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# High Performance Computing with GAMS

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GAMS Software GmbH

## A PROJECT BY



Deutsches Zentrum  
für Luft- und Raumfahrt  
German Aerospace Center



- Introduction
- Model Annotation
- Distributed Model Generation
- Outlook

# Introduction

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## Algebraic Modeling Language

Facilitates to formulate mathematical optimization problems similar to algebraic notation

→ Simplified model building:

*Model is executable algebraic description of optimization problem.*

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$$\begin{aligned}
 & \sum_{p \in P: rp_{r,p}} \text{POWER}_{t,r,p} \\
 + & \sum_{r2 \in R: net_{r2,r}} (\text{FLOW}_{t,r2,r}) - \sum_{r2: net_{r,r2}} \text{FLOW}_{t,r,r2} \\
 + & \sum_{s \in S: rs_{r,s}} (\text{STORAGE\_OUTFLOW}_{t,r,s} - \text{STORAGE\_INFLOW}_{t,r,s}) \geq \text{demand}_{t,r} \quad \forall t \in T, r \in R
 \end{aligned}$$

```

eq_power_balance(t,r) ..
    sum(rp(r,p),    POWER(t,r,p))
+ sum(net(r2,r),  FLOW(t,net)) - sum(net(r,r2), FLOW(t,net))
+ sum(rs(r,s),    STORAGE_OUTFLOW(t,r,s) - STORAGE_INFLOW(t,r,s)) =g= demand(t,r);
    
```

## Algebraic Modeling Language

Facilitates to formulate mathematical optimization problems similar to algebraic notation

→ Simplified model building

### **Declarative elements**

- **Similar to mathematical notation**
- **Easy to learn - few basic language elements: sets, parameters, variables, equations, models**
- **Model is executable (algebraic) description of the problem**

## Algebraic Modeling Language

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### **Declarative elements**

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- **Model is executable (algebraic) description of the problem**

### **Procedural elements**

- **Control Flow Statements (e.g. loops, for, if,...),**
- **Build complex problem algorithms within GAMS**
- **Simplified interaction with other systems**
  - Data exchange
  - GAMS process control

- Broad Range of application areas

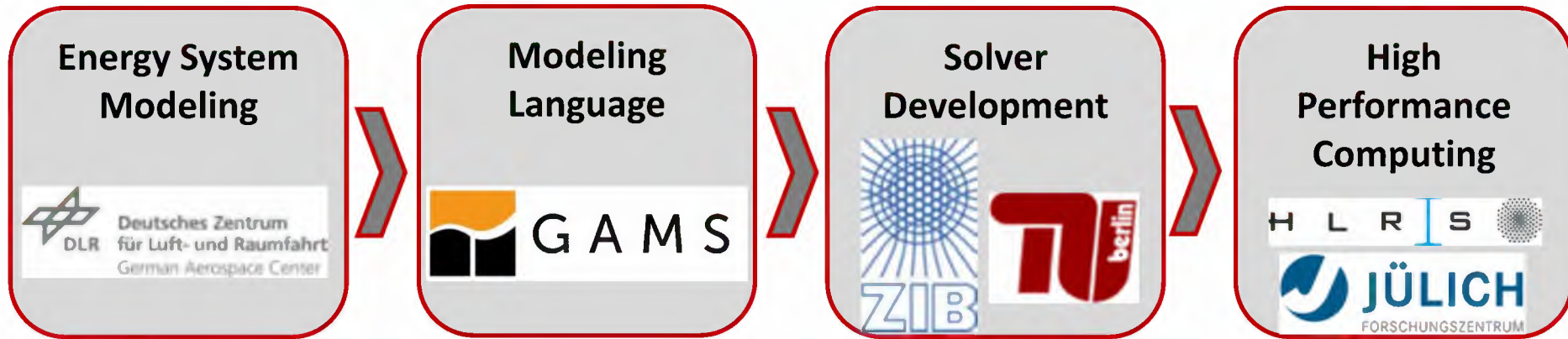
<b>Agricultural Economics</b>	<b>Applied General Equilibrium</b>
<b>Chemical Engineering</b>	<b>Economic Development</b>
<b>Econometrics</b>	<b>Energy</b>
<b>Environmental Economics</b>	<b>Engineering</b>
<b>Finance</b>	<b>Forestry</b>
<b>International Trade</b>	<b>Logistics</b>
<b>Macro Economics</b>	<b>Military</b>
<b>Management Science/OR</b>	<b>Mathematics</b>
<b>Micro Economics</b>	<b>Physics</b>



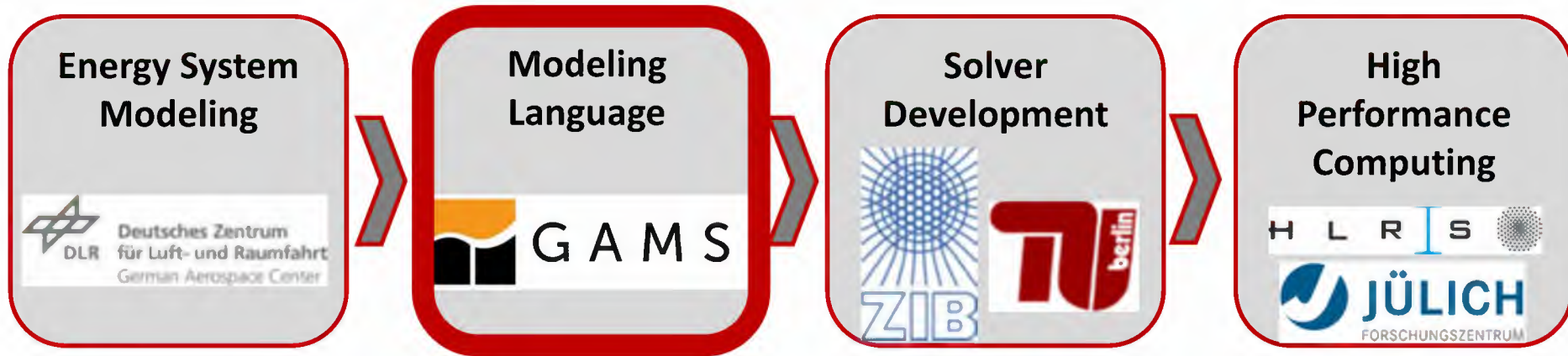
- Broad Range of application areas

Agricultural Economics	Applied General Equilibrium
Chemical Engineering	Economic Development
Econometrics	Energy
Environmental Economics	Engineering
Finance	Forestry
International Trade	Logistics
Macro Economics	Military
Management Science/OR	Mathematics
Micro Economics	Physics

- GAMS is widespread in the ESM community:  
<http://www.energyplan.eu/othertools/>

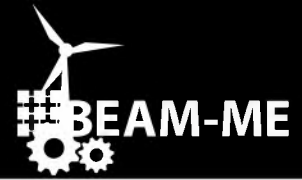


*Goal: Implementation of acceleration strategies from mathematics and computational sciences for optimizing energy system models*



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# Limitations of “standard” Soft- & Hardware



#t	#r	#blocks	#rows (E6)	#cols (E6)	#NZ (E6)	~Mem (GB)	time
730	10	10	0.7	0.8	2.8	2.0	00:01:22
730	10	500	35.0	38.7	142.8	95.7	01:09:36
730	10	2,500	175.3	193.5	713.9	478.8	09:32:55
730	10	4,000	280.5	309.6	1,142.2	767.1	19:22:55
730	10	7,500	526.1	580.5	2,141.2	~1,436.4	–
8,760	10	10	8.4	9.3	34.3	18.2	00:28:57
8,760	10	50	42.1	46.4	171.6	90.4	02:26:25
...							

Test runs were made on JURECA @ JSC

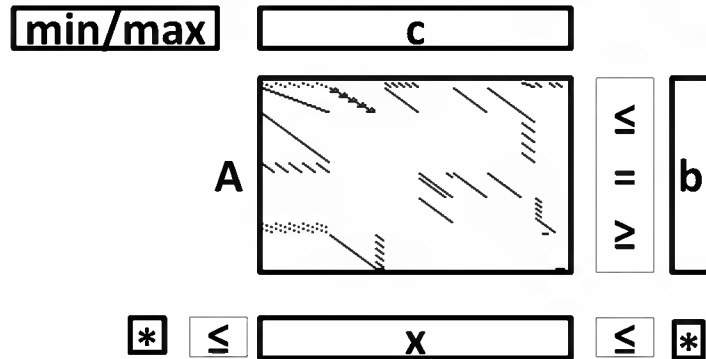
- 2x Intel Xeon E5-2680 v3 (Haswell), 2 x 12 cores @ 2.5GHz
- “fat” node with 1,024 GB Memory
- GAMS 24.8.5 / CPLEX 12.7.1.0
- Barrier Algorithm, Crossover disabled, 24 threads

# Model Annotation

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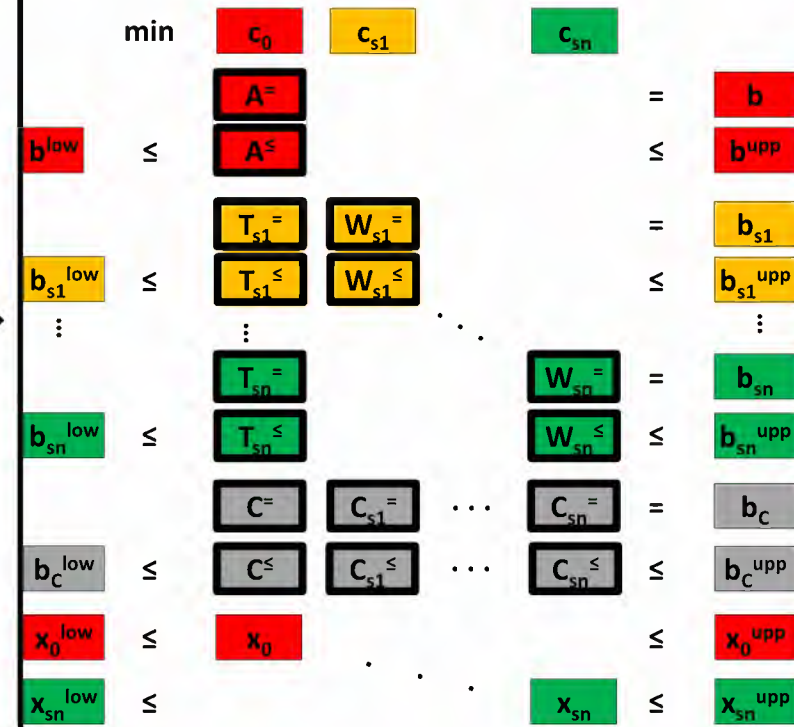
# Model Annotation

Original problem with “random” matrix structure



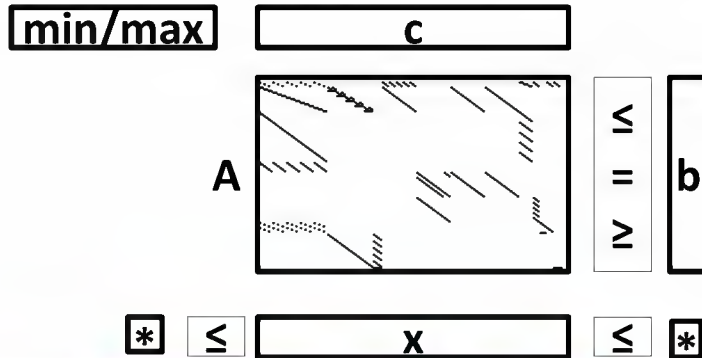
How to  
get  
there?

PIPS exploits matrix block structure



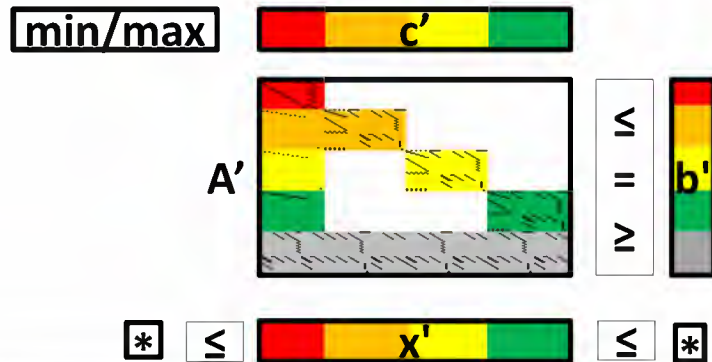
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Original problem with “random” matrix structure

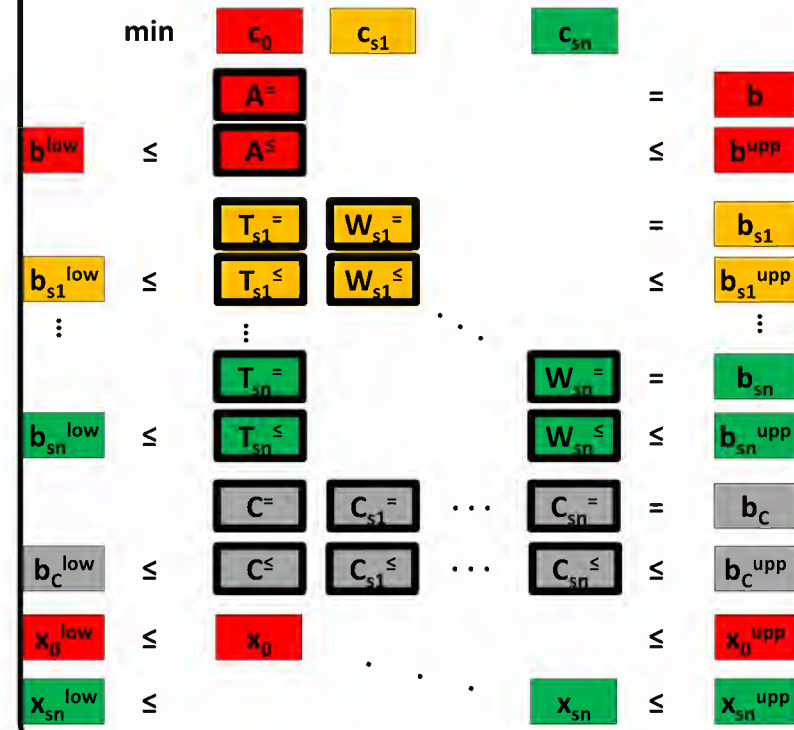


Model  
annotation

Permutation reveals block structure



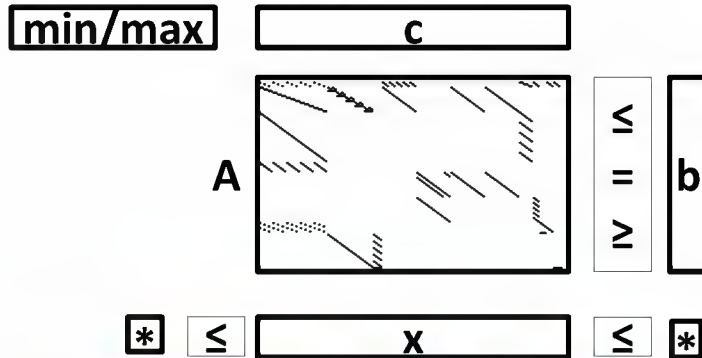
PIPS exploits matrix block structure





# Model Annotation

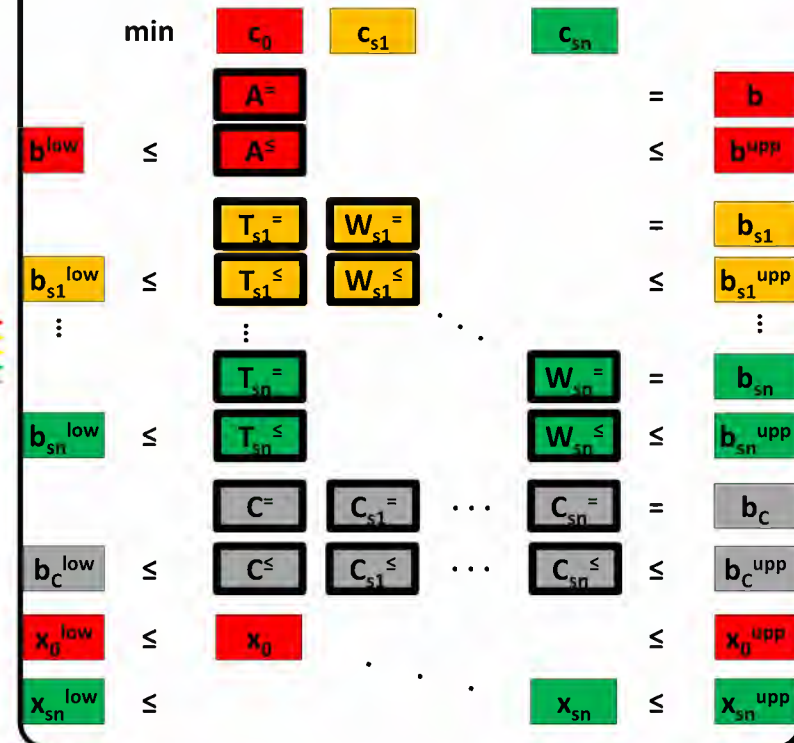
Original problem with “random” matrix structure



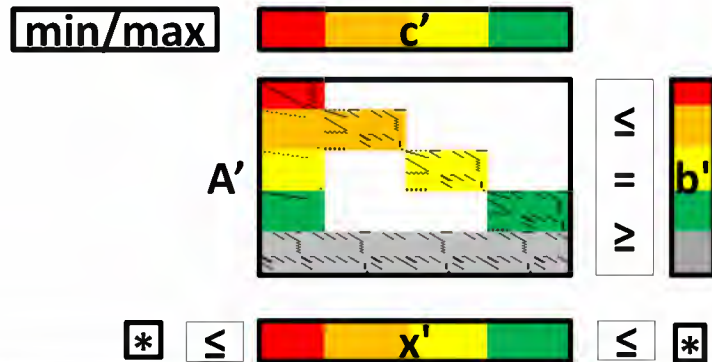
Model  
annotation

Model generation

PIPS exploits matrix block structure



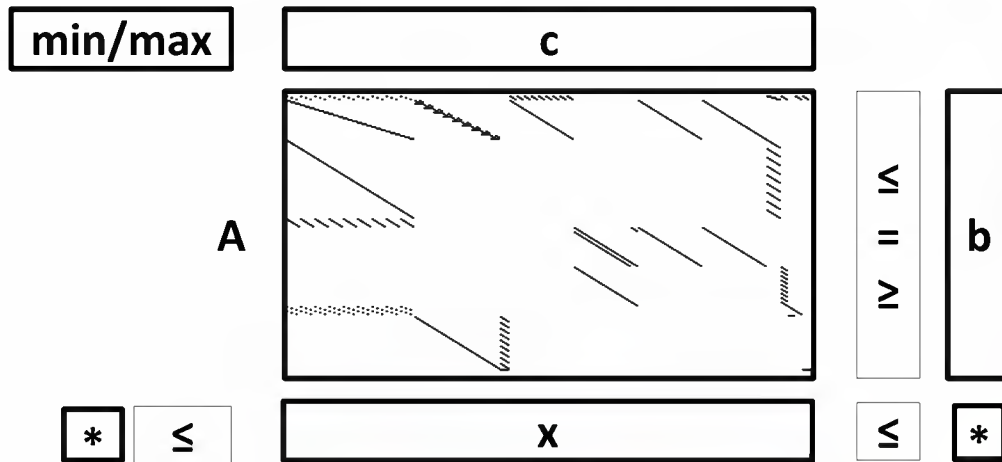
Permutation reveals block structure



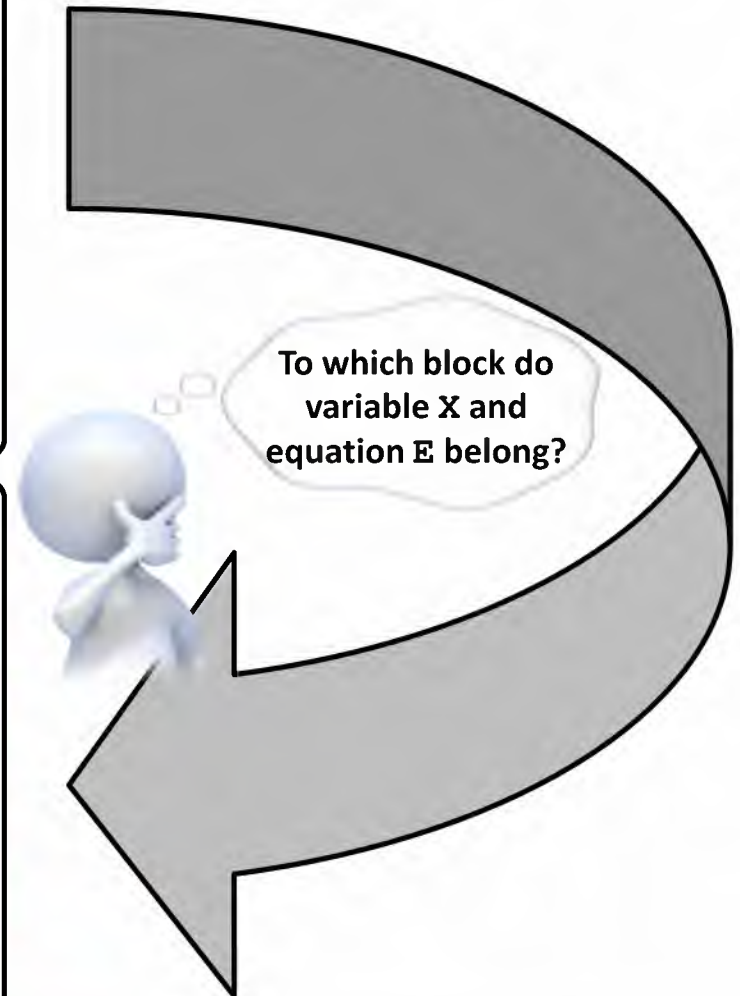
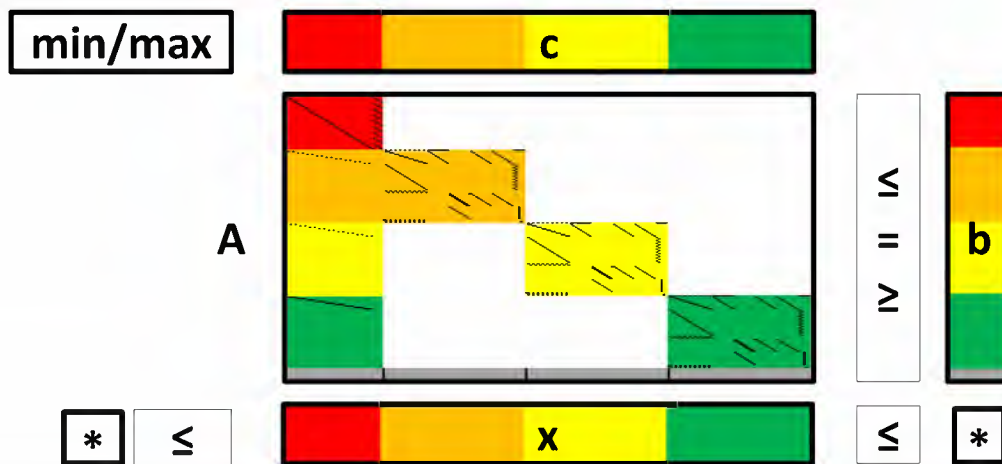


# Model Annotation cont.

Original problem with “random” matrix structure



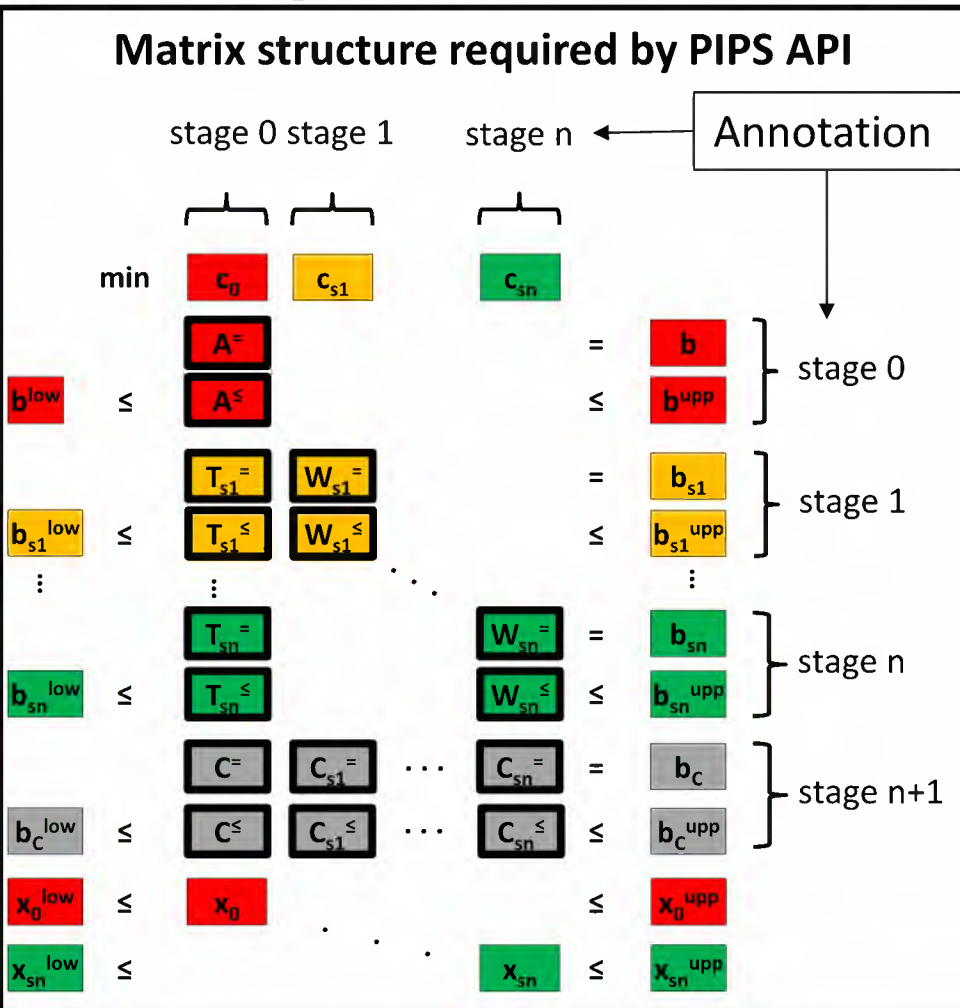
Permutation reveals block structure required by PIPS API



# Model Annotation cont.

## Model Annotation by .Stage

- The .stage attribute is available for variables/equations in GAMS



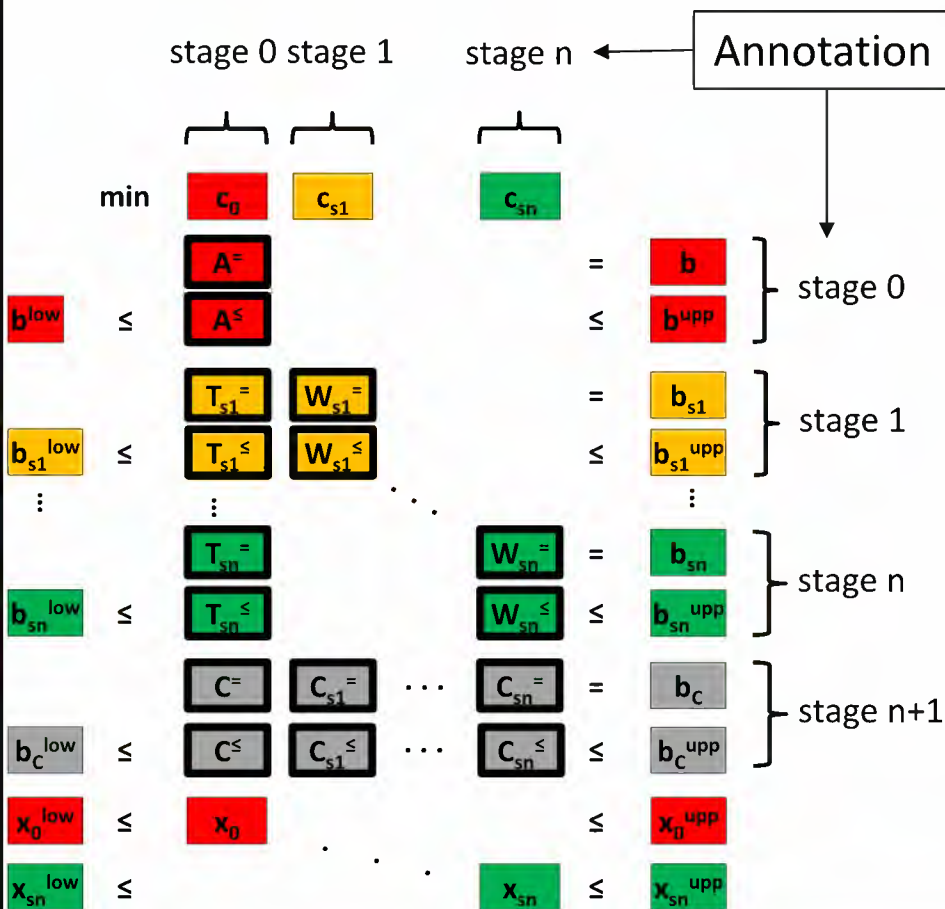
# Model Annotation cont.



## Model Annotation by .Stage

- The .stage attribute is available for variables/equations in GAMS

### Matrix structure required by PIPS API



### Exemplary Annotation for SIMPLE model (regional decomposition)

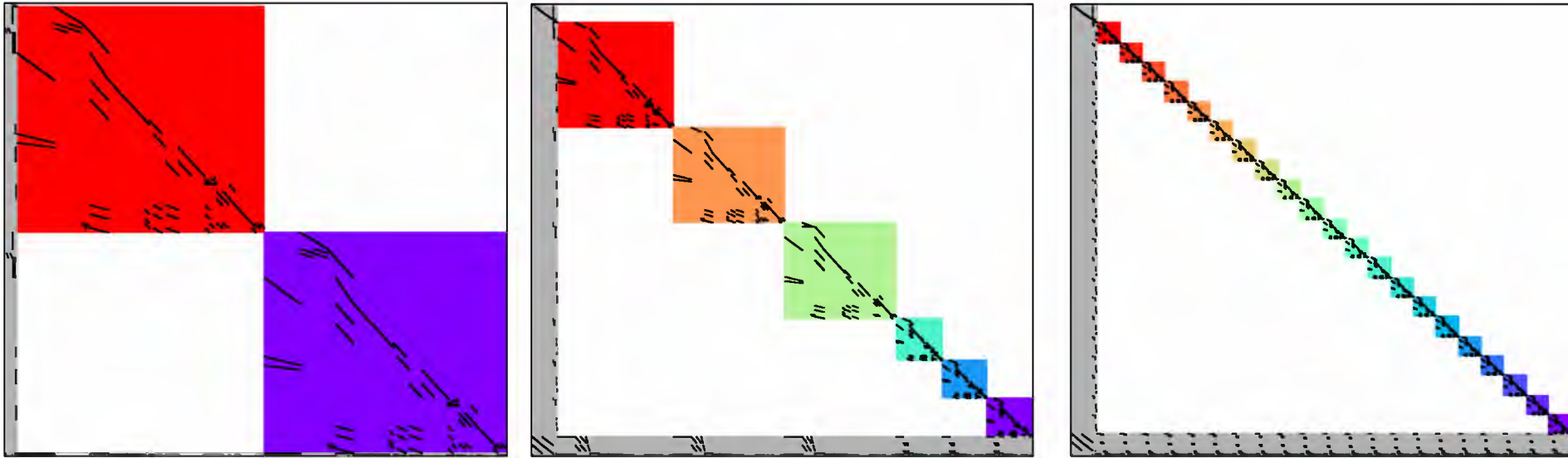
```

Set rr          'regions'
p            'plants'
tt          'time steps'
e          'emissions'
t(tt)      'subset of active time steps'
rp(rr,p)   'region to plant mapping'
net(rr,rr) 'transmission links'

;
Alias(rr,rr1,rr2,rr3);
[...]
* Master variables and equation
FLOW.stage(t,net(rr1,rr2)) = 0;
LINK_ADD_CAP.stage(net(rr1,rr2)) = 0;
[...]
* Block variables and equations
POWER.stage(t,rp(rr3,p)) = ord(rr);
EMISSION.stage(rr3,e) = ord(rr);
[...]
eq_power_balance.stage(t,r) = ord(rr);
eq_emission_region.stage(rr3,e) = ord(rr);
[...]
* Linking Equation
eq_emission_cap.stage(e) = card(rr)+1;
    
```

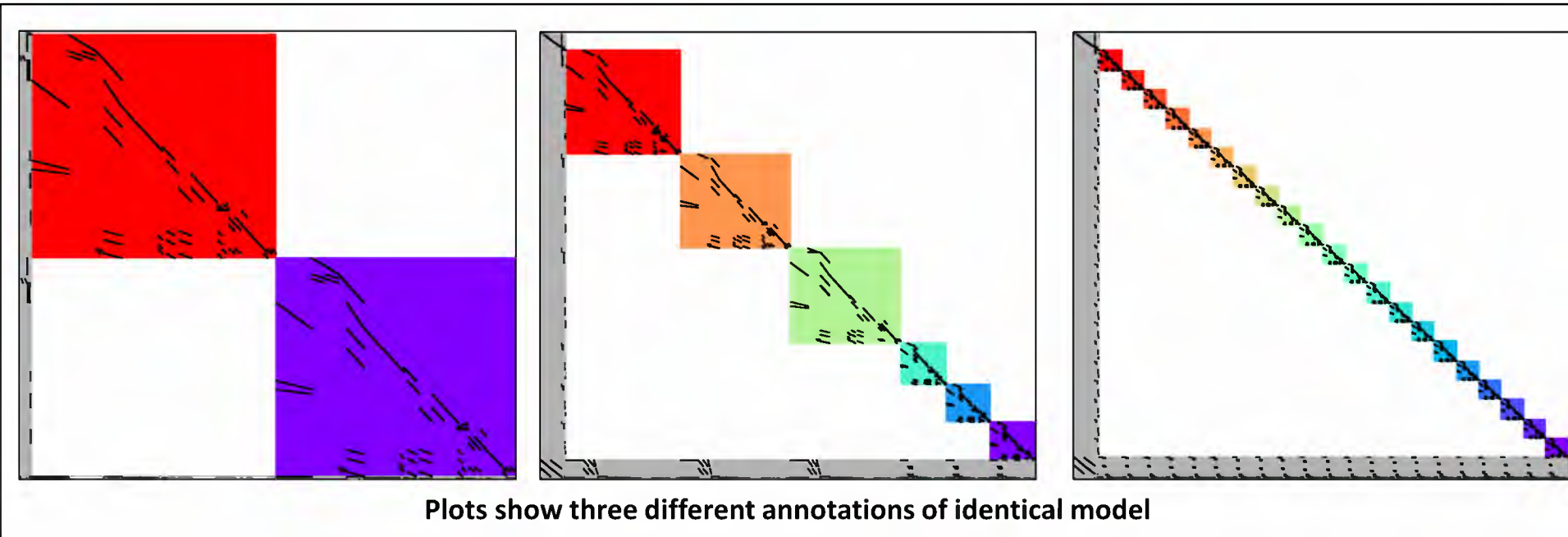
- How to annotate Model depends on how the model should be “decomposed” (by region, time,...)

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Plots show three different annotations of identical model

- How to annotate Model depends on how the model should be “decomposed” (by region, time,...)



- How important are blocks of equal size?

# Distributed Model Generation

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- “Usual Model”: model generation time  $\ll$  solver time
  - For LARGE-scale models the model generation may become significant:
    - due to time consumption
    - due to memory consumption
    - due to hard coded limitations of model size (# non-zeroes  $< \sim 2.1e9$ )
- Distributed “block-wise” model setup in PIPS-IPM
- Model annotation determines block membership of all variables and constraints
- Distributed GAMS processes can generate the separate blocks (model needs to be prepared accordingly!)



# Distributed Model Generation

Consider LP with block-diagonal structure, linking constraints, and linking variables (the kind of problem we want to solve):

	min	$c_0$	$c_{s1}$		$c_{sn}$	
		$A^=$				$=$ $b$
$b^{low}$	$\leq$	$A^{\leq}$				$\leq$ $b^{upp}$
		$T_{s1}^=$	$W_{s1}^=$			$=$ $b_{s1}$
$b_{s1}^{low}$	$\leq$	$T_{s1}^{\leq}$	$W_{s1}^{\leq}$			$\leq$ $b_{s1}^{upp}$
$\vdots$		$\vdots$	$\ddots$			$\vdots$
		$T_{sn}^=$			$W_{sn}^=$	$=$ $b_{sn}$
$b_{sn}^{low}$	$\leq$	$T_{sn}^{\leq}$			$W_{sn}^{\leq}$	$\leq$ $b_{sn}^{upp}$
		$C^=$	$C_{s1}^=$	$\dots$	$C_{sn}^=$	$=$ $b_c$
$b_c^{low}$	$\leq$	$C^{\leq}$	$C_{s1}^{\leq}$	$\dots$	$C_{sn}^{\leq}$	$\leq$ $b_c^{upp}$
$x_0^{low}$	$\leq$	$x_0$				$\leq$ $x_0^{upp}$
$x_{sn}^{low}$	$\leq$				$x_{sn}$	$\leq$ $x_{sn}^{upp}$

# Distributed Model Generation

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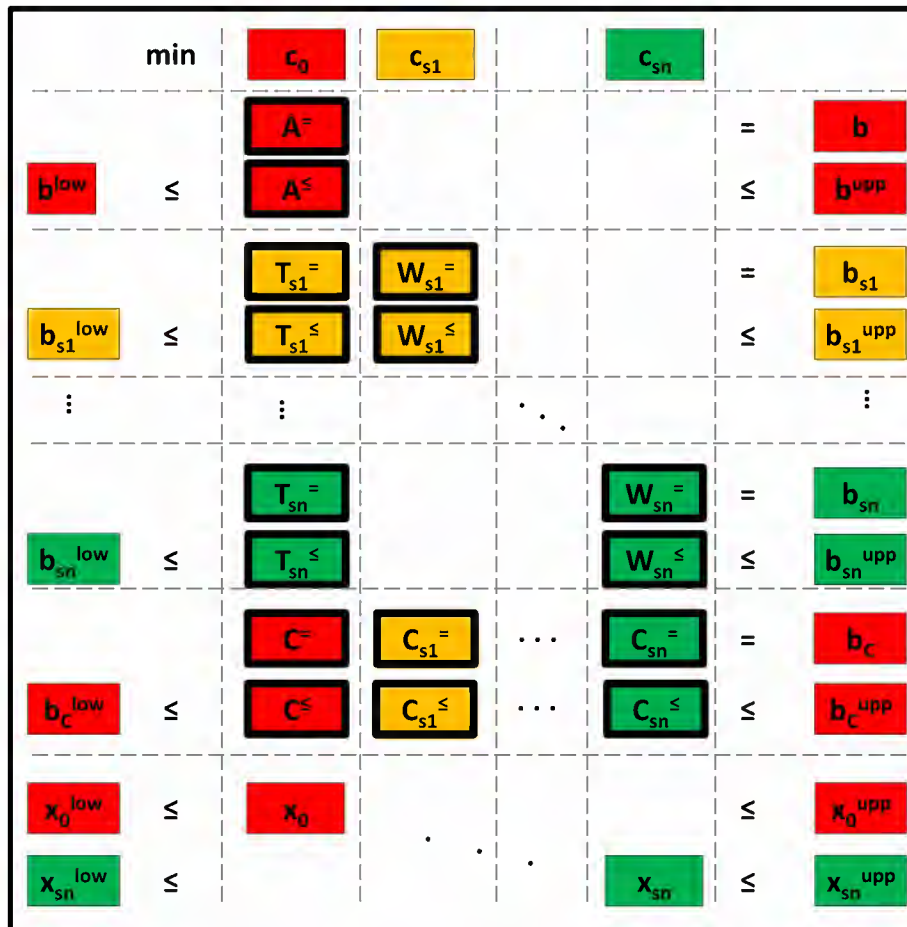
	min	$c_0$	$c_{s1}$		$c_{sn}$	
		$A^=$				$b$
$b^{low}$	$\leq$	$A^{\leq}$				$b^{upp}$
		$T_{s1}^=$	$W_{s1}^=$			$b_{s1}$
$b_{s1}^{low}$	$\leq$	$T_{s1}^{\leq}$	$W_{s1}^{\leq}$			$b_{s1}^{upp}$
$\vdots$		$\vdots$		$\ddots$		$\vdots$
		$T_{sn}^=$			$W_{sn}^=$	$b_{sn}$
$b_{sn}^{low}$	$\leq$	$T_{sn}^{\leq}$			$W_{sn}^{\leq}$	$b_{sn}^{upp}$
		$C^=$	$C_{s1}^=$	$\dots$	$C_{sn}^=$	$b_c$
$b_c^{low}$	$\leq$	$C^{\leq}$	$C_{s1}^{\leq}$	$\dots$	$C_{sn}^{\leq}$	$b_c^{upp}$
$x_0^{low}$	$\leq$	$x_0$				$x_0^{upp}$
$x_{sn}^{low}$	$\leq$				$x_{sn}$	$x_{sn}^{upp}$

Parallel generation of  $n+1$  blocks  
&  
Distribution to MPI processes as  
needed by PIPS -IPM

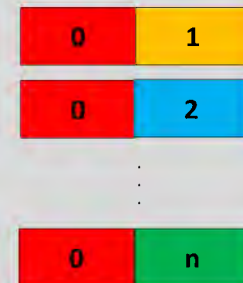
0	1
0	2
$\vdots$	
0	$n$

# Distributed Model Generation

Consider LP with block-diagonal structure, linking constraints, and linking variables (the kind of problem we want to solve):



Parallel generation of  $n+1$  blocks  
&  
Distribution to MPI processes as  
needed by PIPS -IPM



→ Time to generate  $n+1$  blocks in parallel  $\ll$  time to generate monolithic model

# Outlook

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- Model generation and solution are currently separated
  - Integrate those steps into one user friendly process
  - Better user control of GAMS/PIPS
    - options (algorithmic, limits, tolerances)
- Analyze IO bottlenecks (generation/solving)
- Annotation can be adapted for other Decomposition approaches (e.g. CPLEX Benders)
- GAMS-MPI/Embedded Code:
  - Implementation of Benders Decomposition in GAMS for ESM using the GAMS embedded code facility with Python package mpi4py to work with MPI (see talk of L. Westermann, WC-02)

<sup>1</sup> [https://www.gams.com/latest/docs/UG\\_EmbeddedCode.html](https://www.gams.com/latest/docs/UG_EmbeddedCode.html)

# Project BEAM-ME



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