrow \text{STARTUP}(h, p) = 1 \]
\[ \Rightarrow \text{STARTUP}(h, p) \geq \text{ONOFF}(h, p) - \text{ONOFF}(h-1, p) \]
\[ \sum_{h} \text{STARTUP}(h, 'GT') \leq 8 \]
Exercise 5: Add “cool down” time for GT

- GT has to stay off for at least 8 hours when shut down

\[ \text{ONOFF}(h, p) = 0 \land \text{ONOFF}(h - 1, p) = 1 \Rightarrow \text{SHUTDOWN}(h, p) = 1 \]
\[ \Rightarrow \text{SHUTDOWN}(h, 'GT') \geq \text{ONOFF}(h - 1, 'GT') - \text{ONOFF}(h, 'GT') \]

\[ \text{SHUTDOWN}(h, 'GT') = 1 \]
\[ \Rightarrow \text{STARTUP}(h_2, 'GT') = 0 \mid h \leq h_2 < h + 8 \]
\[ \Rightarrow \sum_{h_2 \mid h_2 \geq h \land h_2 < h + 8} \text{STARTUP}(h_2, 'GT') \leq 1 - \text{SHUTDOWN}(h, 'GT') \]
# GAMS on the Web

## Download
- [www.gams.de](http://www.gams.de)
- [www.gams.com](http://www.gams.com)

## Help and Support
- **Support Wiki**: [http://support.gams-software.com](http://support.gams-software.com)
- **Interfaces Wiki**: [http://interfaces.gams-software.com](http://interfaces.gams-software.com)
- **User Group**: [http://www.gams.com/maillist/gams_l.htm](http://www.gams.com/maillist/gams_l.htm)
- **Google Group**: [http://groups.google.de/group/gamsworld](http://groups.google.de/group/gamsworld)

## Search all GAMS Websites
- [http://www.gams.com/search.htm](http://www.gams.com/search.htm)
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