GAMS
Transportation Model

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What is a Model?

- Mathematical Programming (MP) Model
  - List of Equations

- Collection of several intertwined MP Models
  - Data Preparation
  - Data Calibration
  - “Solution” Module (e.g. sequential, parallel, loop)
  - Report Module
A Transportation Model

Minimize Transportation cost
subject to Demand satisfaction at markets
Supply constraints
Mathematical Algebra

\[
\sum_{(c,p) \in \mathcal{N}} t_{\text{cost}} \cdot \text{dist}(c, p) \cdot x^c_p \rightarrow \min \\
\sum_{(c,p) \in \mathcal{N}} x^c_p \leq \sup(c) \quad \forall c \\
\sum_{(c,p) \in \mathcal{N}} x^c_p \geq \text{dem}(p) \quad \forall p \\
x^c_p \geq 0 \quad \forall c, p : (c, p) \in \mathcal{N}
\]
GAMS Algebra

Variables

\[ x(i,j) \] shipment quantities in cases
\[ z \] total transportation costs in thousands of dollars;

Positive Variable \( x \);

Equations

\[-\text{cost} \quad \text{define objective function} \]
\[-\text{supply}(i) \quad \text{observe supply limit at plant } i \]
\[-\text{demand}(j) \quad \text{satisfy demand at market } j \]

\[ \text{cost} \quad z = \sum_{(i,j)} c(i,j) * x(i,j) \]
\[ \text{supply}(i) \quad \sum_{j} x(i,j) = a(i) \]
\[ \text{demand}(j) \quad \sum_{i} x(i,j) = g\ b(j) \]

Model transport /all/ ;
A few Word about GAMS Syntax

- **Symbols:**
  - Sets
  - Parameters
  - Variables
  - Equations
  - Models
  - ASCII Output Files

```plaintext
Sets
  i  canning plants / seattle, san-diego /;
Parameters
  a(i)  capacity of plant i in cases
        / seattle  350
          san-diego 600 /;
Variables
  x(i,j)  shipment quantities in cases;
Equations
  supply(i) observe supply limit at plant i;
Model
  transport /all/ ;
File
  fx  some file / c:\t\text.txt /;
```

- **Statements**
  - Declarations
  - Data Assignments
  - Equation Definition
  - Programming Flow Control
  - Option statement

```plaintext
Parameter c(i,j);
c(i,j) = f * d(i,j) / 1000 ;
supply(i) ..  sum(j, x(i,j)) =l= a(i);
loop(i, put fx i.tl);
option reslim=10;
```
Using the GAMS IDE to build a model

- IDE Project Management
- Documentation
  - User’s Guide
  - McCarl User’s Guide
  - Solver Manuals
- Model Library
- Solver Selection
- Option Editor
- Listing file
  - Tree view
  - Error navigation
- Spell checking
Hands-on! Transportation Model
Types of Variables

- **Continuous Variables**
  - Free/Positive/Negative
  - Lower and/or upper bound

- **Binary Variables**
  - Either 0 or 1

- **Integer Variables**
  - by default ranging between 0 and 100

- **Semicont/Semiint Variables**
  - 0 or above a given minimum

- **Special Ordered Set Variables (SOS1, SOS2)**
Binary Variables

• Powerful Tool to model yes/no decisions

• Models with discrete variables (MIP)
  – Solved using Branch-and-Cut algorithms (lots or LPs)
  – Theoretically difficult problem class
  – Practical:
    • mixed bag
    • Art of Modeling

• Example: Minimum Shipment
  – Ship at least 100 tons or don’t ship
Minimum Shipment

- Continuous Variable $x$ (shipment)

- Binary Variable $ship$ (indicator for minimum shipment):
  - $ship=1$ if $x \geq 100$
  - $ship=0$ if $x = 0$

- Coupling Constraints:
  - $x \geq 100 \times ship$
  - $x \leq bigM \times ship$

- How big do we have to make bigM?
Implement Min/Max Shipments (MIP)

Parameter repl(i,j,*)  Shipments between plants and markets
rep2(*) Objective value;

repl(i,j,'lp') = x.l(i,j);
rep2('lp') = z.l;

scalars xmin / 100 /
    xmax / 275 /;

binary variables ship(i,j) decision variable to ship

equations minship(i,j) minimum shipments
       maxship(i,j) maximum shipments;

minship(i,j).. x(i,j) =g= xmin*ship(i,j);
maxship(i,j).. x(i,j) =l= xmax*ship(i,j);

model m2 min shipments / all /;
solve m2 using mip minimizing z;
repl(i,j,'mip') = x.l(i,j);
rep2('mip') = z.l;

option mip=coincbc
solve m2 using mip minimizing z;
repl(i,j,'mip-coincbc') = x.l(i,j);
rep2('mip-coincbc') = z.l;
display repl,rep2;
Economy of Scales: Cost = Volume$^{\text{factor}}$
Implement Nonlinear Cost (NLP)

* nonlinear cost
  equation nlcost nonlinear cost function;
  scalar beta;

  nlcost.. z =e= sum((i,j), c(i,j)*x(i,j)**beta);

  model m3 / nlcost, supply, demand /;

  beta = 1.5;
  solve m3 using nlp minimizing z;
  rep1(i,j,'nlp-convex') = x.l(i,j);
  rep2('nlp-convex') = z.l;

  beta = 0.6;
  solve m3 using nlp minimizing z;
  rep1(i,j,'nlp-concave') = x.l(i,j);
  rep2('nlp-concave') = z.l;

  option nlp=baron;
  solve m3 using nlp minimizing z;
  rep1(i,j,'nlp-baron') = x.l(i,j);
  rep2('nlp-baron') = z.l;

  display rep1, rep2;
Implement Min/Max and Nonlinear (MINLP)

* min/max and nonlinear objective

model m4 / nlcost,supply,demand,minship,maxship /;

option minlp=baron;
solve m4 using minlp minimizing z;
rep1(i,j,'minlp-bar') = x.l(i,j);
rep2('minlp-bar') = z.l;

option minlp=lindoglobal;
solve m4 using minlp minimizing z;
rep1(i,j,'minlp-lin') = x.l(i,j);
rep2('minlp-lin') = z.l;

display rep1,rep2;
Sources of GAMS Information

Download: http://download.gams-software.com/
Contributed Documentation: http://www.gams.com/docs/contributed
Contributed Software: http://www.gams.com/contrib/contrib.htm
Presentations: http://www.gams.com/presentations
Workshops: http://www.gams.com/courses.htm

Bruce McCarl's Newsletter: http://www.gams.com/maillist/newsletter.htm
GAMS User Group: http://www.gams.com/maillist/gams_l.htm
GAMS Google Group: http://groups.google.de/group/gamsworld

Other relevant sites on the Web: http://www.gams.com/hotlinks.htm
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