A Student-centric Class and Exam Scheduling System at West Point

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## Change in Focus

<table>
<thead>
<tr>
<th>Computation – Past</th>
<th>Model – Present</th>
<th>Application – Future</th>
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<tbody>
<tr>
<td>- Algorithm limits application</td>
<td>- Modeling skill limits applications</td>
<td>- Domain expertise limits application</td>
</tr>
<tr>
<td>- Problem representation is low priority</td>
<td>- Algebraic model representation</td>
<td>- Off-the-shelf graphical user interfaces</td>
</tr>
<tr>
<td>- Large costly projects</td>
<td>- Smaller projects</td>
<td>- Links to other types of models</td>
</tr>
<tr>
<td>- Long development times</td>
<td>- Rapid development</td>
<td>- Models embedded in business applications</td>
</tr>
<tr>
<td>- Centralized expert groups</td>
<td>- Decentralized modeling teams</td>
<td>- Internet/web</td>
</tr>
<tr>
<td>- High computational cost, mainframes</td>
<td>- Low computational cost, workstations</td>
<td>- Users hardly aware of model</td>
</tr>
<tr>
<td>- Users left out</td>
<td>- Machine independence</td>
<td></td>
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<tr>
<td></td>
<td>- Users involved</td>
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</table>
Scheduling US Military Academy West Point

“... each student’s daily activities are a carefully regimented balance of academic, military, and physical requirements.”
USMA is Different

- **Technically**
  - Day1/day2 schedule
  - Special rules (e.g. < 30% athletes in class)
  - Sufficient number of rooms, teachers, …

- **Scheduling around the cadets needs**
  - No conflicting activities
  - Individual schedule of activities is compliant to vast catalogue of *business rules*

Software evaluation did not find an “off the shelf” product that could handle USMA requirements
ATTENDANCE PERIODS

1-DAY CLASS PERIODS

0735 0830 0840 0935 0945 1040 1145 1245 1340 1350 1445 1455 1550

A  LAB  B  LAB  C  LAB  D  NOON MEAL  CMDT’S HOUR  E  F

2-DAY CLASS PERIODS

0735 0830 0840 0935 0945 1040 1145 1245 1340 1350 1445 1455 1550

G  LAB  H  LAB  I  LAB  J  NOON MEAL  DEAN’S HOUR  K  L

R/S/T/U LAB HOURS
Academic Scheduling

• Course scheduling
  – For a given set of course offerings find good schedules for all cadets.

• Term End Exam (TEE) scheduling
  – Scheduling preparation
  – Find good schedules for exam courses and cadets.
Course Scheduling

- Given course hours & capacity
  - MA481,AB,36
  - MA481,CD,18
  - MA481,EF,18
  - PE300,C,180
  - PE300,J,60
  - MA371,F,18

- Given cadet’s course registration
  - 043671XXX,MA481
  - 043671XXX,PE300

- Objective: Find a *good* assignment of cadet’s course requests to course hours
  - 043671XXX,MA481,CD
  - 043671XXX,PE300,J
Problems with a Model

- There is no solution subject to all constraints/rules for real data
- Infeasibilities
  - Individual Cadet Infeasibilities
  - System Infeasibility (e.g. Capacity)
- Goal Programming:
  - Relax constraints/rules by penalizing violations
  - How to Select penalties for constraint violations
  - Penalty depend on individual Cadet
An Optimization Model

\[
\begin{align*}
\min & \quad \sum_{ro} (p_{1,ro} \times \pi_{1,ro} + p_{2,ro} \times \pi_{2,ro}) + \sum_{c} (p_{3,c} \times \pi_{3,c} + p_{4,c} \times \pi_{4,c}) \\
\sum_{o} x_{c,ro} &= 1 \quad \text{(for all 8TAP entries)} \\
\sum_{r} x_{c,ro} &\leq 1 + \pi_{3,c} \quad \text{(for all cadets c for all time slots o)} \\
-\sigma - \pi_{4,c} &\leq \sum_{ro \text{ on day-1}} x_{c,ro} - \sum_{ro \text{ on day-2}} x_{c,ro} \leq \sigma + \pi_{4,c} \quad \text{(for all cadets c)} \\
x_{c,ro} &= 0 \quad \text{(for all c, ro where c has activity at o)} \\
\sum_{c} x_{c,ro} &\leq cap_{ro} + \pi_{1,ro} \quad \text{(for all course hours ro)} \\
\sum_{c \text{ freshman&athlete}} x_{c,ro} - 0.6 \sum_{c} x_{c,ro} &\leq \pi_{2,ro} \quad \text{(for all course hours ro)}
\end{align*}
\]

- 60,000 Variables, 500,000 Non-Zeros
- 24 hours CPLEX 6.6 and no integer solution
Decomposition

- **Pre-Scheduling**
  - Filter cadets with no feasible schedule
  - Overcome infeasibility by relaxation/data changes

- **Scheduling**
  - All individual constraints/rules are hard constraints
  - Find assignment that does not exceed capacity (or penalize overloads)
Pre-Scheduling

• One cadet at a time
  – Check feasibility
  – If infeasible produce several infeasible schedules ranked by severeness of infeasibility
    • Hour Conflict
    • Day – Day Balance
    • Last Hour Free
  – Human Intervenes

• Thousands of small MIPs
## Results

- AY 2000/2 parallel tested
- AY 2001/1 deployed

<table>
<thead>
<tr>
<th></th>
<th>Legacy System + human deconflicter</th>
<th>New System</th>
</tr>
</thead>
<tbody>
<tr>
<td>Individual Relaxations</td>
<td>203/304/116</td>
<td>58/25/4</td>
</tr>
<tr>
<td>Capacity Overloads</td>
<td>12/54</td>
<td>9/21</td>
</tr>
<tr>
<td>Number of Schedulers</td>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td>Time to produce Schedule</td>
<td>4 Weeks</td>
<td>1 Day</td>
</tr>
</tbody>
</table>
# Term End Exam Scheduling

## Term End Exam Courses

- **Morning Period:**
  - CE371
  - CH384
  - CS383
  - HI366

- **Afternoon Period:**
  - CE404
  - LG484
  - LS362
  - MS350

## Exam Scheduling Details

- Approximately 4000 cadets
- Approximately 20000 exams

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<thead>
<tr>
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<th>2</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>CE371</td>
<td>CH101</td>
<td>EV203</td>
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<tr>
<td>CH384</td>
<td>CS408</td>
<td>PH203</td>
</tr>
<tr>
<td>CS383</td>
<td>EE301</td>
<td>PL300</td>
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<tr>
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<td>EN302</td>
<td>LR204</td>
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<tr>
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<td>LF382</td>
<td>CE403</td>
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<tr>
<td>LG484</td>
<td>SE388</td>
<td>CS380</td>
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<tr>
<td>LS362</td>
<td>SS388</td>
<td>SS201</td>
</tr>
<tr>
<td>MS350</td>
<td>...</td>
<td>...</td>
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Overcoming Conflicts

• Schedule with conflicts

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<tr>
<td>MS350</td>
<td>CE403</td>
<td>...</td>
</tr>
</tbody>
</table>

Cadet’s 8TAP:

| PL300 |
| CE372 |
| CE403 |
| CS380 |
| EV180 |
| HI302 |

• Makeup/ahead for an exam course:
  - An additional exam offering for a small group of cadets who can not go to the primary exam offering

• Resolve conflicts by adding makeup/ahead
• Given exam courses
  - MA481 CE371 CH100 ...
• Given exam periods
  - p1, p2, p3, ... p12
• Given cadet’s exam course ‘requests’
  - 043671571,CE403
  - 043671571,CE380
• Find an assignment of exam course sessions (primaries, makeups) to periods and cadet’s requests to exam courses sessions.
  - CE403,prim,p12 CE403,mkup,p4 CE380,prim,p4 ...
  - 043671571,CE403,p4 043671571,CE380,p4 ...
• Objective: Minimize the total number of makeups
An Optimization Model

- **Variables**
  - $x(c, r, p)$: course/period cadet request
    - Value: 250.000
  - $y(r, s, p)$: course session to period
    - Value: $|S|\times 3.000$
  - $z(r, p)$: primary indicator
    - Value: 3.000

- **Constraints**
  - Conflict
    - Value: 50.000
  - Assign
    - Value: 20.000
  - PrimEnroll
    - Value: 3.250
  - Consecutive
    - Value: 36.000
  - Exams per day
    - Value: 6.000
  - Inclusive
    - Value: $|I|\times 12$
  - Exclusive
    - Value: $|S||I|\times 12$
  - Fixed, Prohibit, No makeup, Finished
  - Coupling of $x$ and $y$
    - Value: $\sum x(c, r, p) \leq enroll(r) \cdot y(r, s, p) \cdot |S|\times 3.000$
Solution Approach

• Heuristic based on a collection of medium sized optimization models produces conflict free schedules and automatically relaxes constraints.

• Improvement module starts with a good/mediocre solution and a set of relaxed constraints and tries to
  – Improve number of makeups
  – Reinforce relaxed constraints
Solution Improvement

- Decompose the problem
  - Assignment of cadet request to exam course session
  - Assignment of exam sessions to periods

- Given a feasible schedule – iterate until no progress

20000 Exams

Cadet

CE380, primary
CE380, makeup

CE403, primary
CE403, makeup

HI302, primary
HI302, makeup

1 2 6

250 courses

250 courses

Timetable

250 courses
Feasibility Study

- TEE last application of legacy system
  - Mainframe, Cobol, ~1980
  - Maintenance + on-site personnel: $500,000/year
- By March 2001: decision for renewal

TEE Schedule for AY2001/2 (End of May 2001)
- Chuck + Legacy system
  - Partial schedule, approx. 90 makeups (4 Weeks)
- Chuck + GAMS TEE scheduler
  - Complete schedule, no conflicts, 60 makeups (10 minutes)
  - The improver module produced schedule with 40 makeups
More Computational Results

- Three data sets 01/2, 02/1 (early), 02/1
- Constraint violations ‘OK’

<table>
<thead>
<tr>
<th>Year</th>
<th>Courses</th>
<th>Periods</th>
<th>Requests</th>
<th>Makeups</th>
</tr>
</thead>
<tbody>
<tr>
<td>01/2</td>
<td>226</td>
<td>12</td>
<td>18937</td>
<td>38</td>
</tr>
<tr>
<td>02/1 early</td>
<td>213</td>
<td>12</td>
<td>18512</td>
<td>49</td>
</tr>
<tr>
<td>02/1</td>
<td>252</td>
<td>11</td>
<td>21175</td>
<td>61</td>
</tr>
</tbody>
</table>
Before

CADET BASIC TRAINING

During the Flexed Arm Hang event in the Warrior Competition, a new cadet grimaces as he hangs there but does not let go of the chin-up bar.

CADET WARRIORS

West Point
After
Conclusions

• Two Student-Centric Scheduling Problems
  – Course Scheduling
  – TEE scheduling
• Math. Programming Approaches
• Successful Applications
• Running at USMA without model changes for several years (changes in hardware, interface, newer solver versions, …)