Economic Equilibrium Analysis
with GAMS/MPSGE

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GAMS/MPSGE: A Mathematical Programming System for General Equilibrium Analysis

• specifically designed for applied general equilibrium analysis, including both models represented as systems of equations and those which involved *complementarity* between inequalities and bounded variables.

• system is particularly useful for large, complex models based on benchmark equilibrium datasets

• GAMS/MPSGE provides a highly structured framework for inexperienced analysts, yet GAMS/MPSGE models can be customized through the use of *auxiliary variables*.
Economic Models in the Policy Arena

Economic models produce results which can play a central role in political dialogue.

Although economic models are based on formal mathematics, it is important to recognize that the origins of economic analysis are in social philosophy rather than physical science.

Everyone participates in economic transactions, so non-specialist audiences can be influenced by populist appeals to “common sense”.
Economic Equilibrium Ideas

*Agents* in an economic model include consumers, producers and governments, collectively representing all participants in market transactions in a given economy.

A central concept in economic equilibrium models is that agents *optimize subject to constraints*.

*Systematic errors* are logically inconsistent with individually rational choice.

A typical starting point for *dynamic* economic models is that consumers are fully informed and hold *consistent expectations of the future*. This approach differentiates economic models from models of physical systems.
GAMS/MPSGE Equilibrium Framework

Variables

\( p \in \mathbb{R}^N \)  Prices for all goods and factors (possibly indexed by commodity, sector, region, household, time period etc.)

\( y \in \mathbb{R}^M \)  Production activity levels (also indexed)

\( M \in \mathbb{R}^H \)  Income levels for each consumer in the model
Given Functions and Data

\( \tilde{p}_j \) is a vector of producer prices, net (gross) of applicable taxes for outputs (inputs, resp.)

\( r_j(\tilde{p}_j) \) is the unit revenue function for sector \( j \)

\( c_j(\tilde{p}_j) \) is the unit cost function for sector \( j \)

\( d_{ih}(p, M_h) \) is the demand function for household \( h \), derived from budget-constrained utility maximization. By definition, these functions satisfy Walras' law:

\[
\sum_i p_i d_{ih}(p, M_h) = M_h.
\]

\( \omega_{ih}, \theta_{jh} \) are matrices of commodity endowments and tax revenue allocations.
Dual Equilibrium: Zero Profit

\[ c_j(\tilde{p}_j) \geq r_j(\tilde{p}_j) \quad \perp \quad y_j \geq 0 \]

Primal Equilibrium: Market Clearance

\[ \sum_j \left( \frac{\partial r_j}{\partial \tilde{p}_{ij}} - \frac{\partial c_j}{\partial \tilde{p}_{ij}} \right) y_j + \sum_h \omega_{ih} \geq \sum_h d_{ih}(p, M_h) \quad \perp \quad p_i \geq 0 \]

Income Balance

\[ M_h = \sum_i p_i \omega_{ih} + \sum_j \theta_{jh} \ y_j \left[ \sum_i (\tilde{p}_{ij} - p_{ij}) \left( \frac{\partial r_j}{\partial \tilde{p}_{ij}} - \frac{\partial c_j}{\partial \tilde{p}_{ij}} \right) \right] \]
Calibrated Functions

Data for specification of functions is typically based on the first two terms in a Taylor approximation:

\[ \bar{a}_{ij}^U = \frac{\partial c_i}{\partial \bar{p}_i} \bar{p} \] benchmark producer inputs (the “use matrix”)

\[ \bar{a}_{ij}^M = \frac{\partial r_j}{\partial \bar{p}_i} \bar{p} \] benchmark producer output (the “make matrix”),

\[ \bar{d}_{ih} = d_{ih}(\bar{p}, \bar{M}_h) \] benchmark consumer demands at reference prices

\[ \omega_{ih}, \theta_{jh} \] benchmark initial endowments and tax shares.
Mission for Public Income – Colombian Fedesarrollo


- 56 production sectors

- 6 categories of labor:
  
  ufs  Urban formal salaried work
  ufn  Urban formal non-salaried work
  utc  Urban traditional contract work
  umc  Urban modern contract work (consulting)
  rsw  Rural salaried work (organized farming work)
  rnw  Rural non-salaried work (farming)
- 10 representative households

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<th>$/day</th>
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## Tax Revenue

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<tr>
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<th>%</th>
<th>Base</th>
<th>Tax Rates (%)</th>
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<td></td>
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<tr>
<td>State/Local</td>
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<td>200</td>
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<tr>
<td>Individual Income</td>
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<td>4</td>
<td>51</td>
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<td>Subsidies</td>
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<td><strong>Total</strong></td>
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<td>Local Govt. Income:</td>
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<td>Soc. Security Expend:</td>
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Equilibrium Framework

Key:
- Physical flow of goods:
- Financial flows of factor earnings, tax payments and transfers:
Model Formulation

Sets:

\[ s, g \quad \text{Sectoral and commodity identifiers} \]
\[ h \quad \text{Households} \]
\[ \ell \quad \text{Labor types} \]

Activity Levels:

\[ Y_s \quad \text{Production activity level} \]
\[ A_g \quad \text{Aggregate supply to domestic and export markets} \]
\[ C_h \quad \text{Aggregate consumption demand by household } h \]
\[ K \quad \text{Capital stock} \]
\[ I \quad \text{Investment} \]
\[ G \quad \text{Public demand} \]
Prices:

- $e$ Real exchange rate
- $r^K$ Rental price of capital
- $w_\ell$ Wage rate for labor type $\ell$
- $p^Y_g$ Supply price of good $g$ (gross of indirect taxes)
- $p^A_g$ Market price of good $g$ (gross of excise and VAT)
- $p^C_h$ Consumer price index
- $p^G_h$ Public provision price index
- $p^I$ Investment cost index

Other variables:

- $\kappa$ Capital stock
- $G_B$ Government external balance
Leontief Demand and Supply Coefficients

\( a_{gs}^M \) Output of good \( g \) per unit activity of sector \( s \), the *make matrix*.

\( a_{gs}^U \) Input of good \( g \) per unit activity of sector \( s \), the *use matrix*.

\( a_g^G \) Demand for good \( g \) per unit of government activity

\( a_g^I \) Demand for good \( g \) per unit of aggregate investment

\( a_{gg'}^\mu \) Trade and transport margin net demand per unit aggregate supply of good \( g' \)
Cost and Revenue Functions

\[ c_s^Y(\bar{w}_s, r^K) \quad \text{Unit cost of value-added in sector } s \ (Y_s) \]
\[ R_g^A(\bar{p}_r^A, \bar{p}_g^X) \quad \text{Unit revenue per unit of aggregate supply } (A_g) \]
\[ c_g^A(p_g^Y, \bar{p}_g^M) \quad \text{Unit cost of aggregate supply } (A_g) \]
\[ c_h^C(\bar{w}_h, p^A) \quad \text{Unit cost of final demand for leisure and goods } (C_h) \]
Arbitrage (Zero-Profit) Conditions

- Domestic production ($Y_s$):
  \[ \sum_g p^Y_{gs}a^M_{gs} = \sum_g p^A_{gs}a^U_{gs} + c_s^Y(\tilde{w}_{ls}, r^K) \]

- Aggregate Supply ($A_g$):
  \[ R^A_g(p^A_g, p^X_g) = c^A_g(p^Y_g, p^M_g) + \sum_{g'} p^A_{g'}a^\mu_{gg'} \]

- Consumption Cost ($C_h$):
  \[ p^C_h = c^C_h(p^A, \tilde{w}_{lh}) \]

- Cost of Investment ($I$):
  \[ p^I = \sum_g a^I_g p^A_g \]
• Cost of Public Provision \((G)\):

\[ p^G = \sum_g a_g^G p^A_g \]
Market Clearance Conditions

- Domestic Output
  \[ \sum_s Y_s a_{gs}^M = A_g \frac{\partial c_g^A}{\partial p_g^Y} \]

- Domestic Demand
  \[ A_g = \sum_{g'} a_{gg'}^U a_{g'}^A + \sum_s a_{gs}^Y Y_s + \sum_h \frac{\partial c_h^C}{\partial p_g^A} + G a_g^G + I a_g^G \]

- Labor Markets
  \[ \sum_h \bar{L}_{\ell h} = \sum_h \bar{C}_h \frac{\partial c_h^C}{\partial \bar{w}_{\ell h}} + \sum_s Y_s \frac{\partial c_s^Y}{\partial \bar{w}_{\ell s}} \]

- Capital Market
  \[ \kappa \sum_h \bar{K}_h = \sum_s Y_s \frac{\partial c_s^Y}{\partial r_K} \]
● Household Demand

\[ C_h = \frac{M_h}{c_h^C} \]

● Investment-Savings

\[ I = \sum_h S_h + S_G \]

● Current account

\[ B_G + \sum_h B_h + \sum_g A_g \frac{\partial R_g^A}{\partial p_g^X} = \sum_g A_g \frac{\partial c_g^A}{\partial p_g^M} \]
**Income Balance**

- **Household Income**

\[
M_h = \sum_\ell \bar{L}_{\ell h} \bar{w}_{\ell h} + \kappa \bar{r}^K \bar{K}_h + e \bar{B}_h - \bar{T}_h - p^I \bar{S}_h
\]

- **Government**

\[
M_G = \sum_g A_g \frac{\partial c^A_g}{\partial \bar{p}^M_g}(\bar{p}^M_g - p^M_g) + \\
+ \sum_g A_g \frac{\partial R^A_g}{\partial \bar{p}^X_g}(\bar{p}^X_g - p^X_g) + \\
+ \sum_s Y_s \frac{\partial c^Y_s}{\partial \bar{w}_{\ell s}}(\bar{w}_{\ell s} - w_{\ell s}) + \\
+ \sum_{\ell h} \left(L_{\ell h} - \frac{\partial c^C_h}{\partial \bar{w}_{\ell h}}\right)(w_{\ell s} - \bar{w}_{\ell h}) + \ldots
\]
Auxiliary Constraints

- Steady-state model ($\kappa$):
  \[ q = \frac{\tilde{r}^K}{p^I} = 1 \]

- Budget Balance ($B_G$)
  \[ G = 1 \]
Data Management in GAMS

* Units of the 1997 SAM are Millions of 1997 Pesos
* (current price). After scaling, we get Billions of Pesos

set r SAM Rows /11*210/;
alias (r,c);
sam(r,c) = sam(r,c)/1000;

parameter samchk Check of SAM consistency;
samchk(r) = round(sum(c, sam(r,c)-sam(c,r)), 5);
display samchk;

set Goods(r) /11*68/, Sectors(r) /70*127/, Labor(r) /128*133/, Capital(r) /134/, Households(r) /186*195/, Government(r) /135*158,159*185,197*199/, Firms(r) /196,200*201,202*206/, Row(r) /69,207/, Investment(r) /208*210/;
## Aggregated Social Accounts

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<th>Sector</th>
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<th>S</th>
<th>L</th>
<th>C</th>
<th>H</th>
<th>G</th>
<th>F</th>
<th>R</th>
<th>I</th>
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<td>74.3</td>
<td>7.6</td>
<td>0.1</td>
<td>6.2</td>
<td>10.7</td>
<td>4.8</td>
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<td>Government</td>
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<td>2.1</td>
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<td></td>
<td></td>
<td></td>
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<td></td>
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<td>Firms</td>
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<td>24.5</td>
<td>9.5</td>
<td>2.1</td>
<td>15.3</td>
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<tr>
<td>row</td>
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<td>1.3</td>
<td>2.8</td>
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<td></td>
<td>5.4</td>
<td>-0.2</td>
<td>14.5</td>
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$model:sam97

$sectors:
  y(s) ! Production
  x(g) ! Domestic supply
  a(s) ! Aggregate supply
  gov ! Public expenditure
  inv ! Investment
  con(h) ! Household consumption ...

$commodities:
  py(g) ! Output price
  pa(g) ! Aggregate price
  pc(h) ! Household consumption
  w(l) ! Wage rates
  rk ! Return to capital (gross of tax)
  pfx ! Foreign exchange ...

$consumers:
  ra(h) ! Households
  govt ! Government
Production Functions

$\text{prod}:y(s) \quad s:0 \quad va:1 \quad t:0
  \begin{align*}
  o: & \text{py}(g) \quad q: \text{make}(s,g) \quad a: \text{govt} \quad t: (tcm(s) + tif(s) - \text{sub}(s) + ty(s)) \\
  i: & \text{pa}(g) \quad q: \text{use}(g,s) \quad a: \text{govt} \quad t: \text{vat}(s) \\
  i: & \text{w}(l) \quad q: \text{l}d0(l,s) \quad p: \text{pl}0(l,s) \quad a: \text{govt} \quad t: \text{pyrl}(l,s) \quad va: \\
  i: & \text{rk} \quad q: \text{kd}0(s) \quad \text{va:} \\
  \end{align*}

$\text{prod}:x(g) \quad t:4
  \begin{align*}
  o: & \text{pd}(g) \quad q: \text{d}0(g) \quad p:1 \\
  o: & \text{pfx} \quad q: \text{x}0(g) \quad p: \text{px}0(g) \quad a: \text{govt} \quad t: \text{crt}(g) \\
  i: & \text{py}(g) \quad q: \text{y}0(g) \quad p: \text{py}0(g) \quad a: \text{govt} \quad t: \text{tp}(g) \\
  \end{align*}

$\text{prod}:a(g) \quad s:0 \quad dm:4
  \begin{align*}
  o: & \text{pa}(g) \quad q: \text{a}0(g) \quad p: \text{pa}0(g) \quad a: \text{govt} \quad t: \text{txs}(g) \quad t: \text{vat}(g) \\
  i: & \text{pmg}(gg) \quad q: \text{margin}(gg,g) \\
  i: & \text{pd}(g) \quad q: \text{d}0(g) \quad dm: \\
  i: & \text{pfx} \quad q: \text{m}0(g) \quad p: \text{pm}0(g) \quad dm: \quad a: \text{govt} \quad t: \text{tm}(g) \\
  \end{align*}
Dual Equilibrium: Zero Profit

\[ c_j(\tilde{p}_j) \geq r_j(\tilde{p}_j) \perp y_j \geq 0 \]

Primal Equilibrium: Market Clearance

\[ \sum_j \left( \frac{\partial r_j}{\partial \tilde{p}_{ij}} - \frac{\partial c_j}{\partial \tilde{p}_{ij}} \right) y_j + \sum_h \omega_{ih} \geq \sum_h d_{ih}(p, M_h) \perp p_i \geq 0 \]

Income Balance

\[ M_h = \sum_i p_i \omega_{ih} + \sum_j \theta_{jh} y_j \left[ \sum_i (\tilde{p}_{ij} - p_{ij}) \left( \frac{\partial r_j}{\partial \tilde{p}_{ij}} - \frac{\partial c_j}{\partial \tilde{p}_{ij}} \right) \right] \]
Equilibrium Framework

Key:

Physical flow of goods:

Financial flows of factor earnings, tax payments and transfers:
Corresponding GAMS Algebra

* Demand for domestic input into Armington production:

\begin{align*}
\text{DEF}_A_Y(d,i,re) & \equiv (\text{vd}(d,i,re))\ldots \\
A_A_Y(d,i,re) & = E = \\
& \quad \text{vd}(d,i,re) \times \\
& \quad \left\{ \left( (1 - \thetaa_m(d,i,re)) \times \text{py}(i,re)^{(1 - \text{esubdm}(i,re))} \\
& \quad \quad + \quad \thetaa_m(d,i,re) \times \text{PM}(i,re)^{(1 - \text{esubdm}(i,re))} \\
& \quad \right)^{(1/(1 - \text{esubdm}(i,re)))} \right\} \div \text{py}(i,re) \\
& \times \text{esubdm}(i,re); \\
\end{align*}
Constraints Associated with Auxiliary Variables

$constraint: tau$
  gov =e= 1;

$constraint: kf$
  pinv =e= pinv0 * rkf;
## Proportional Tax Increases: Steady-State Model

### Value-Added Taxes

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<th>ACF(σ)</th>
<th>Revenue</th>
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<tbody>
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<td>Pesos(T)</td>
</tr>
<tr>
<td>×1.2</td>
<td>1.29</td>
<td>1.36</td>
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<td></td>
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<tr>
<td>×1.6</td>
<td>1.33</td>
<td>1.40</td>
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<td></td>
<td>3.2</td>
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<tr>
<td>×2.0</td>
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### Import Tariffs

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<td>Pesos(T)</td>
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<td>×1.6</td>
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<tr>
<td>×2.0</td>
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Tax Revenue Yield in the Refined Petroleum Market

Tax Rate = $t_0$, Tax Revenue = $T_0$

Tax Rate = $t_0 \times 2$, Tax Revenue = $T_0 \times (1 + \text{yield})$