Exam scheduling at United States Military Academy West Point

Frederik Proske

Operations Research Analyst
GAMS Software

Agenda

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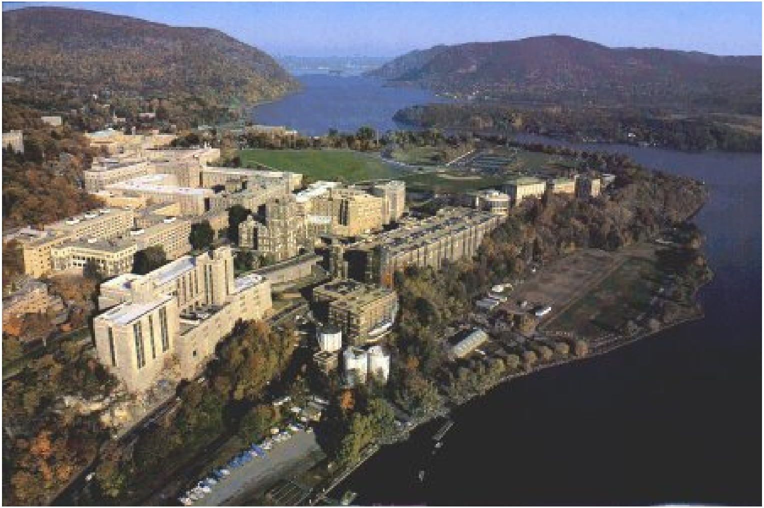
1. The Examination Timetabling Problem (ETP) at USMA West Point
2. Algorithm
3. Implementation in GAMS
4. Computational results
5. Conclusion

About USMA



Source: http://www.usma.edu

The ETP at USMA

- More than 21 000 term end exams need to be scheduled to a fixed number of 11 time slots
- Each cadet can only attend one exam per period (hard constraint)
- Number of periods is not sufficient to generate clash-free schedule
- Some courses will therefore need an extra exam version called makeup exam

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Current objectives at USMA

- Makeups also used for improving other objectives at USMA
- Multiple objectives like minimizing number of makeups or minimizing the violations of various soft constraints

→ Aim is to find a good balance between the different objectives

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Two formulations for the ETP

- Hierarchical approach: separate objective function for each objective
  - → Modules solved successively
- A linear and nonlinear IP model were formulated for each objective of the ETP
- Linear model is iteratively decomposed into smaller subproblems each solved using CPLEX
- Nonlinear model solved with local search based solver: *LocalSolver*

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Two formulations for the ETP

- Linear version of the constraints to ensure that no cadet takes more than one exam per period:

$$\sum_{r \in \mathcal{CR}_c} xy_{c,r,p} \leq 1 \qquad \forall c,p \tag{1}$$

$$x_{c,r,\underbrace{m}} + y_{r,m,p} - 1 \leq xy_{c,r,p} \qquad \forall c,r,m,p \tag{2}$$

'1': base exam
'2': makeup exam

- Nonlinear version:

$$\sum_{r \in \mathcal{CR}_c} max(0, y_{r,'1',p} - \sum_{\substack{m2 \in \mathcal{RM}_r \\ m2 \geq 2}} x_{c,r,m2}) + \sum_{r \in \mathcal{CR}_c} \sum_{\substack{m2 \in \mathcal{RM}_r \\ m2 \geq 2}} x_{c,r,m2} \cdot y_{r,m2,p} \leq 1 \qquad \forall c,p \tag{3}$$

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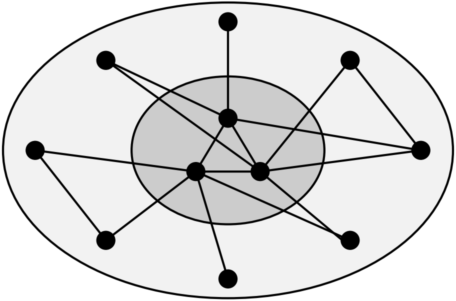
Decomposition strategies

Decomposition based on vertex degree



■ group 1 □ group 2 ● course

– common cadets

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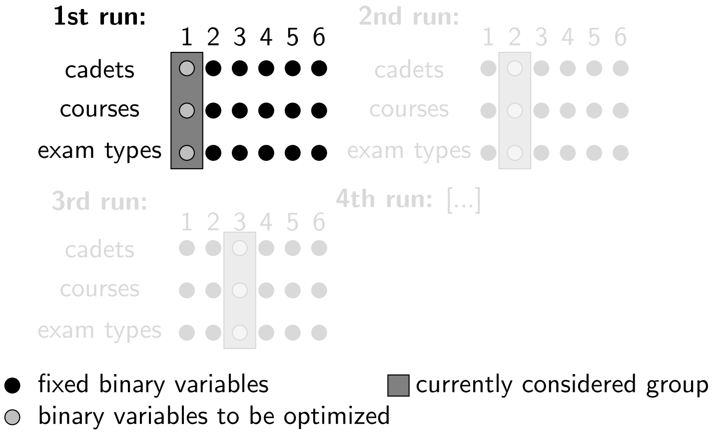

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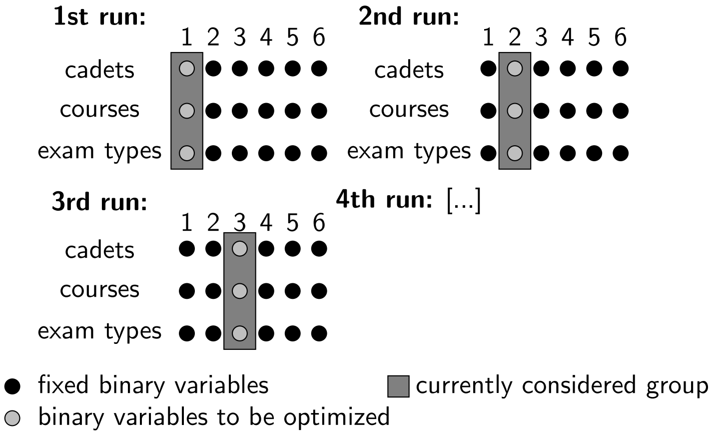
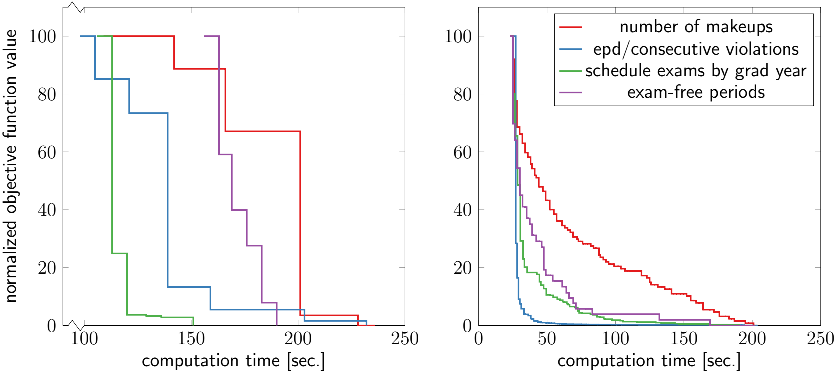


Solving the ETP

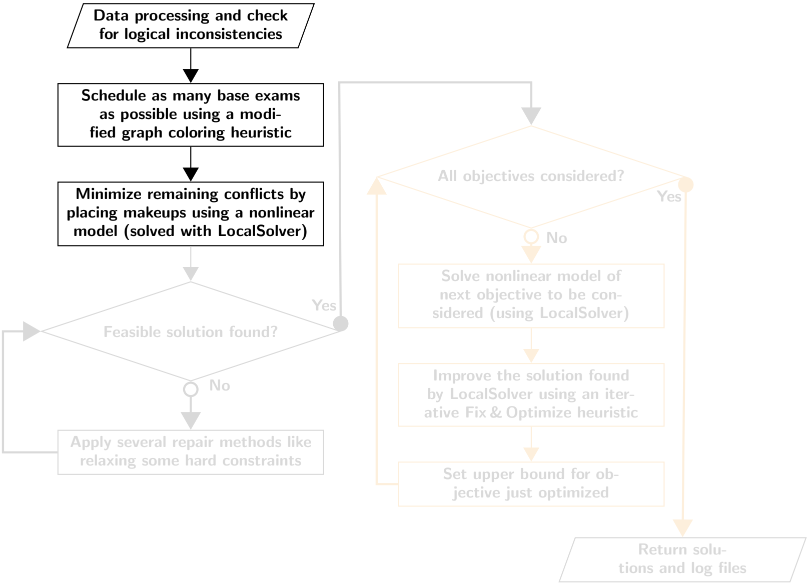
Qualitative trend of the objective function value over time for the two solution methods



Fix&Optimize heuristic

LocalSolver

The complete algorithm



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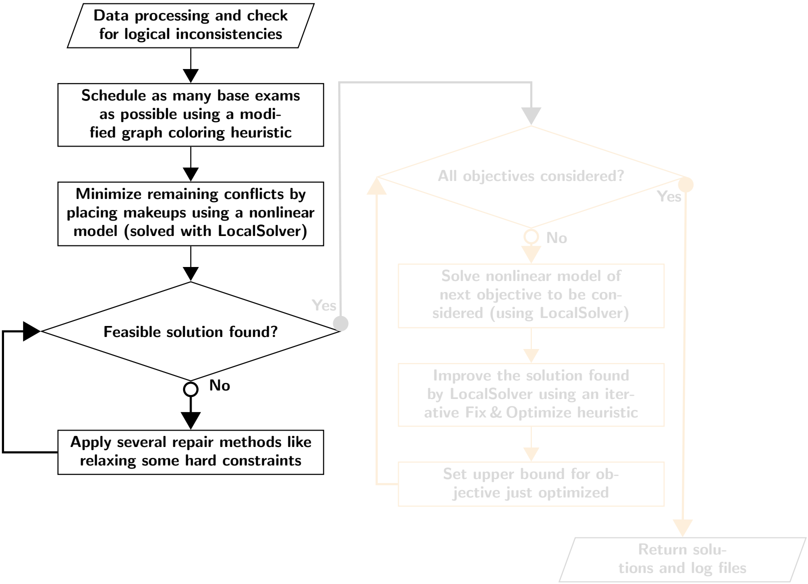

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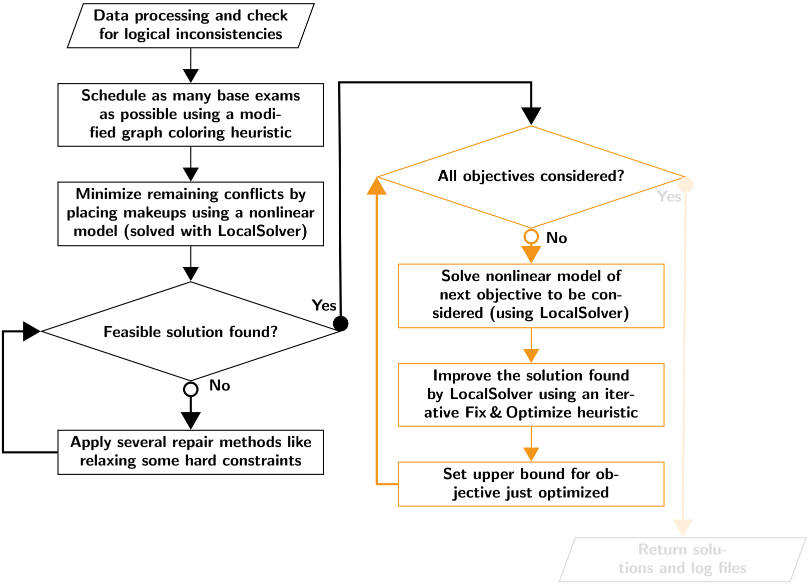

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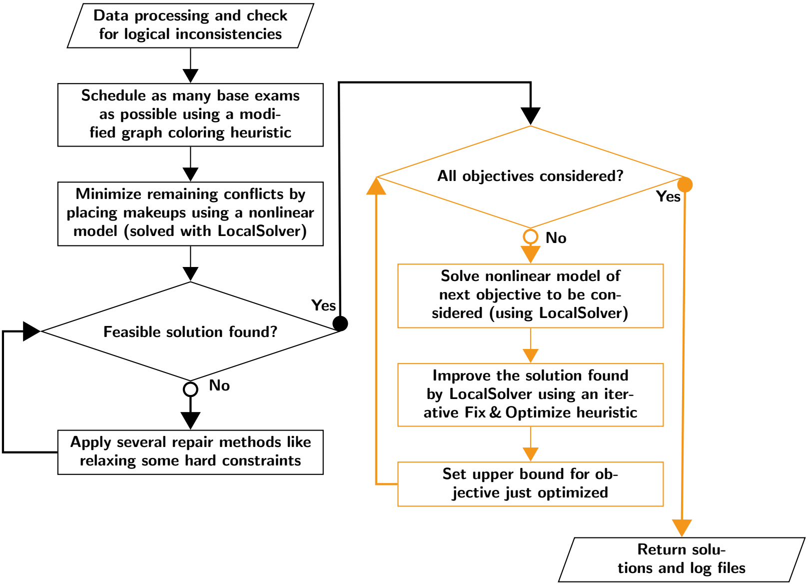

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Linear model formulation

```
noConflicts(c,p)..
   sum(cr(c,r), xy(cr,p)) =l= 1;

combineXY(cr(c,r),m,p)..
   x(cr,m) + y(r,m,p) - 1 =l= xy(cr,p);

                         ⋮

solve capacity minimizing objMaxCapVio use mip;

                         ⋮

solve makeup minimizing objMu use mip;
```

Nonlinear model formulation

```
ls_assignToPeriod(c,p)..
  sum(cr(c,r), max[0, y(r,'0',p)
                        - sum(rm(r,mx), x(cr,mx))])
  + sum((cr(c,r),mx), x(cr,mx) * y(r,mx,p))
        =l= 1;

                            ⋮

solve ls_cadetConflicts minimizing objConflicts use
   minlp;
```

Fix&Optimize heuristic

Decomposition based on vertex degree

```
loop(vdGroup,
* free all variables
  x.lo(cr(c,r),m) = 0;
  x.up(cr(c,r),m) = 1;
  y.lo(rm,p) = 0;
  y.up(rm(r,m),p)$[not prohibit(r,p)] = 1;
* fix all exams of other vdGroups
  y.fx(rm(r,m),p)$[not rSet(vdGroup,r)] = y.l(rm,p);
  solve makeup minimizing objMu use mip;
);
```

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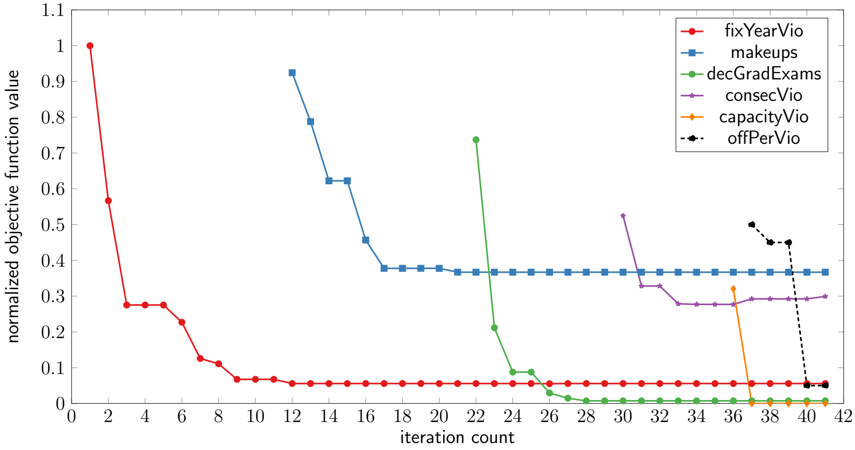
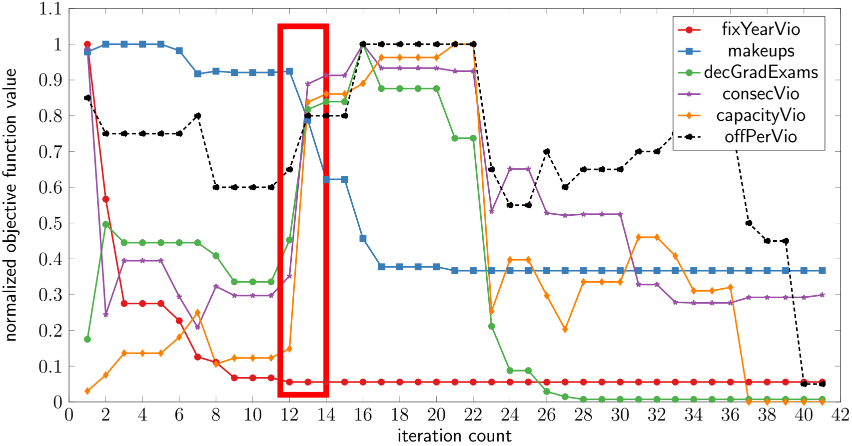
Computational results

objective functions	F&O	*LocalSolver*	combination
makeups	114	172	126
epd/consecutive violations	0	25	0
exams for December Graduates	3	0	0
cadet specific exam-free periods	1	1	0
schedule exams by grad year	389	470	425
exam-free periods	330	636	183
capacity violations	0	0	0
computation time [sec.]	1323	(884)	1156

Overview (normalized)



Conflicting objectives



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- Exam timetabling problem with a special structure
- Algorithm based on decomposition (divide and conquer) as well as local search techniques
- Local search (in particular *LocalSolver*) consistent in finding initial solution (no hard constraints violated)
- Fix&Optimize heuristic for improving solution by exploiting problem structure (MIPs solved using CPLEX)

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- Model formulation in GAMS similar to algebraic notation
- Switching between solvers with one line of code (currently more than 25 solvers supported)
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