GAMS

Leibniz Universität Hannover

Exam scheduling at United States Military Academy West Point

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OR 2017 BERLIN

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Agenda

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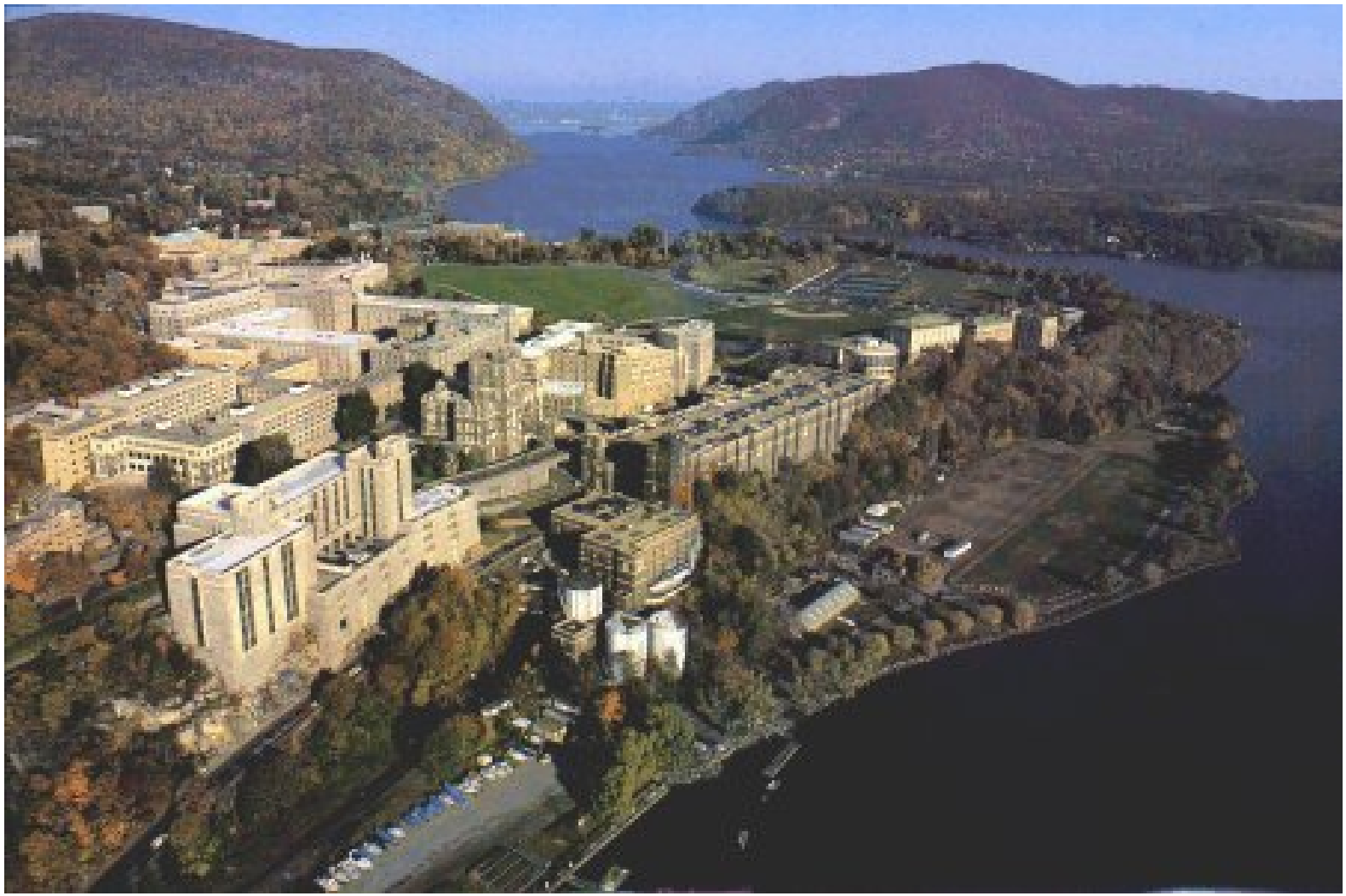
About USMA



Source: http://www.usma.edu

The ETP at USMA

- More than $21\,000$ term end exams need to be scheduled to a fixed number of 11 time slots
- Each cadet can only attend one exam per period (hard constraint)
- Number of periods is not sufficient to generate clash-free schedule
- Some courses will therefore need an extra exam version called makeup exam

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Current objectives at USMA

- Makeups also used for improving other objectives at USMA
- Multiple objectives like minimizing number of makeups or minimizing the violations of various soft constraints

→ Goal is to find a good balance amongst different objectives

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Two formulations for the ETP

- Hierarchical approach: separate objective function for each objective
  - $\rightarrow$ Modules solved successively
- A linear and nonlinear IP model were formulated for each objective of the ETP
- Linear model is iteratively decomposed into smaller subproblems each solved using CPLEX
- Nonlinear model solved with local search based solver: LocalSolver

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Two formulations for the ETP

- Linear version of the constraints to ensure that no cadet takes more than one exam per period:

$$\sum_{rr \in \mathcal{CR}_{cc}} xy_{cc,rr,pp} \leq 1 \qquad \forall cc, pp \tag{1}$$

$$x_{cc,rr,\underbrace{mm}} + y_{rr,mm,pp} - 1 \leq xy_{cc,rr,pp} \qquad \forall cc, rr, mm, pp \tag{2}$$

'1': base exam

'2': makeup exam

- Nonlinear version:

$$\sum_{rr \in \mathcal{CR}_{cc}} max(0, y_{rr,'1',pp} - \sum_{\substack{mm2 \in \mathcal{RM}_{rr} \\ mm2 \geq 2}} x_{cc,rr,mm2}) + \sum_{rr \in \mathcal{CR}_{cc}} \sum_{\substack{mm2 \in \mathcal{RM}_{rr} \\ mm2 \geq 2}} x_{cc,rr,mm2} \cdot y_{rr,mm2,pp} \leq 1 \qquad \forall cc, pp \tag{3}$$

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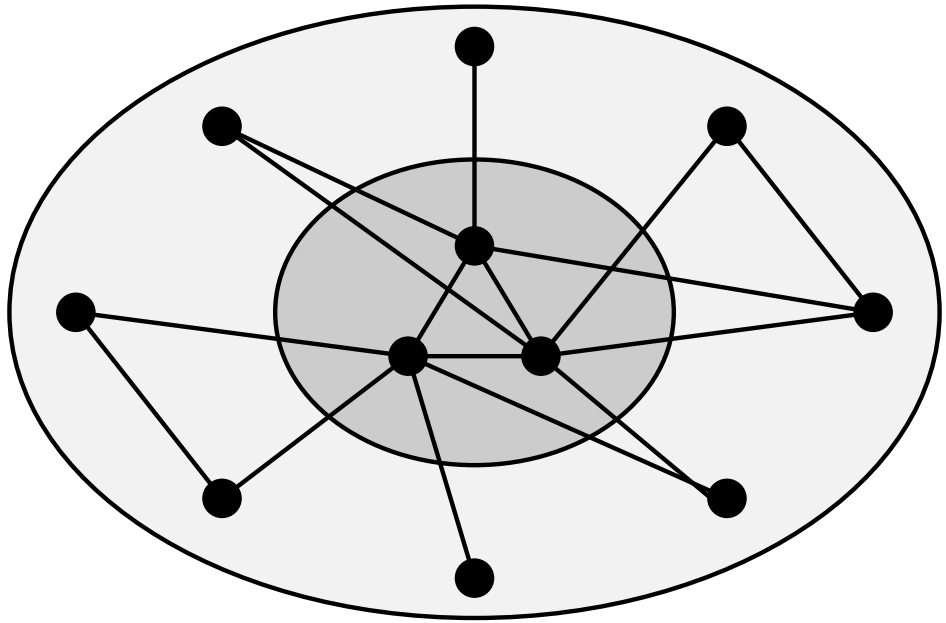
Decomposition strategies

Decomposition based on vertex degree



group 1 group 2 course

common cadets

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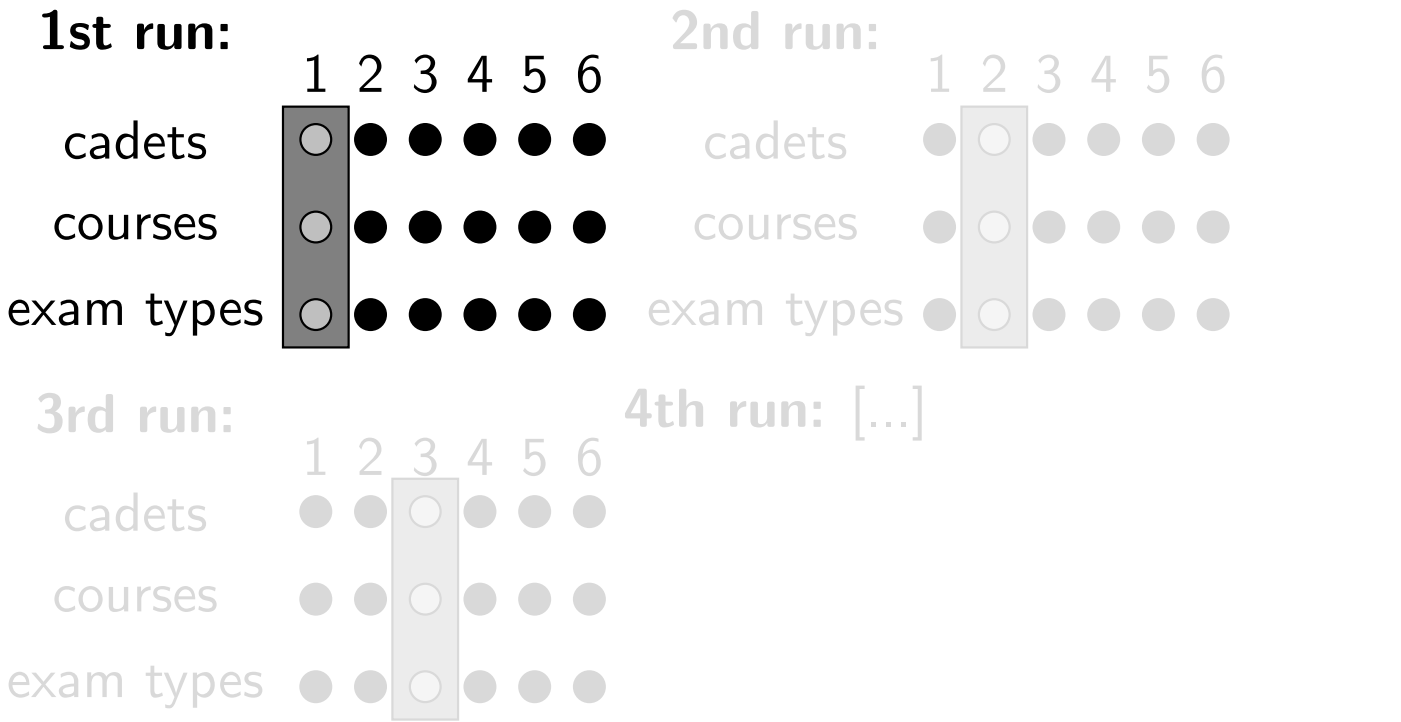

- ● fixed binary variables
- ○ binary variables to be optimized
- ■ currently considered group

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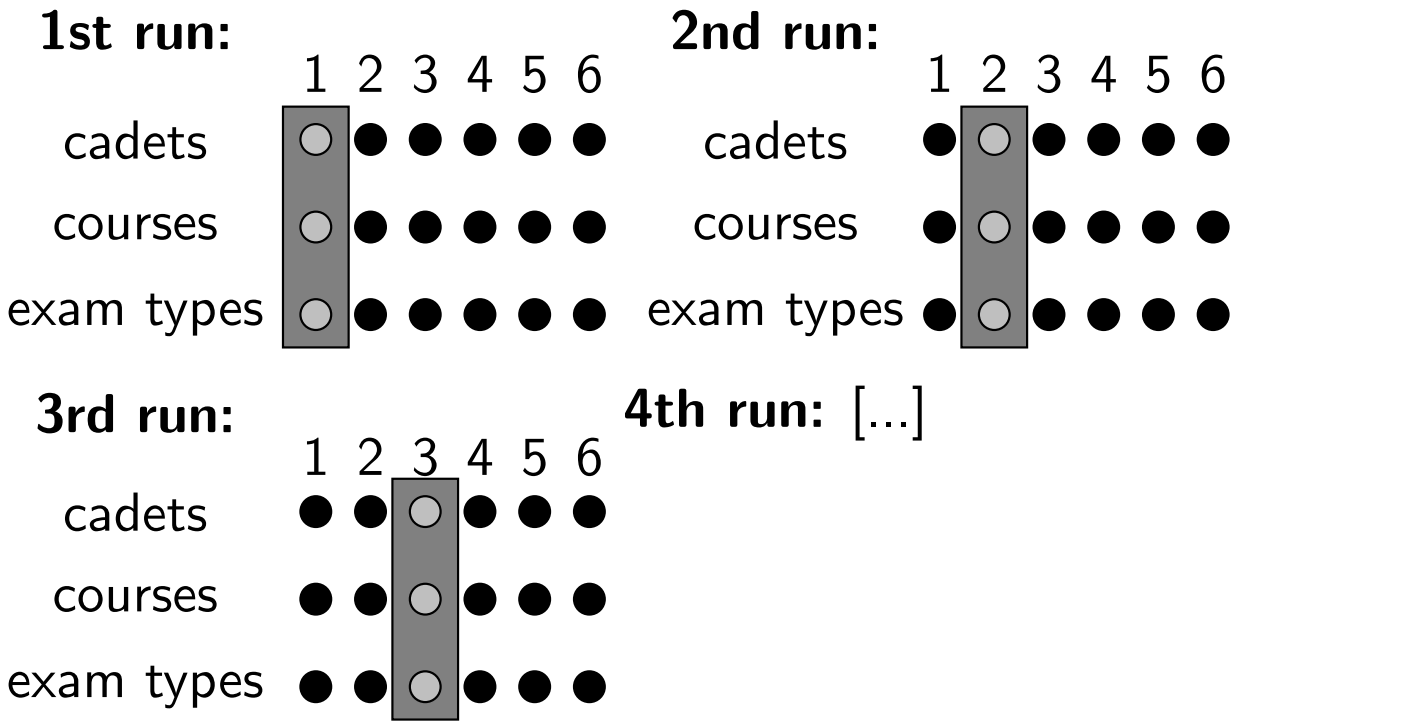

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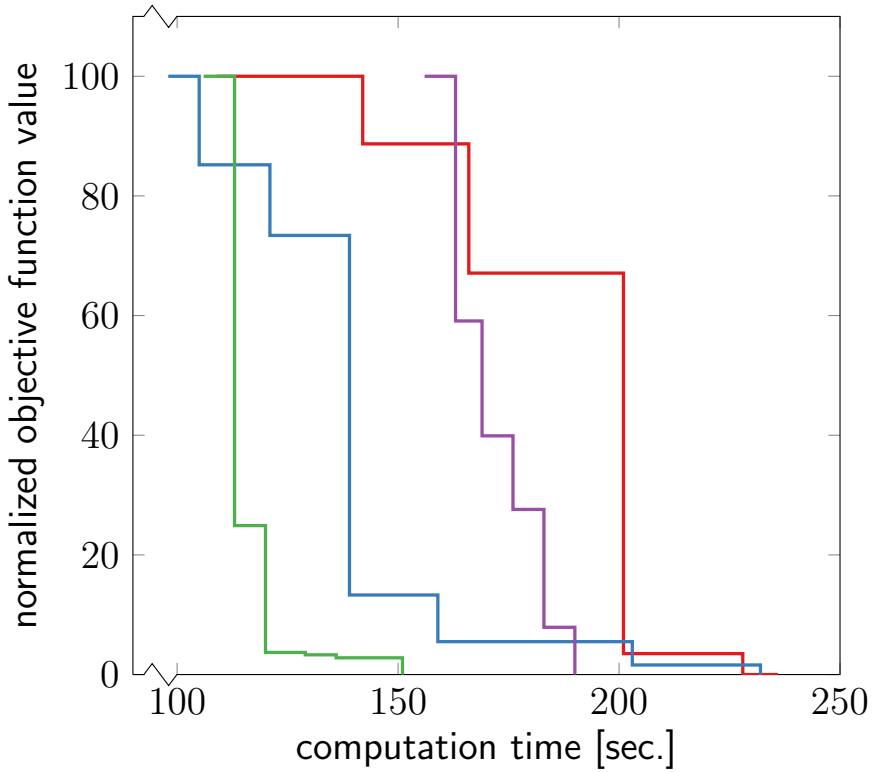
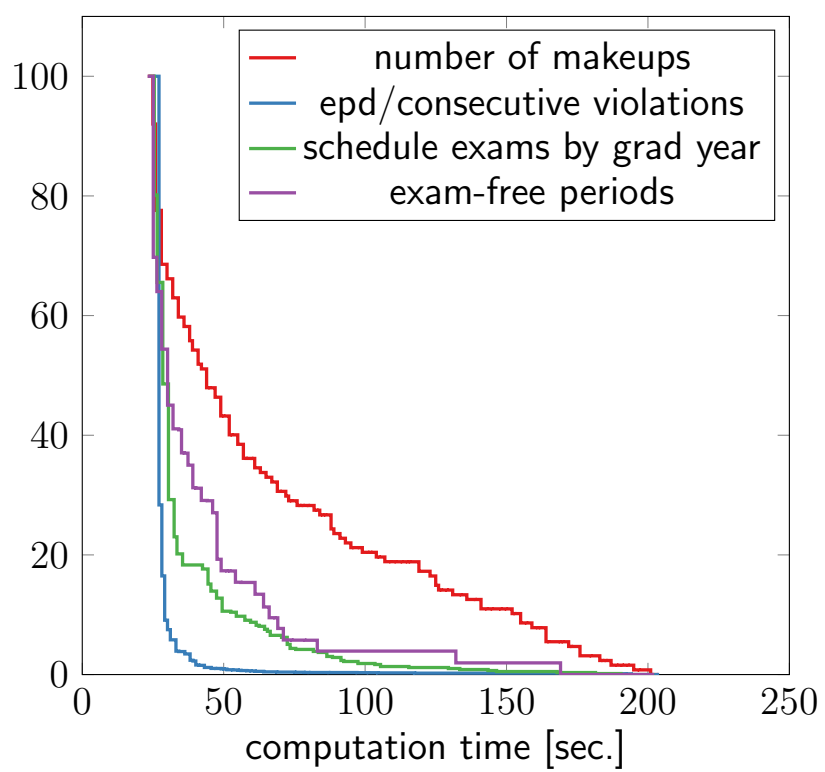
Solving the ETP

Qualitative trend of the objective function value over time for the two solution methods



Fix&Optimize heuristic



LocalSolver

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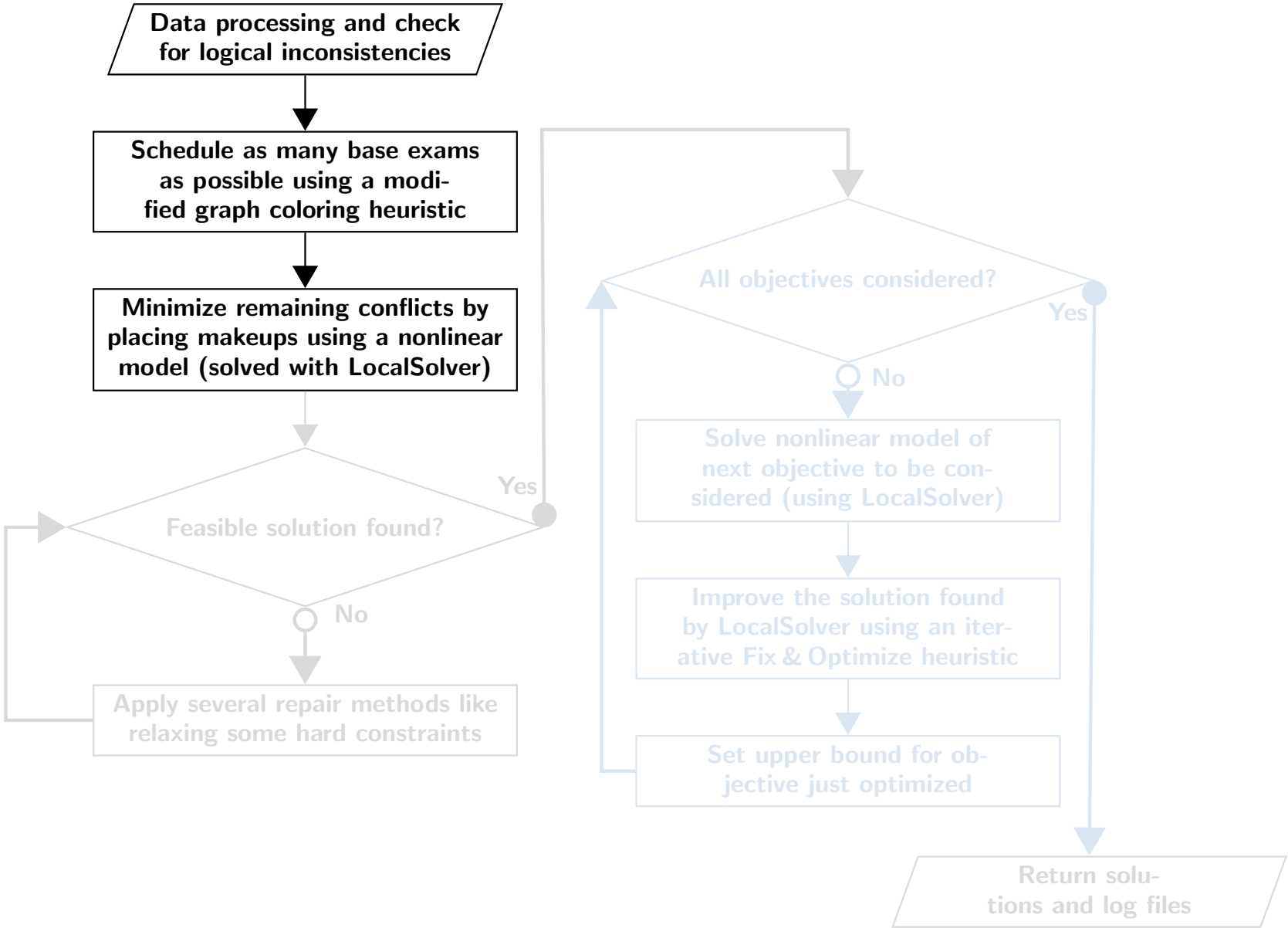
The complete algorithm



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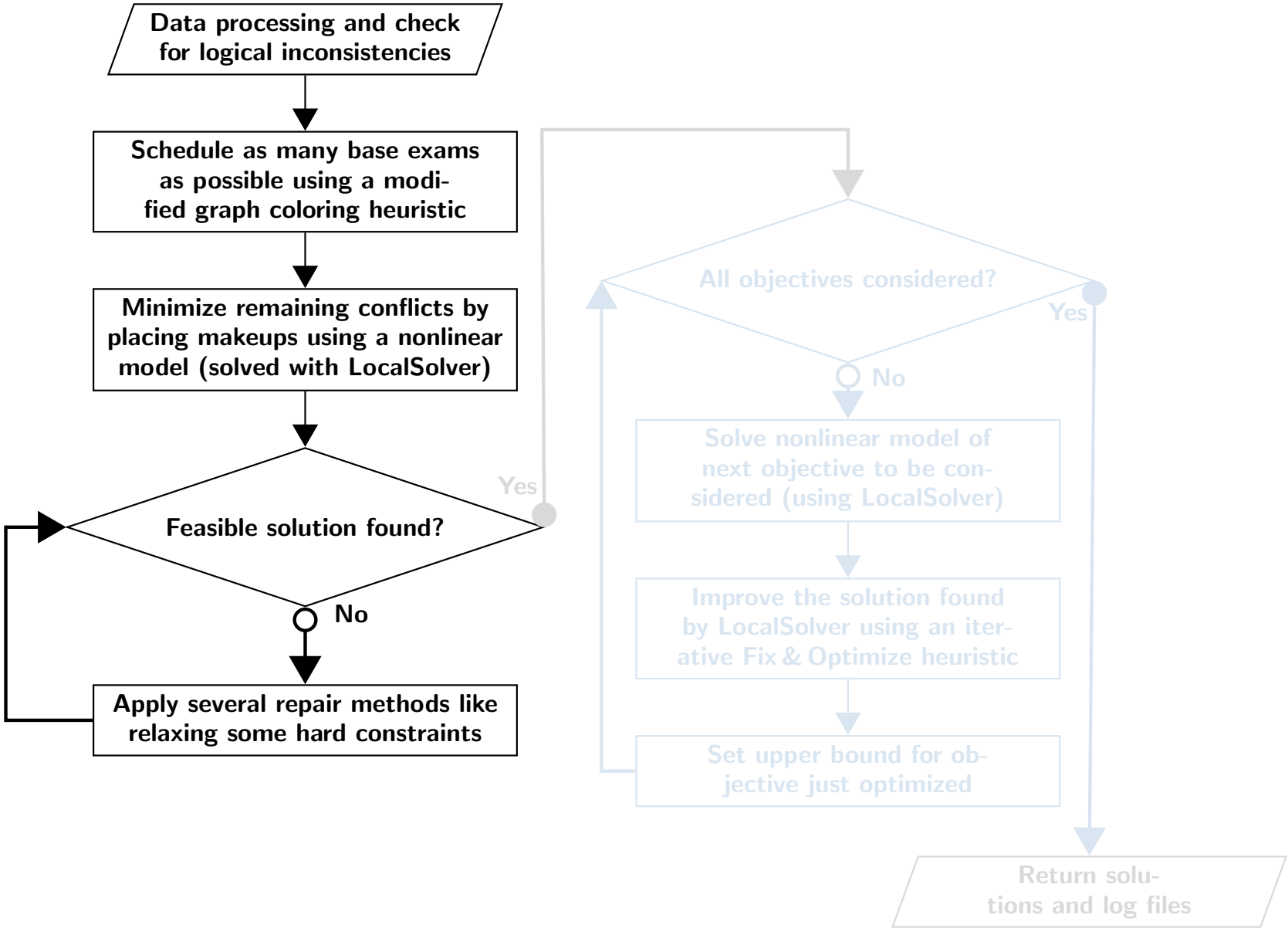

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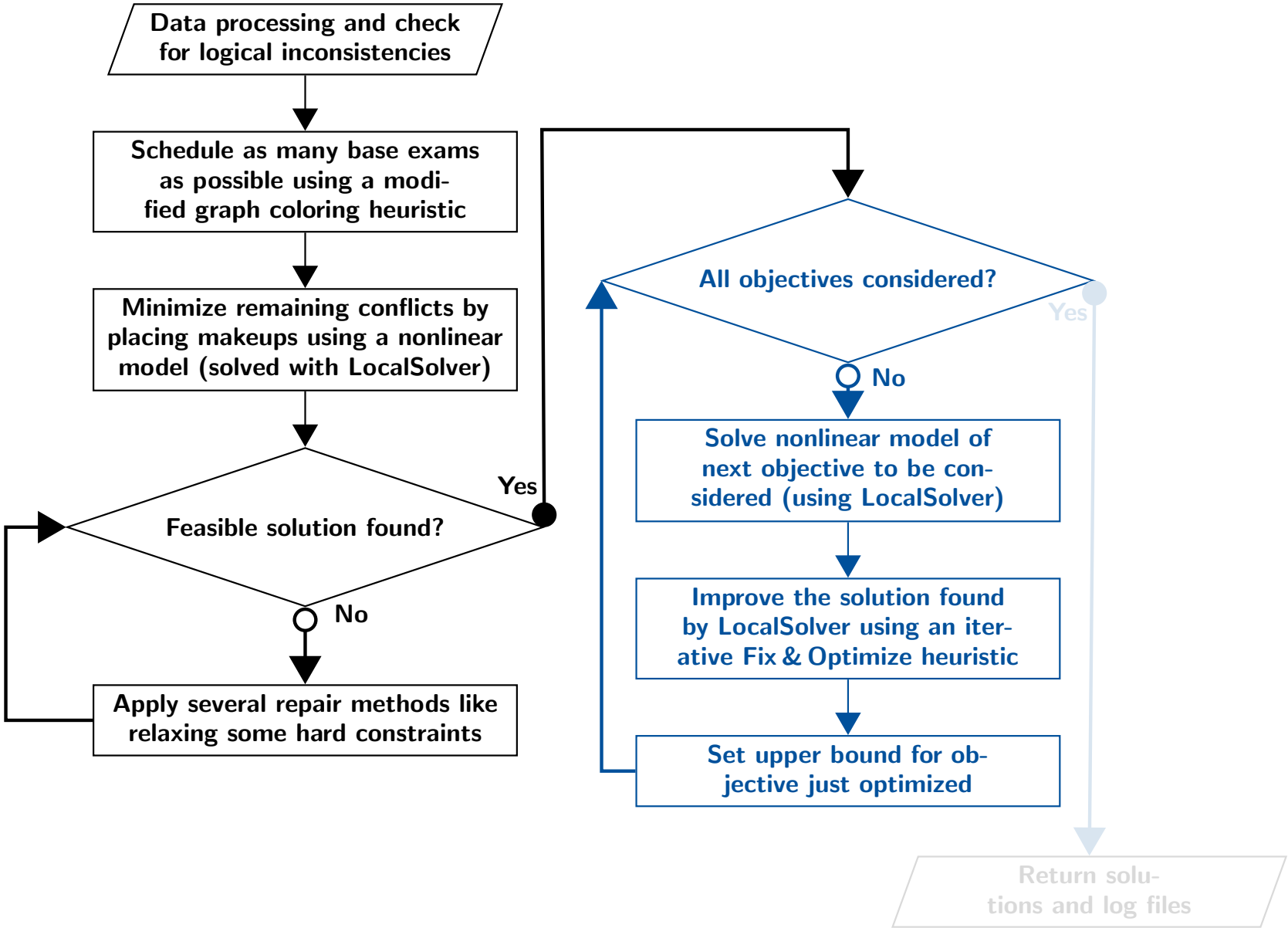

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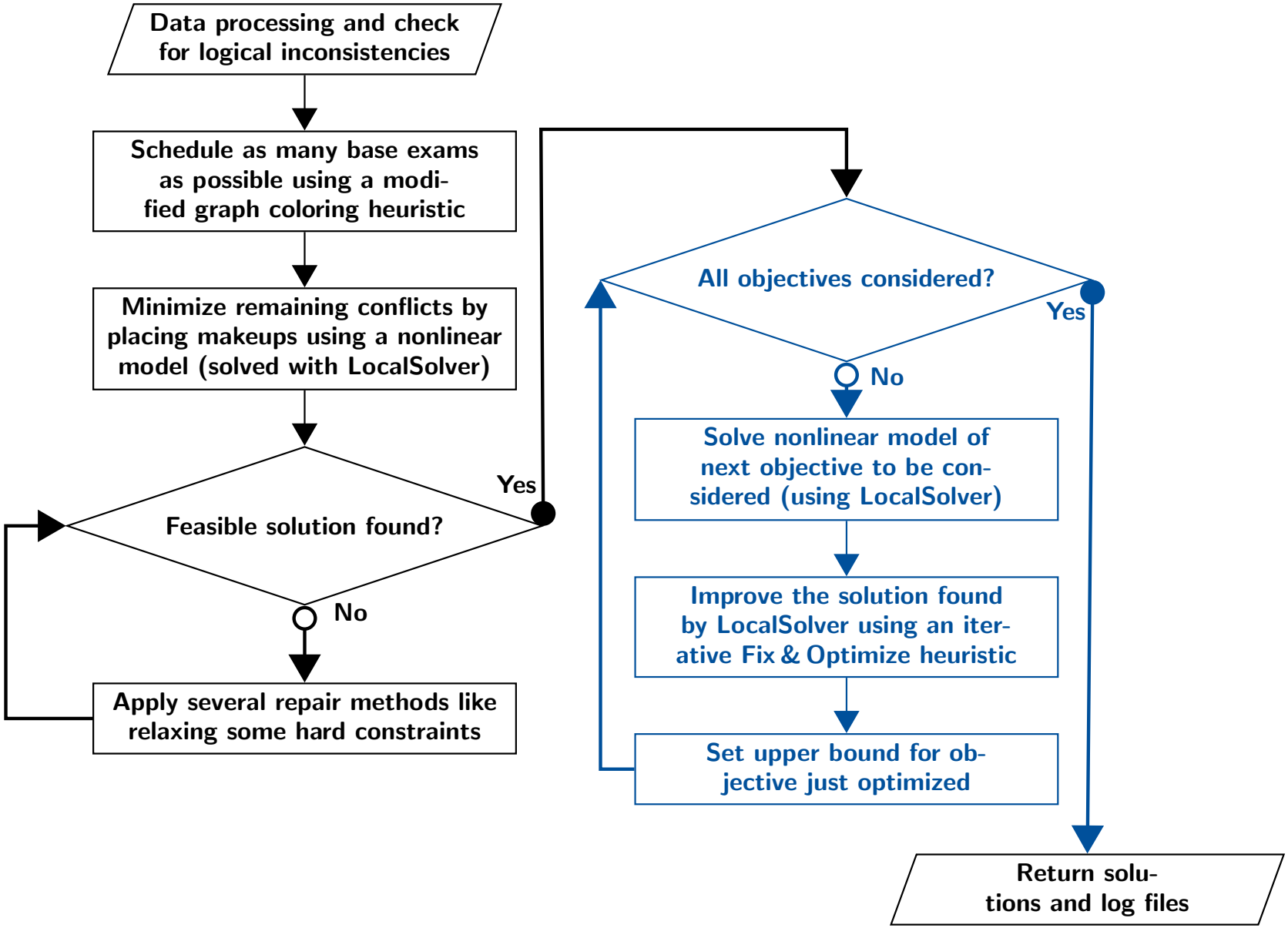

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Computational results

objective functions	F&O	*LocalSolver*	combination
makeups	114	172	126
epd/consecutive violations	0	25	0
exams for December Graduates	3	0	0
cadet specific exam-free periods	1	1	0
schedule exams by grad year	389	470	425
exam-free periods	330	636	183
capacity violations	0	0	0
computation time [sec.]	1323	(884)	1156

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Outlook

- Use completely different solution methods like column generation to compare with the results obtained by our algorithm
- Apply the (modified) algorithm to different ETPs and other timetabling problems in general

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