Optimization of Gamma Knife Radiosurgery

Michael Ferris, Jin-Ho Lim
University of Wisconsin, Computer Sciences

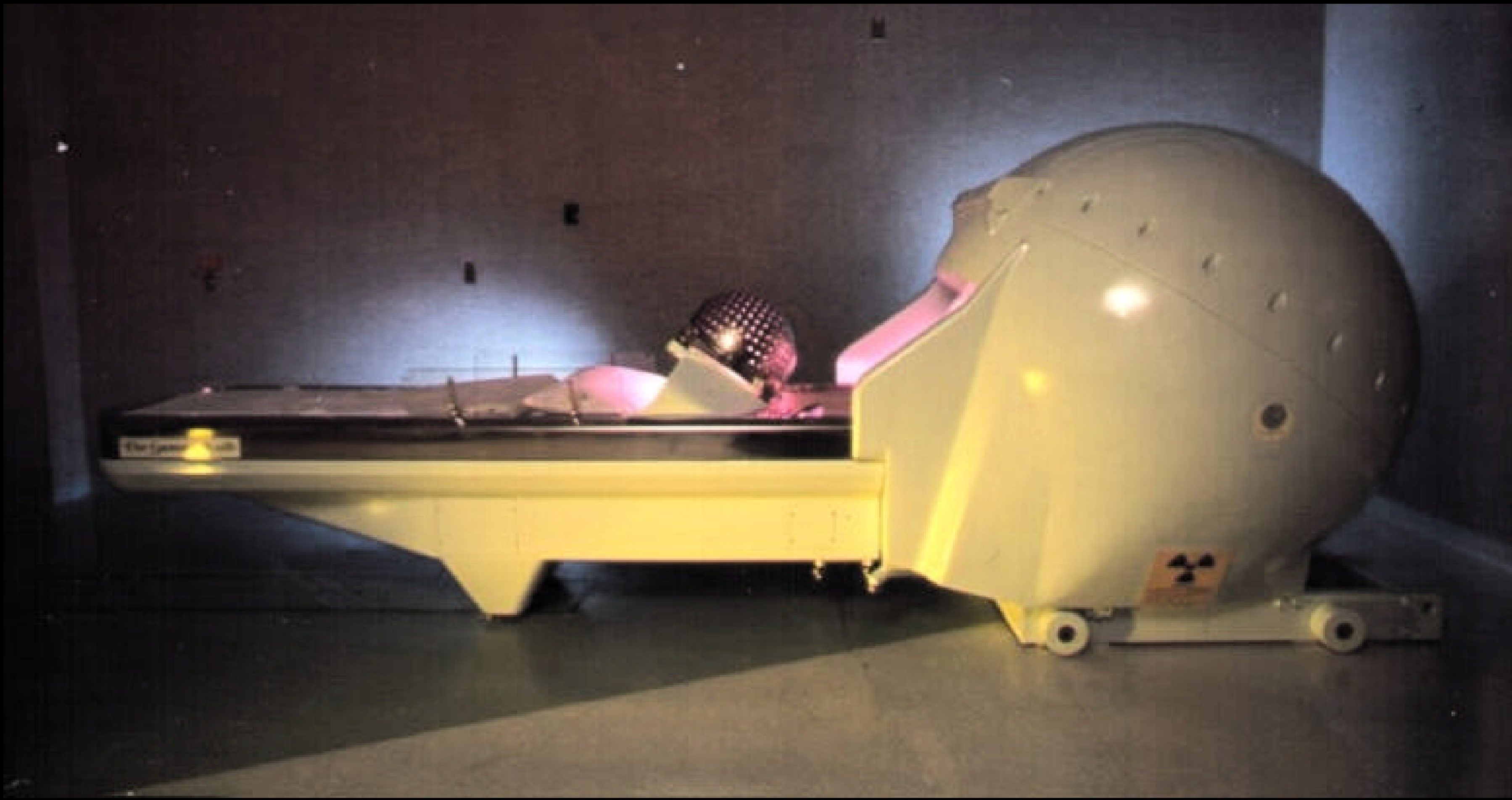
David Shepard
University of Maryland School of Medicine

Supported by Microsoft, NSF and AFOSR

Overview

- Details of machine and problem
- Optimization formulation
  - modeling dose
  - shot/target optimization
- Results
  - Two-dimensional data
  - Real patient (three-dimensional) data

The Gamma Knife



201 cobalt gamma ray beam sources are arrayed in a hemisphere and aimed through a collimator to a common focal point.

The patient’s head is positioned within the Gamma Knife so that the tumor is in the focal point of the gamma rays.

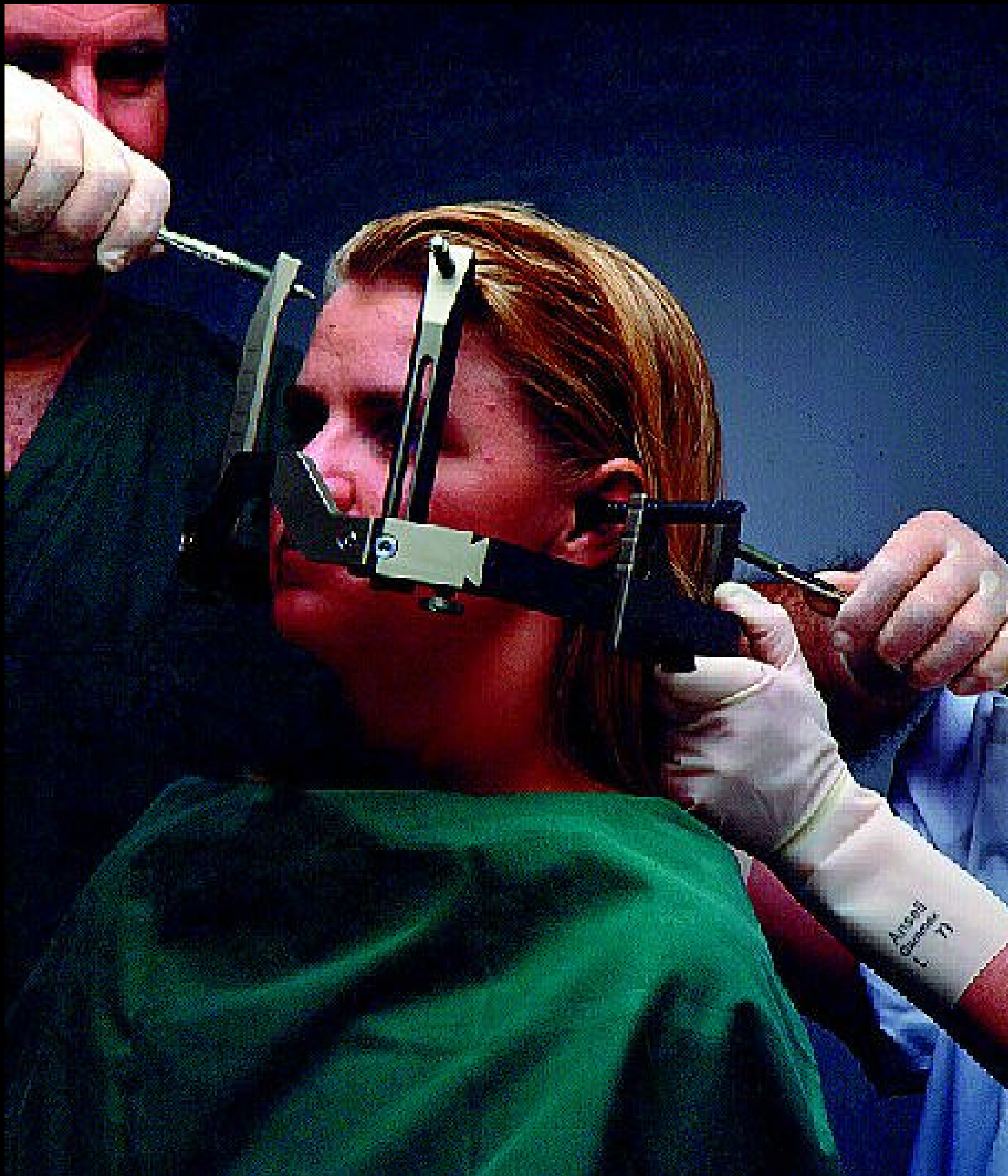
What disorders can the Gamma Knife treat?

- Malignant brain tumors
- Benign tumors within the head
- Malignant tumors from elsewhere in the body
- Vascular malformations
- Functional disorders of the brain
  - Parkinson’s disease

Gamma Knife Statistics

- 120 Gamma Knife units worldwide
- Over 20,000 patients treated annually
- Accuracy of surgery without the cuts
- Same-day treatment

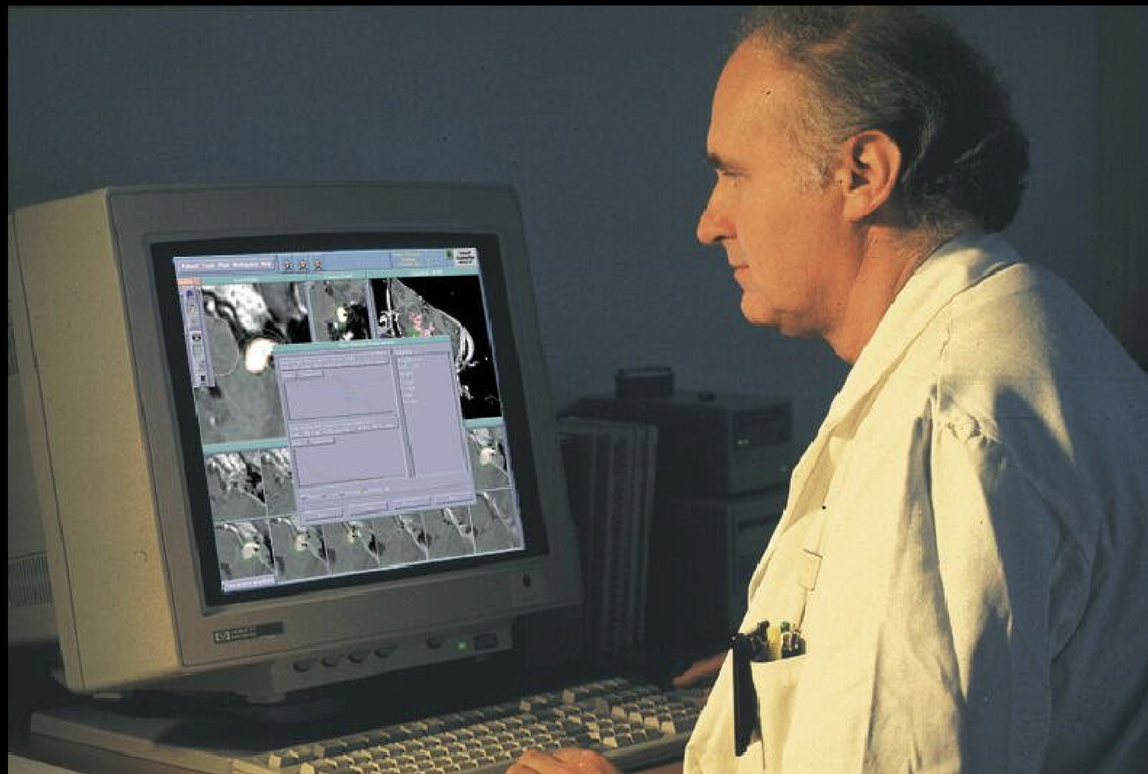
- Expensive instrument



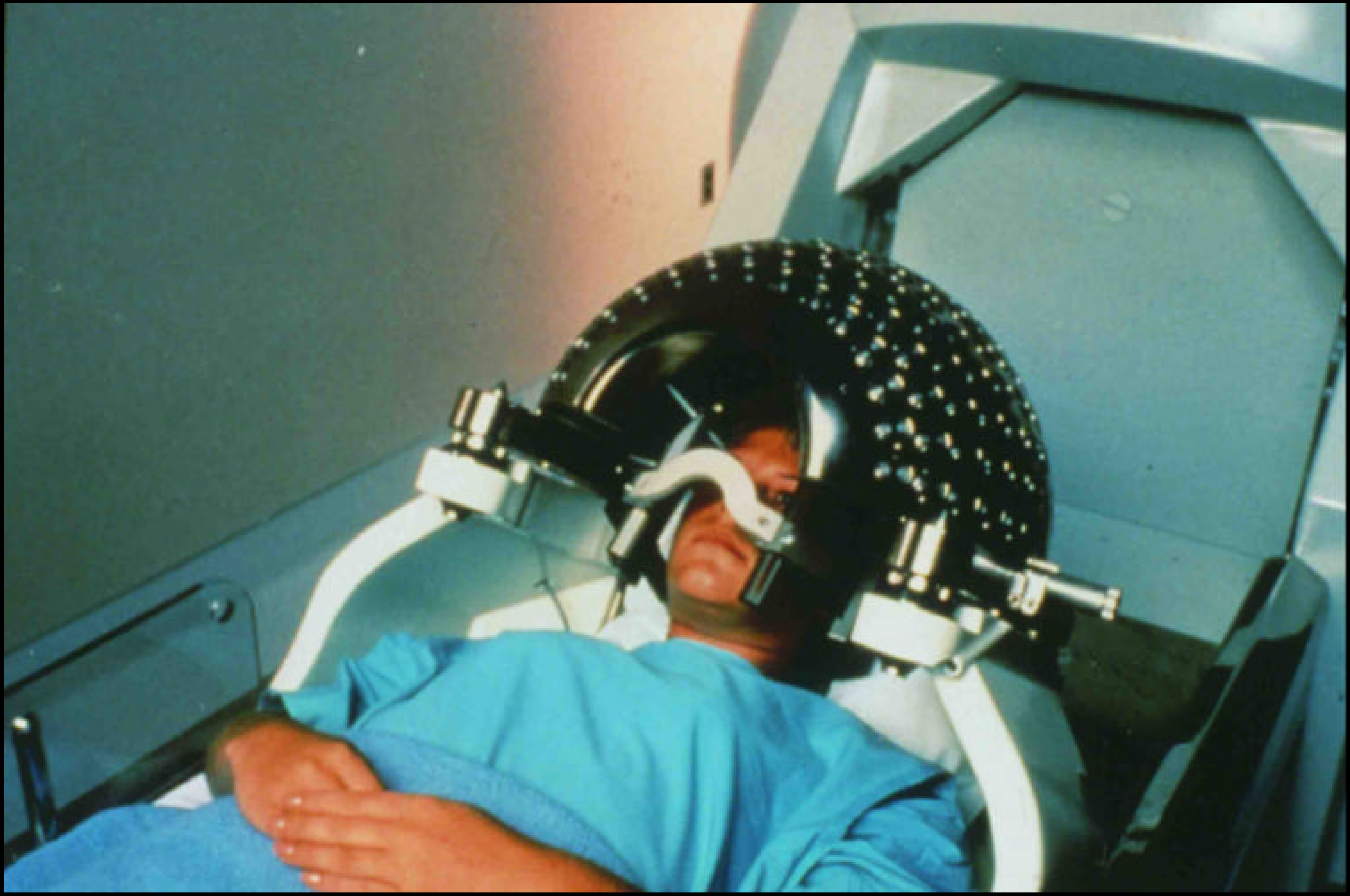
How is Gamma Knife Surgery performed?

Step 1: A stereotactic head frame is attached to the head with local anesthesia.

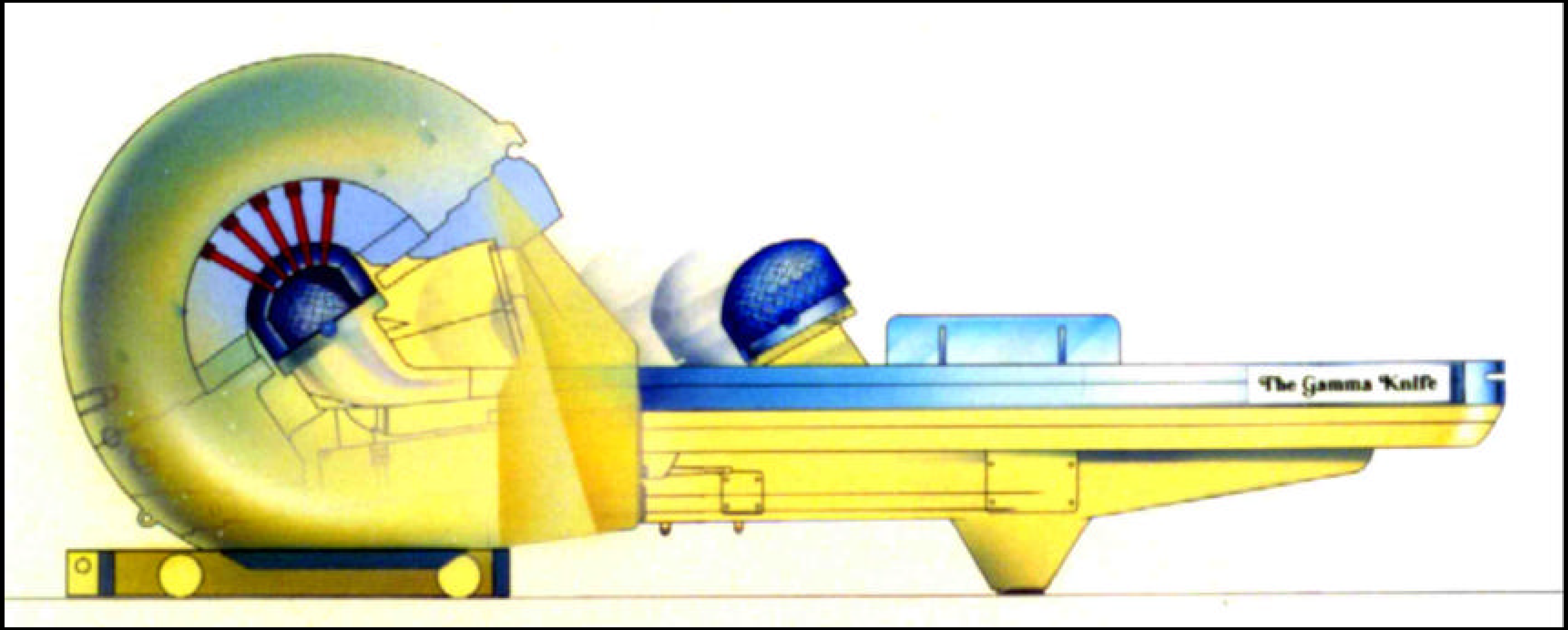
Step 2: The head is imaged using a MRI or CT scanner while the patient wears the stereotactic frame.



Step 3: A treatment plan is developed using the images. Key point: very accurate delivery possible.

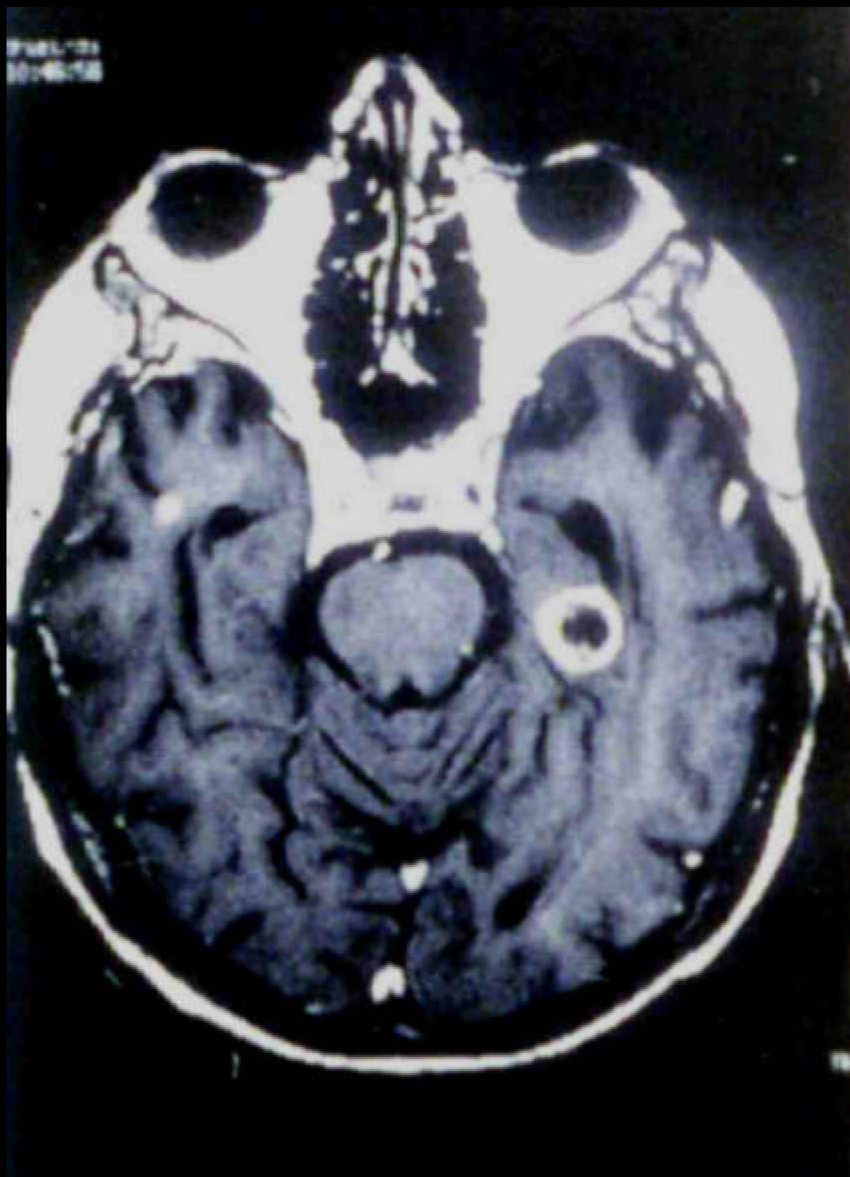


Step 4: The patient lies on the treatment table of the Gamma Knife while the frame is affixed to the appropriate collimator.

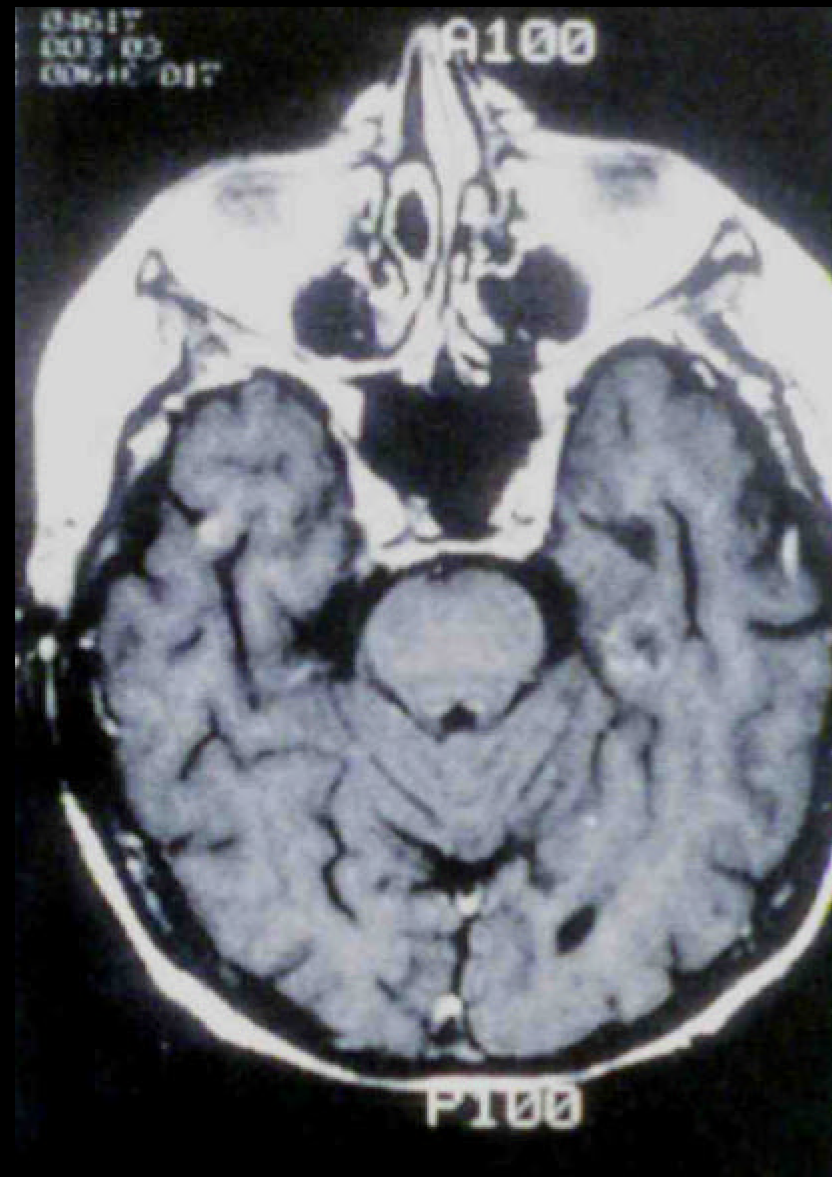


Step 5: The door to the treatment unit opens. The patient is advanced into the shielded treatment vault. The area where all of the beams intersect is treated with a high dose of radiation.

Before



After

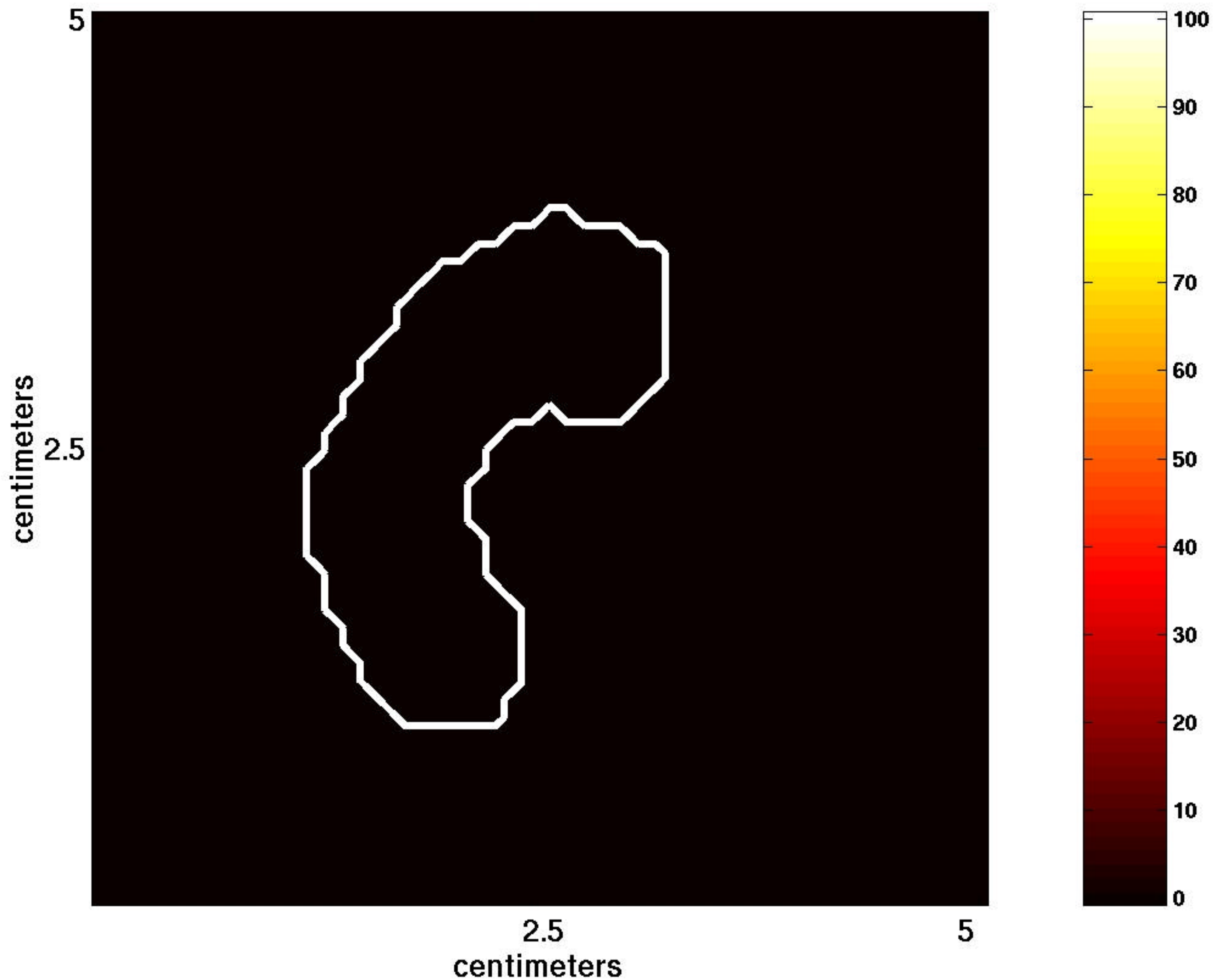



4
4
8
8

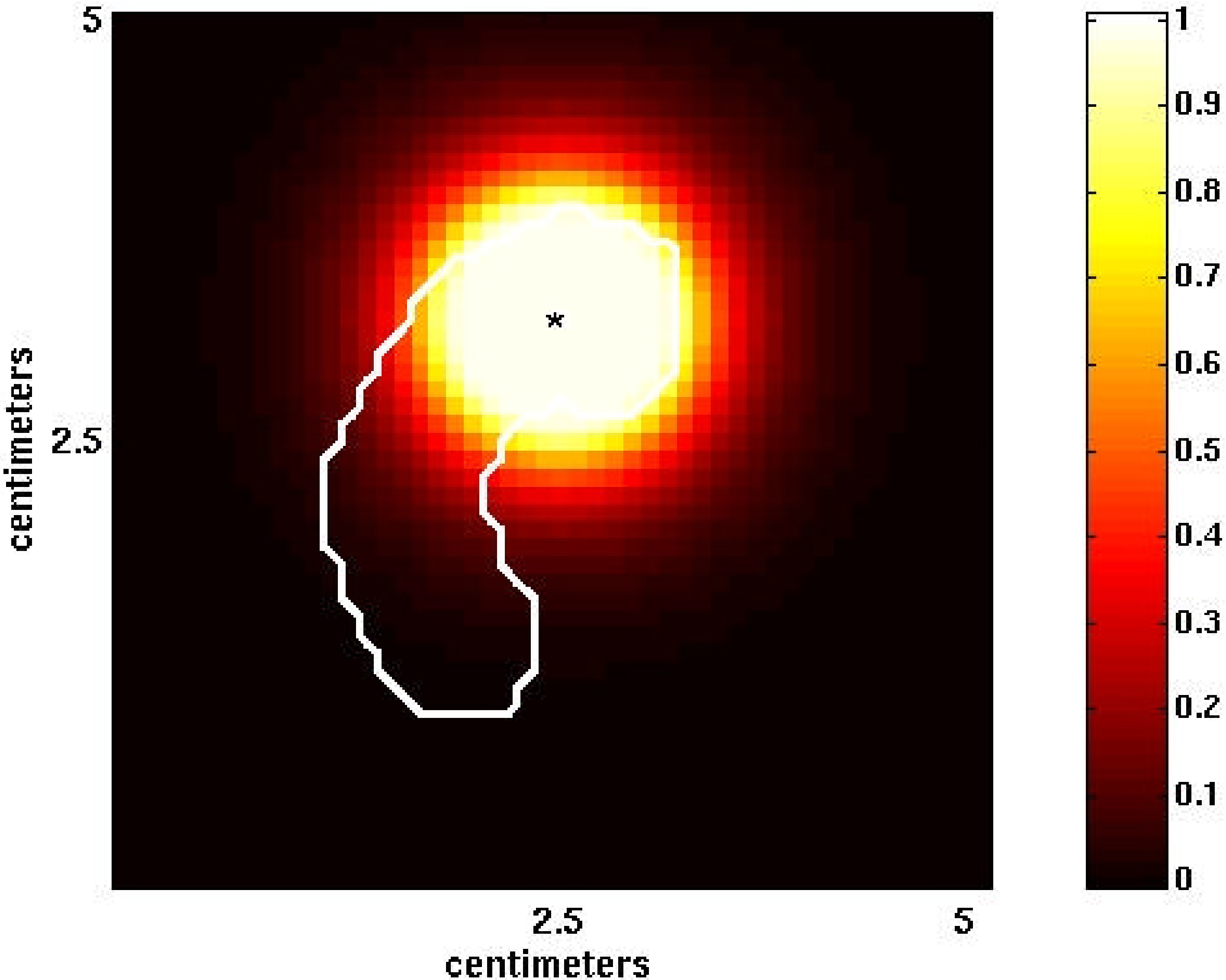
Treatment Planning

- Through an iterative approach we determine:
  - the number of shots
  - the shot sizes
  - the shot locations
  - the shot weights
- The quality of the plan is dependent upon the patience and experience of the user

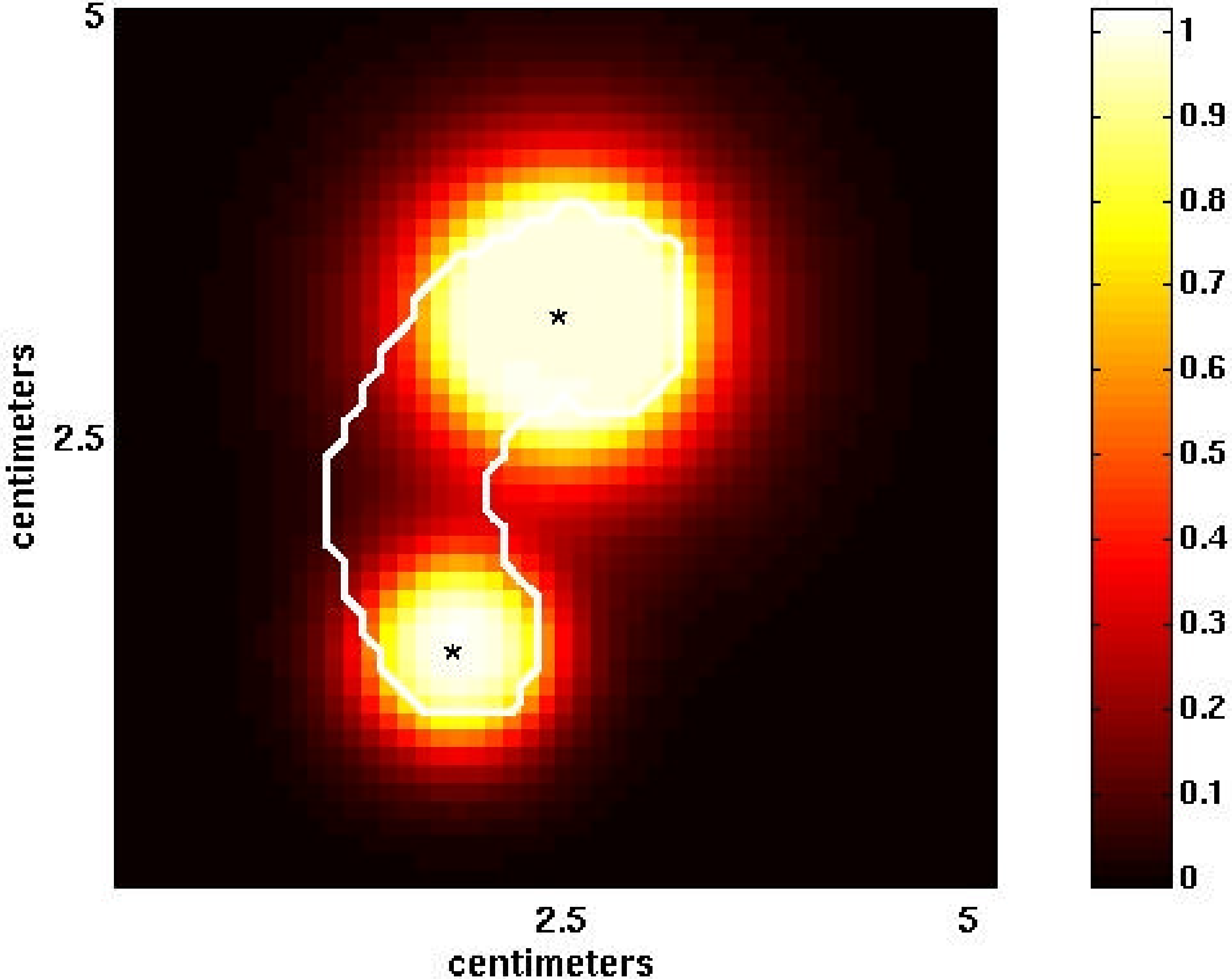
Target


5
2.5
centimeters
2.5
5
centimeters
100
90
80
70
60
50
40
30
20
10
0

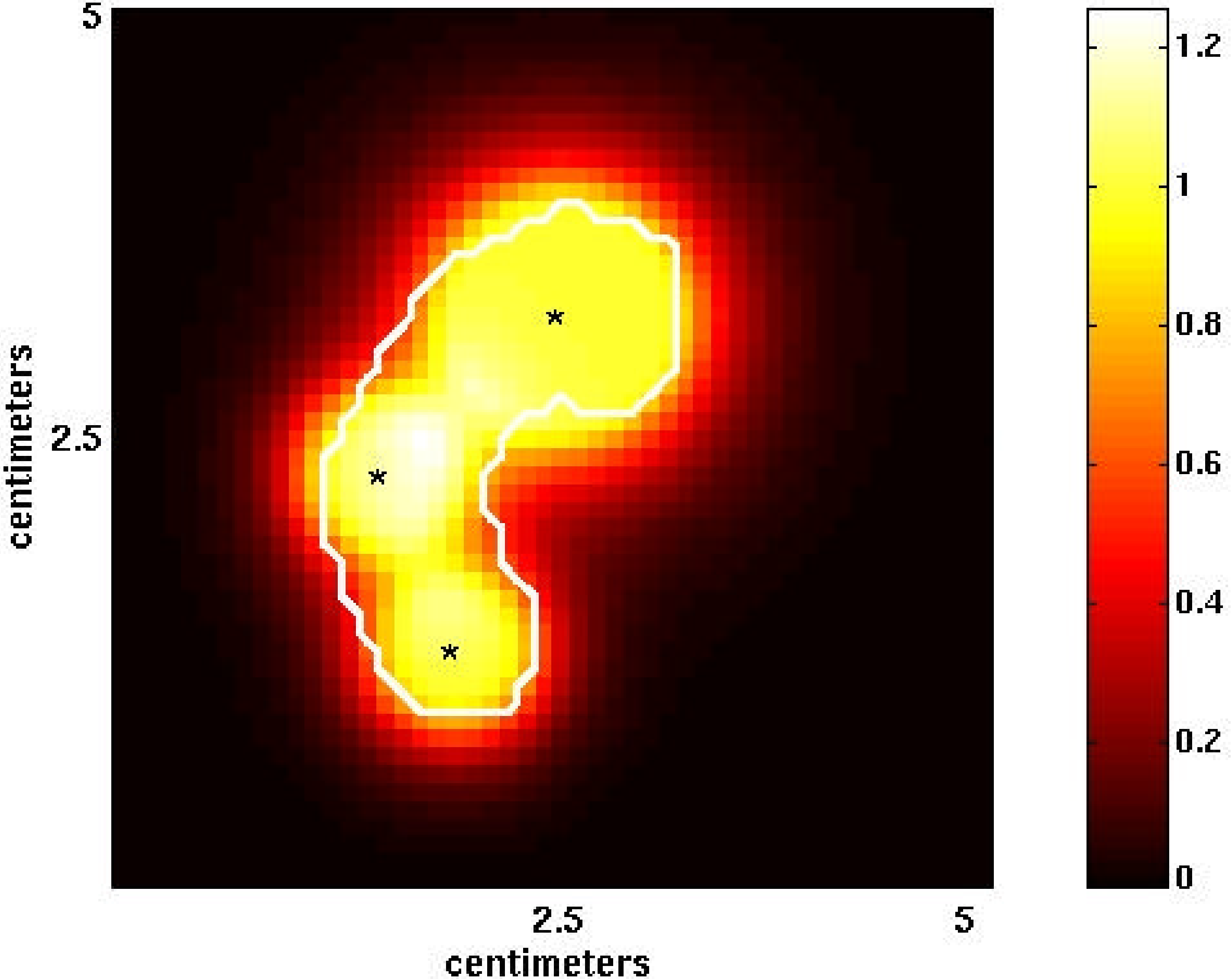
1 Shot

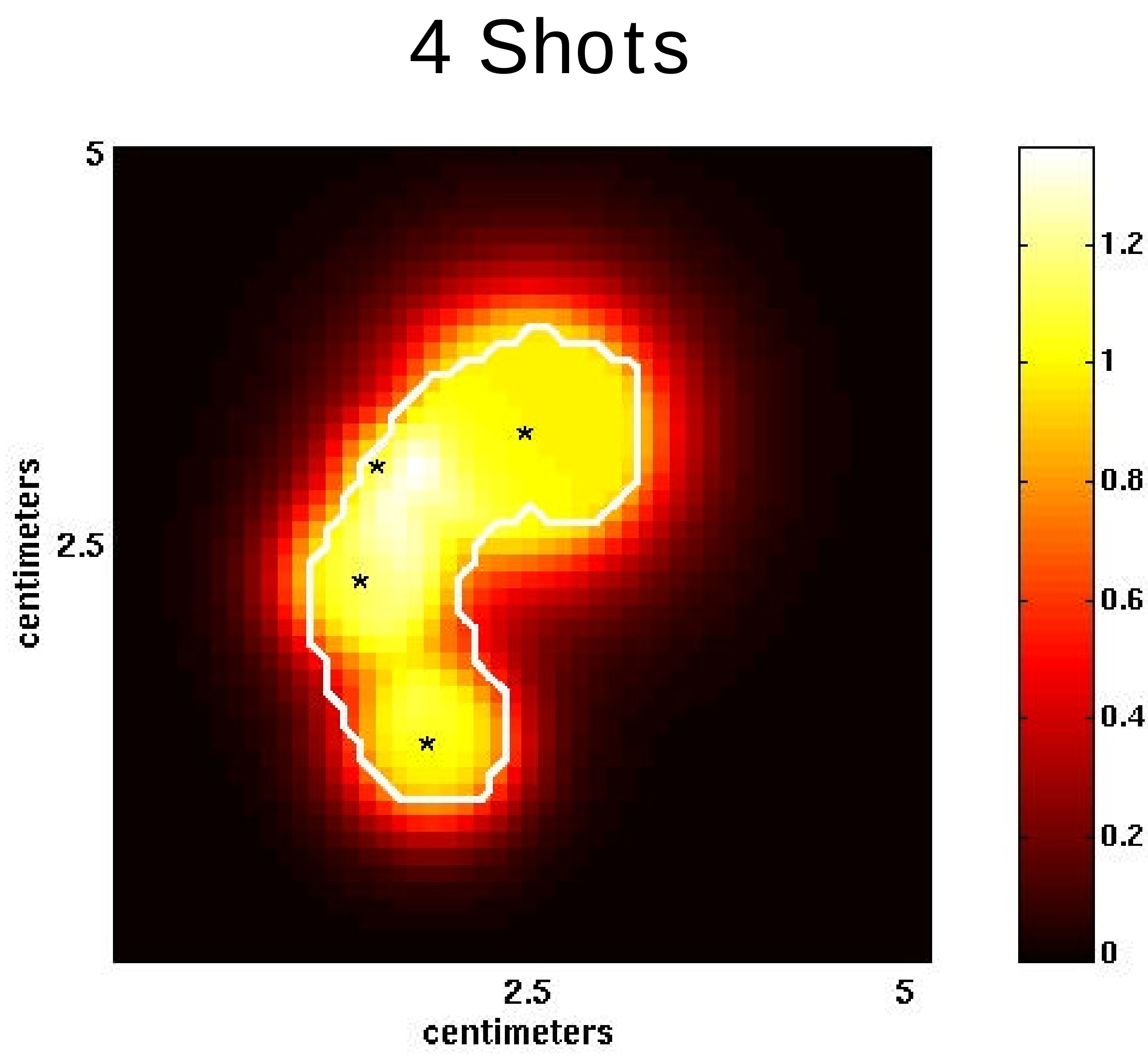


2 Shots

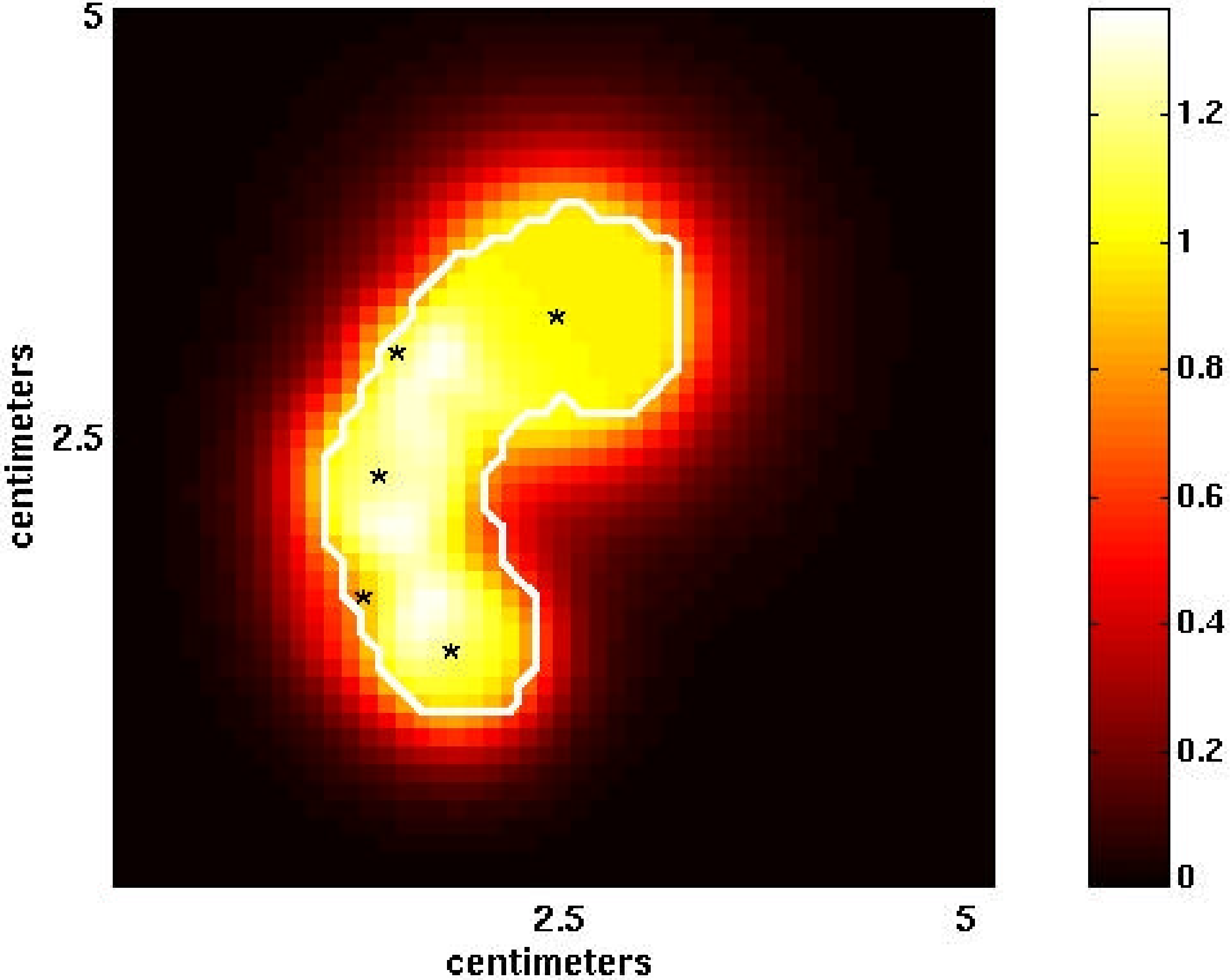


3 Shots




4 Shots
5
2.5
centimeters
2.5
5
centimeters
1.2
1
0.8
0.6
0.4
0.2
0

5 Shots



Inverse Treatment Planning

- Develop a fully automated approach to Gamma Knife treatment planning.
- A clinically useful technique will meet three criteria: robust, flexible, fast

- Benefits of computer generated plans
  - uniformity, quality, faster determination

Computational Model

- Target volume (from MRI or CT)
- Maximum number of shots to use
  - Which size shots to use
  - Where to place shots
  - How long to deliver shot for

  - Conform to Target (50% isodose curve)
  - Real-time optimization

Summary of techniques

Method	Advantage	Disadvantage
Sphere Packing	Easy concept	NP-hard Hard to enforce constraints
Dynamic Programming	Easy concept	Not flexible Not easy to implement Hard to enforce constraints
Simulated Annealing	Global solution (Probabilistic)	Long-run time Hard to enforce constraints
Mixed Integer Programming	Global solution (Deterministic)	Enormous amount of data Long-run time
Nonlinear Programming	Flexible	Local solution Initial solution required

Ideal Optimization

$$\min_{t_{s,w}, x_s} Dose(NonTarget)$$

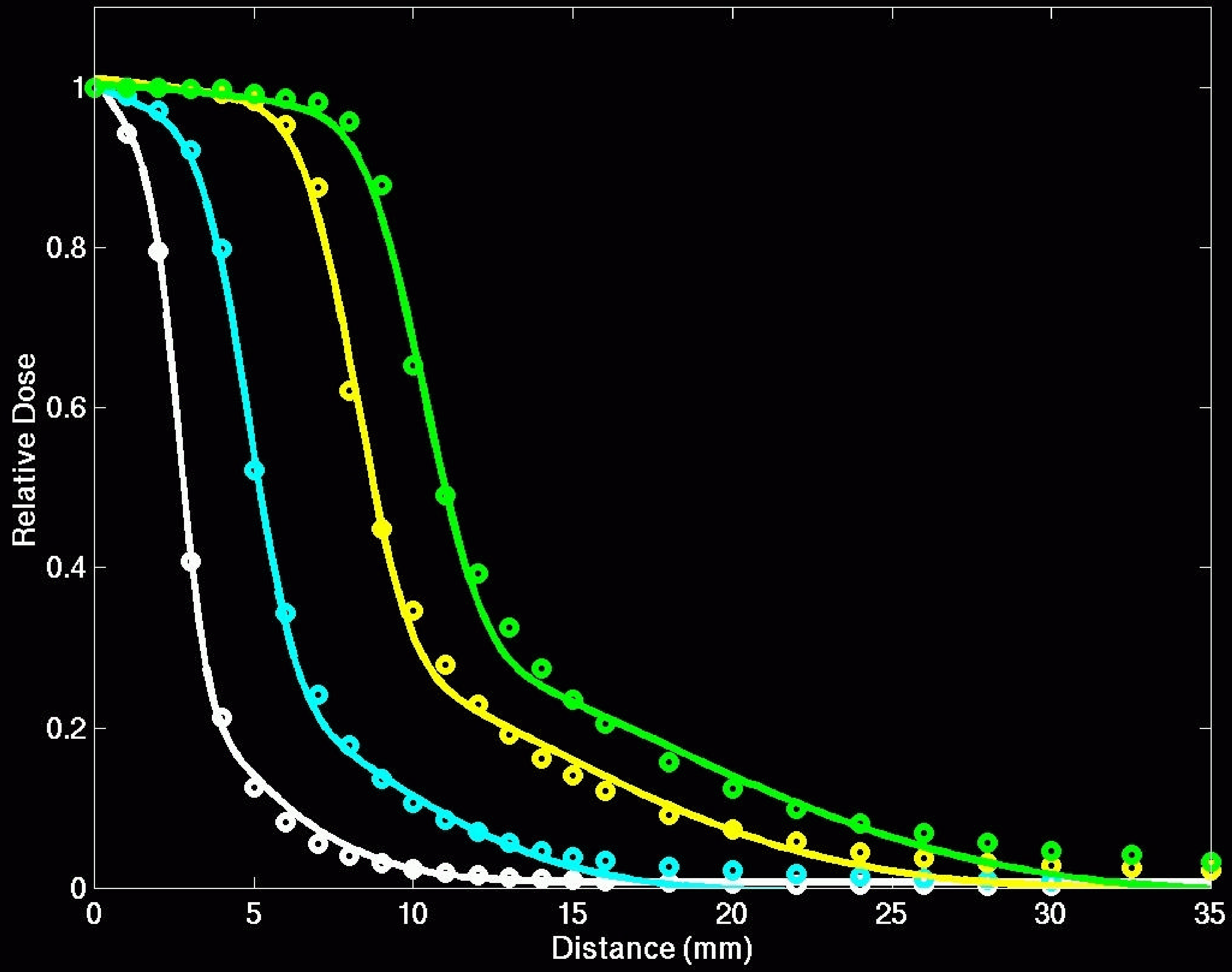
subject to

$$Dose(i) = \sum_{s \in S, w \in W} t_{s,w} D_w(x_s, i)$$

$$0.5 \leq Dose(Target) \leq 1$$

$$t_{s,w} \geq 0$$

$$|S| \leq n$$


Relative Dose
Distance (mm)
0
0.2
0.4
0.6
0.8
1
0
5
10
15
20
25
30
35

Dose calculation

- Measure dose at distance from shot center in 3 different axes
- Fit a nonlinear curve to these measurements (nonlinear least squares)
- Functional form from literature, 10 parameters to fit via least-squares

$$m_1 \, erf(\frac{d_1(x)-r_1}{\sigma_1}) + m_2 \, erf(\frac{d_2(x)-r_2}{\sigma_2})$$

MIP Approach

Choose a subset of locations from S

$$\psi_{s,w} = \begin{cases} 1 & \text{if use shot s of width w} \\ 0 & \text{else} \end{cases}$$

$$D_{s,w}(i) := D_w(x_s, i)$$

$$Dose(i) = \sum_{s \in S, w \in W} t_{s,w} D_{s,w}(i)$$

Features of MIP

- Large amounts of data/integer variables
- Possible shot locations on 1mm grid too restrictive
- Time consuming, even with restrictions and CPLEX
- but ... have guaranteed bounds on solution quality

Data reduction via NLP

Let x_s be variable locations

$s = 1, 2, \ldots, N$

$D_w(x_s, i)$ is nasty nonlinear function

What width shot to use at x_s?

$$\psi_{s,w} = \begin{cases} 1 & \text{if shot s is width w} \\ 0 & \text{else} \end{cases}$$

$$\underline{T}\psi_{s,w} \leq t_{s,w} \leq \overline{T}\psi_{s,w}$$

$$\sum_{s,w} \psi_{s,w} \leq n$$

$$\min_{t_{s,w}, x_s, y_s} \quad Under(Target)$$

$$\text{s.t.} \quad Dose(i) = \sum_{s \in S, w \in W} t_{s,w} D_w(x_s, i)$$

$$Under(i) \geq 1 - Dose(i) \geq 0$$

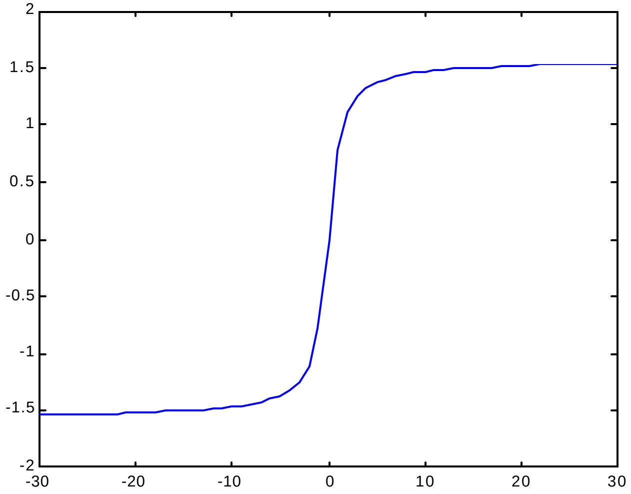
$$Dose(Target) / \Big(\sum_{s,w} t_{s,w} \overline{D_w} \Big) \geq P$$

$$\sum_{s,w} \psi_{s,w} \leq n$$

$$\overline{T} \psi_{s,w} \geq t_{s,w} \geq \underline{T} \psi_{s,w}$$

Iterative approach

- Approximate via "arctan"



$$\sum_{s,w} \frac{2}{\pi} arctan(t_{s,w}) \leq n$$

- First, solve with coarse approximation, then refine and reoptimize

Difficulties

- Nonconvex optimization
  - speed
  - robustness
  - starting point
- Too many voxels outside target
- Too many voxels in the target (size)
- What does the neurosurgeon really want?

$$\min_{t_{s,w}, x_s, y_s} \quad Under(Target)$$

$$\text{s.t.} \quad Dose(i) = \sum_{s \in S, w \in W} t_{s,w} D_w(x_s, i)$$

$$Under(i) \geq 1 - Dose(i) \geq 0$$

$$Dose(Target) / \left(\sum_{s,w} t_{s,w} \overline{D_w} \right) \geq P$$

$$\sum_{s,w} \text{arctan}(t_{s,w}) \leq n \; \pi/2$$

$$t_{s,w} \geq 0$$

Conformity estimation

min $$\sum_{(s,w)\in S\times W} \bar{D}_w t_{s,w}$$

subject to $$Dose(i) = \sum_{(s,w)\in S\times W} t_{s,w} D_w(x_s, i)$$

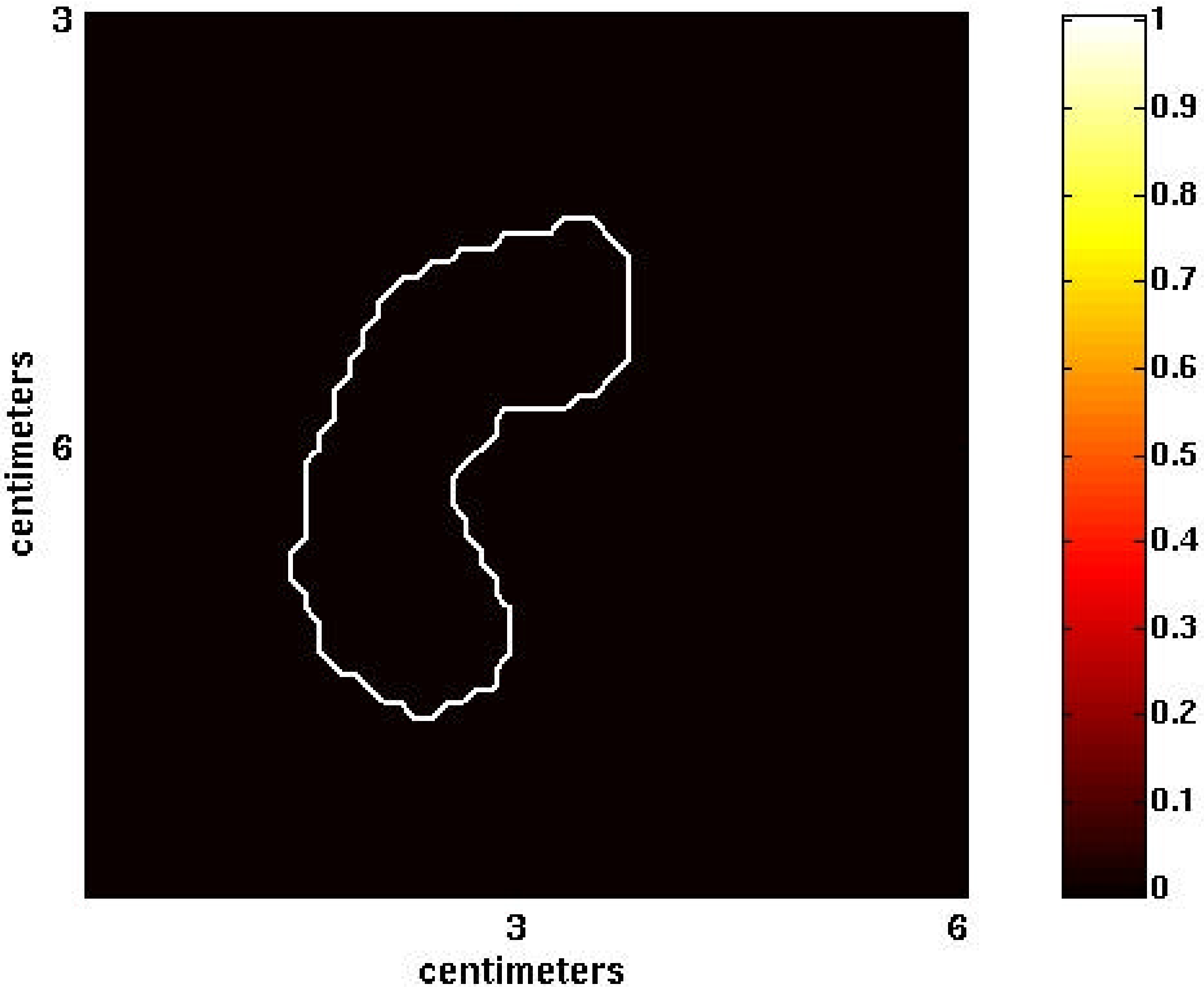
$$0 \leq UnderDose(i) \geq 1 - Dose(i)$$

$$\sum_{i\in Target} UnderDose(i) \leq NP_U$$

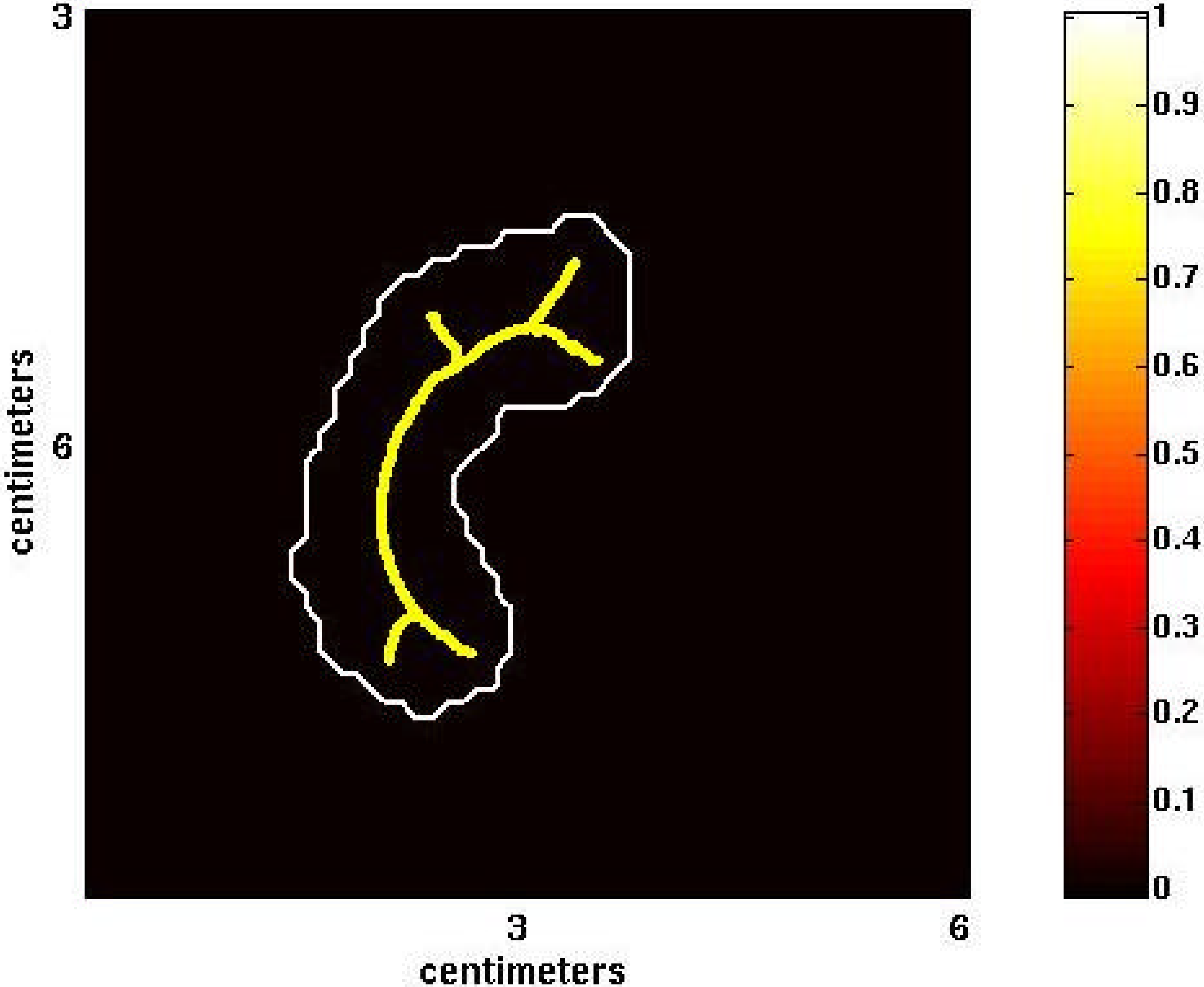
$$\sum_{(s,w)\in\{1,\ldots,n\}\times W} arctan(t_{s,w}) \leq n$$

$$0 \leq t_{s,w} \leq \bar{t}$$

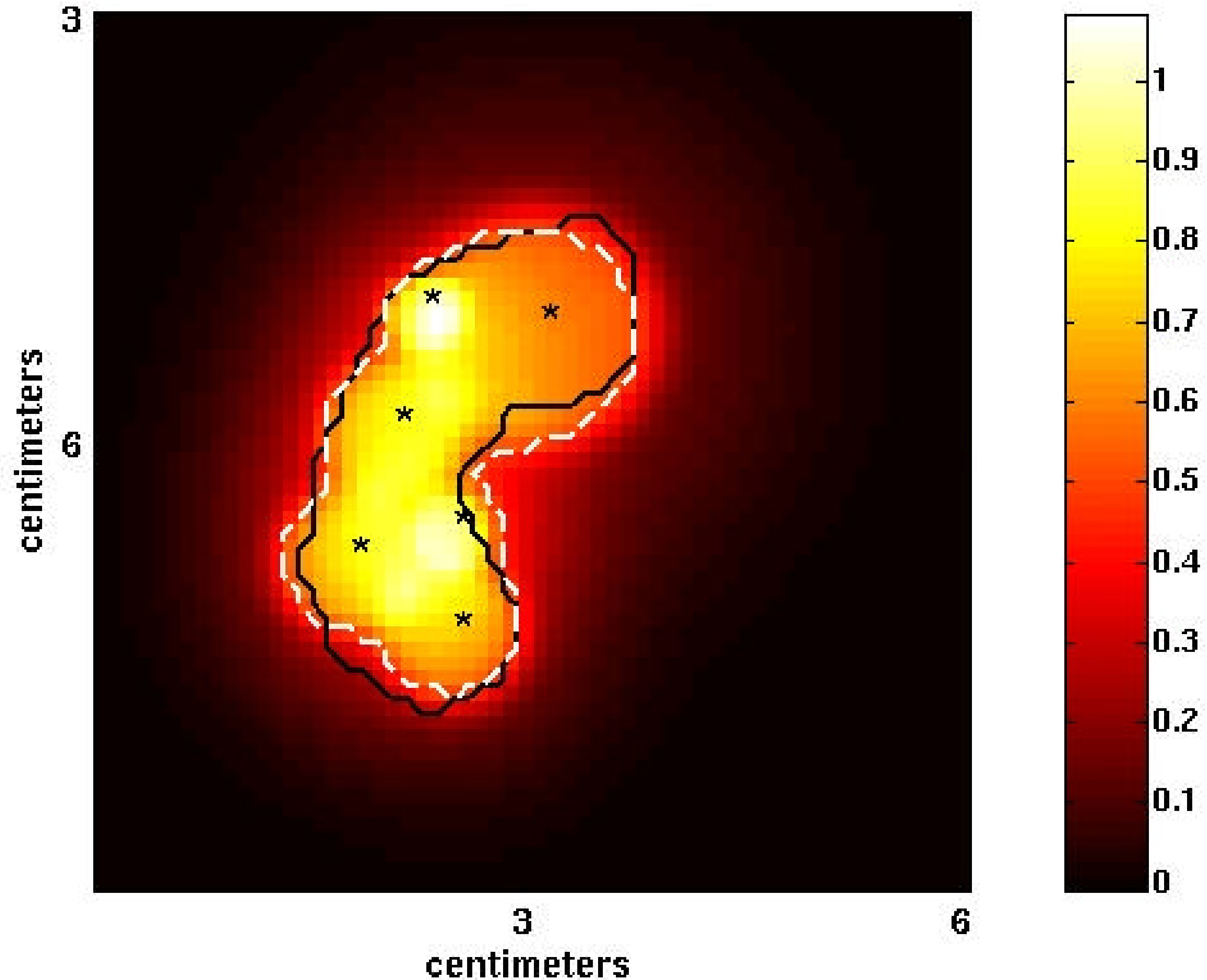
Target



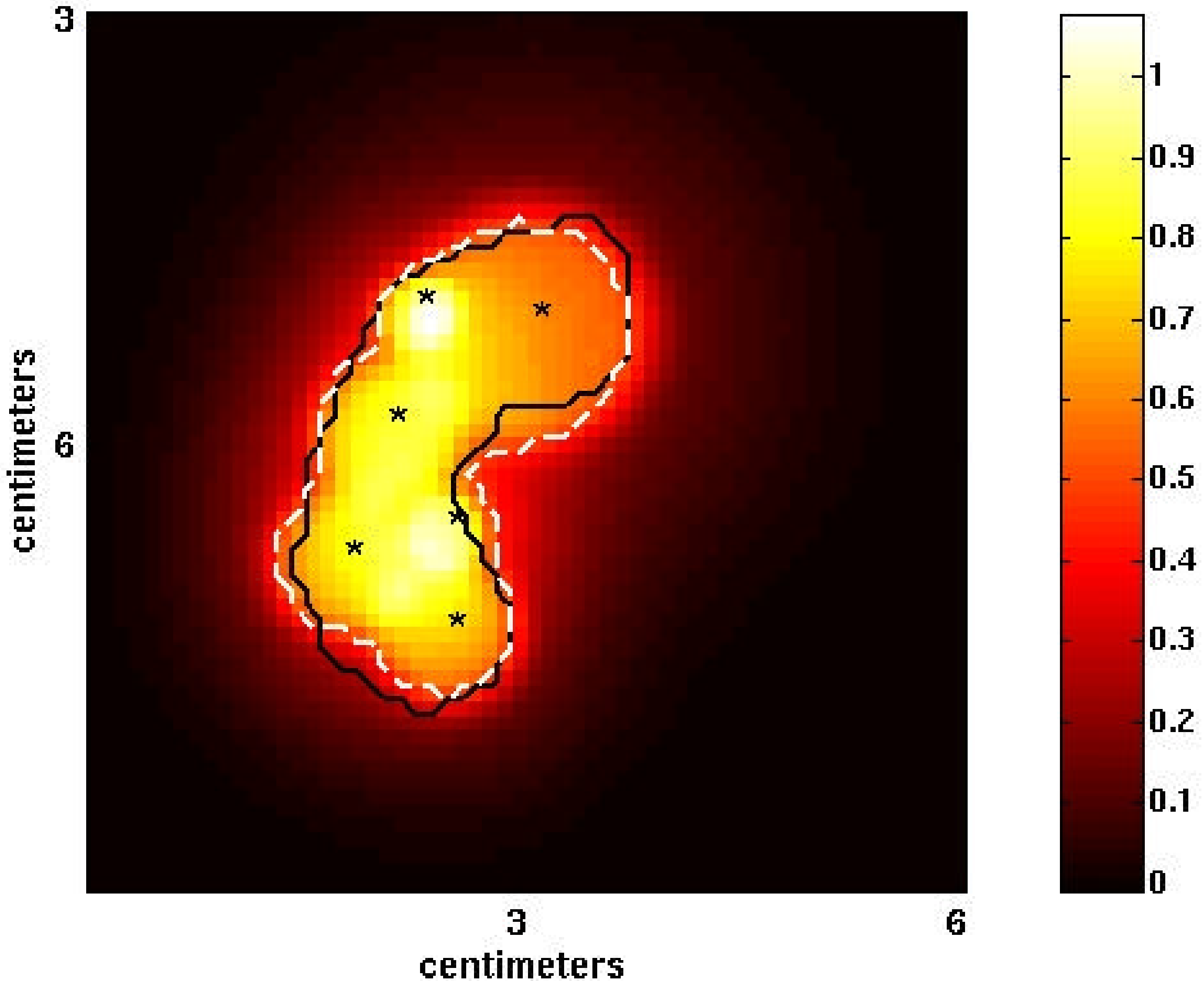
Target Skeleton is Determined



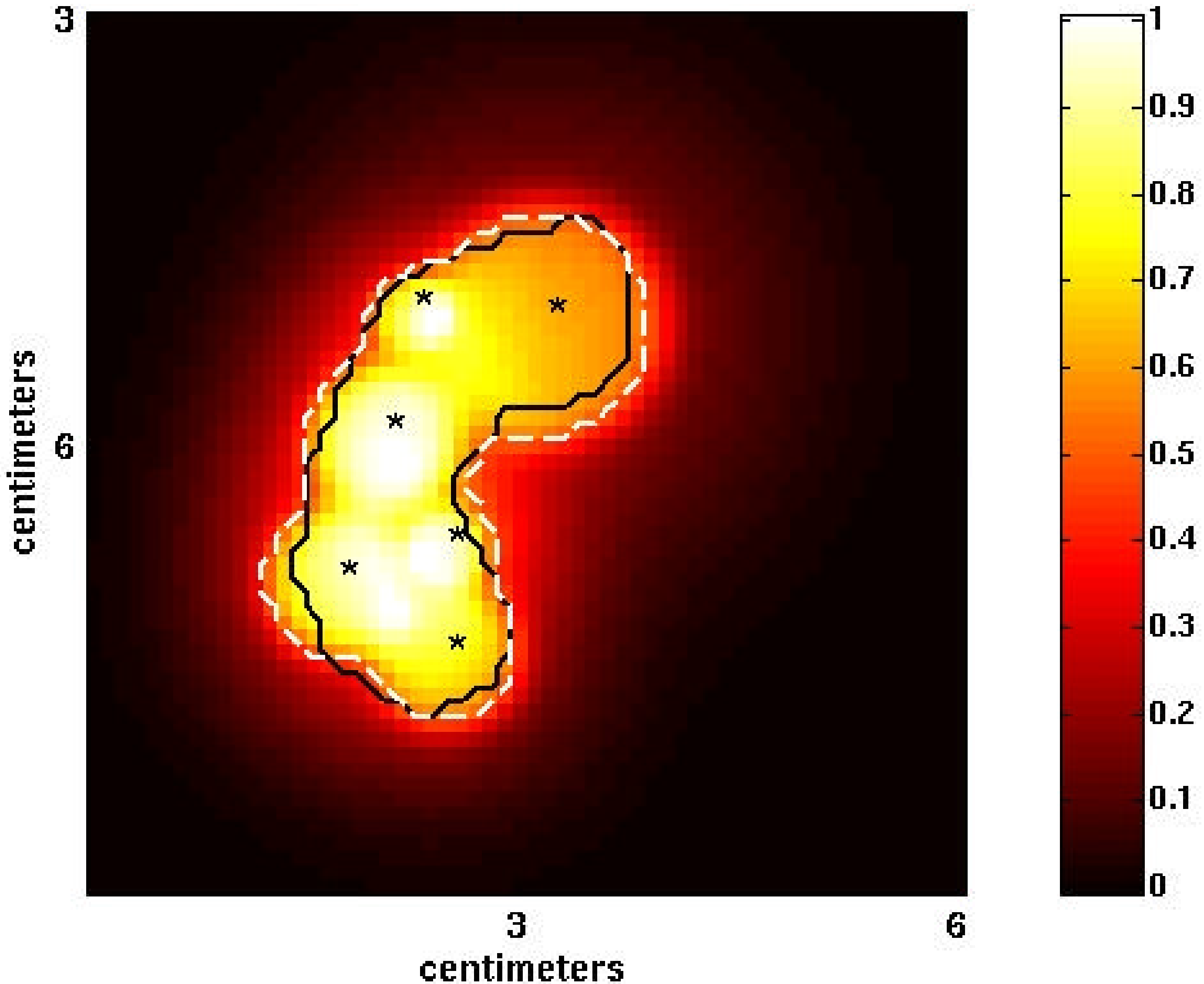
Sphere Packing Result



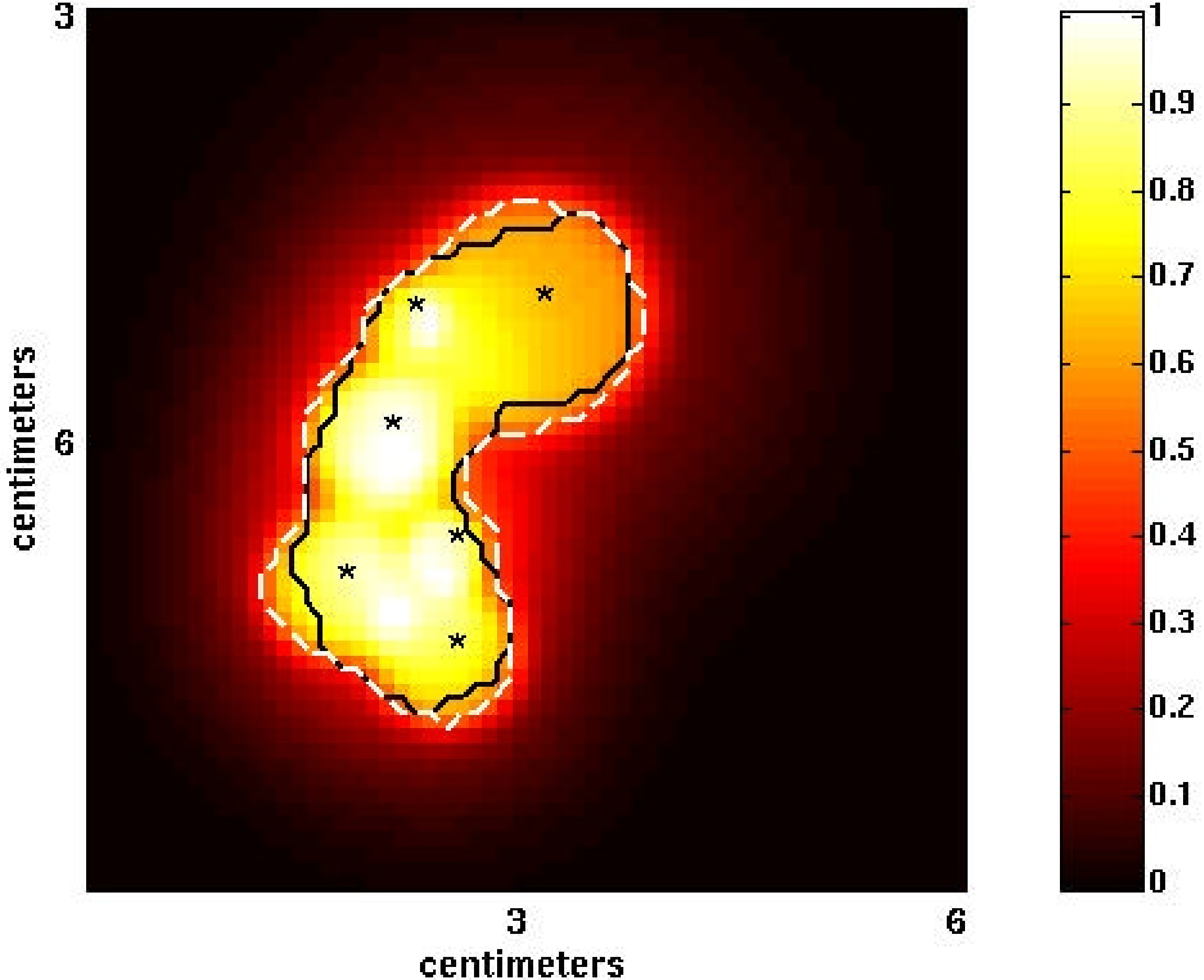
10 Iterations



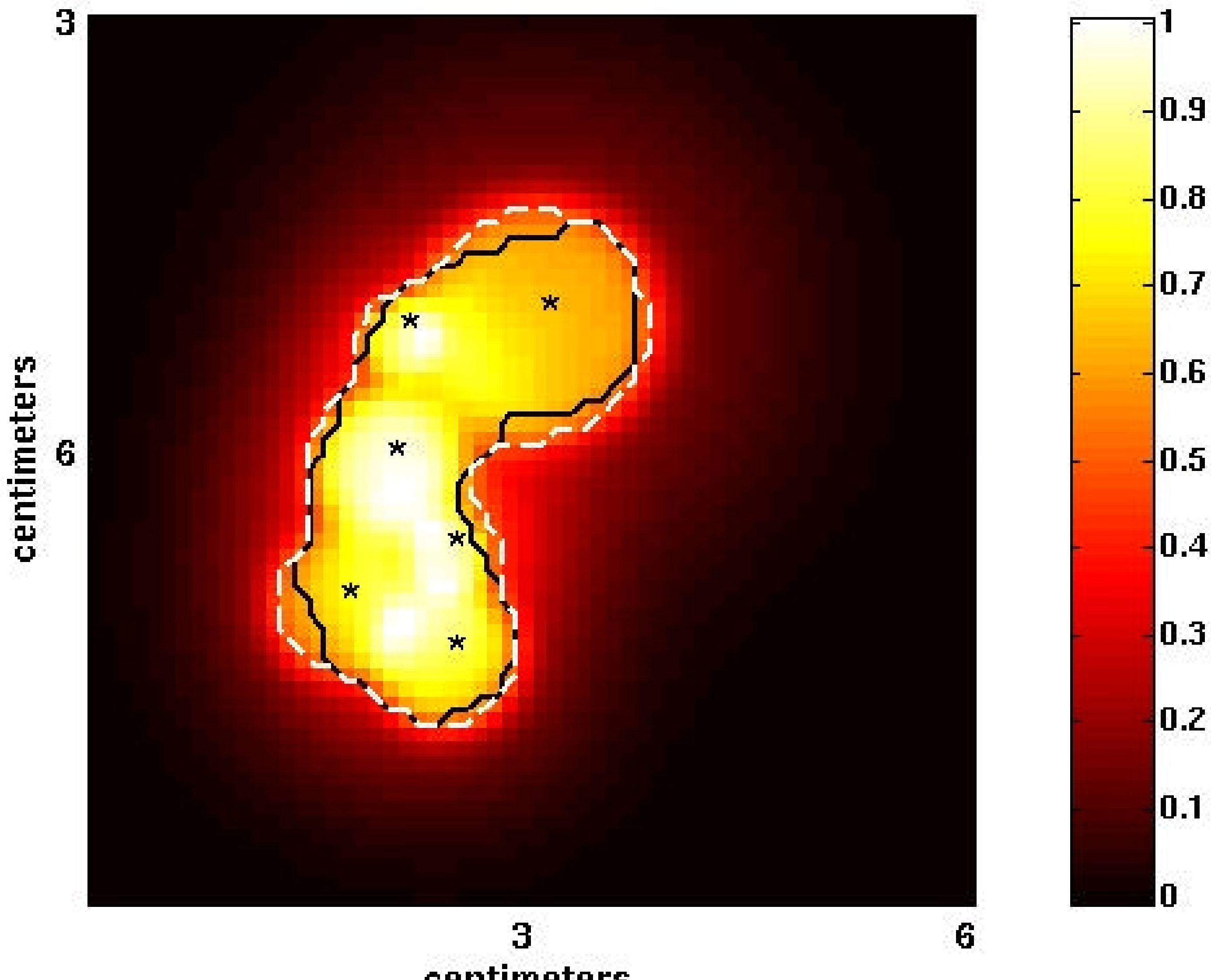
20 Iterations



30 Iterations



40 Iterations



Iterative Approach

- Rotate data (prone/supine)
- Skeletonization starting point procedure
- Conformity subproblem (P)
- Coarse grid shot optimization
- Refine grid (add violated locations)
- Refine smoothing parameter
- Round and fix locations, solve MIP for exposure times

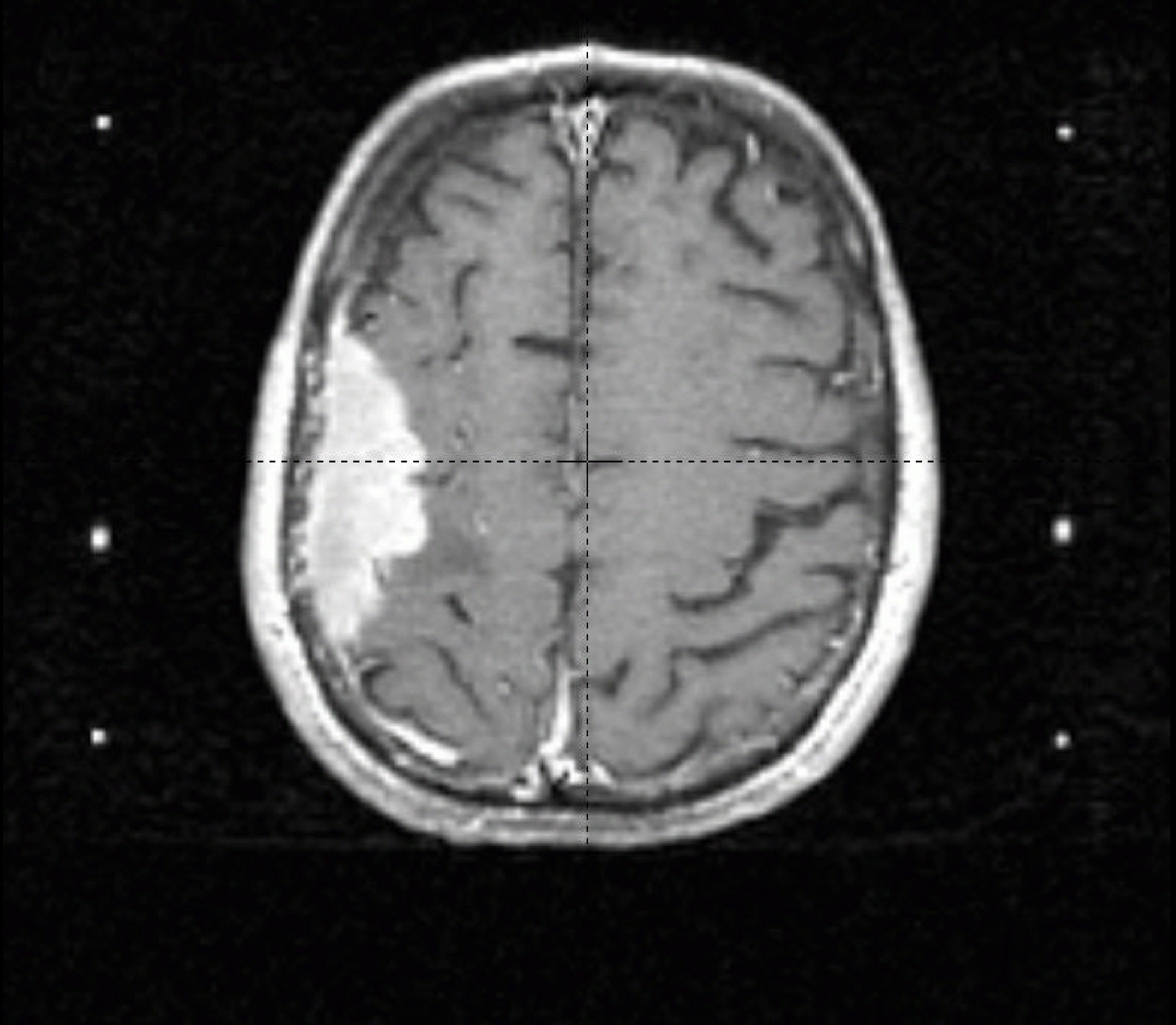
Status

- Automated plans have been generated retrospectively for over 30 patients
- The automated planning system is now being tested/used head to head against the neurosurgeon
- Optimization performs well for targets over a wide range of sizes and shapes

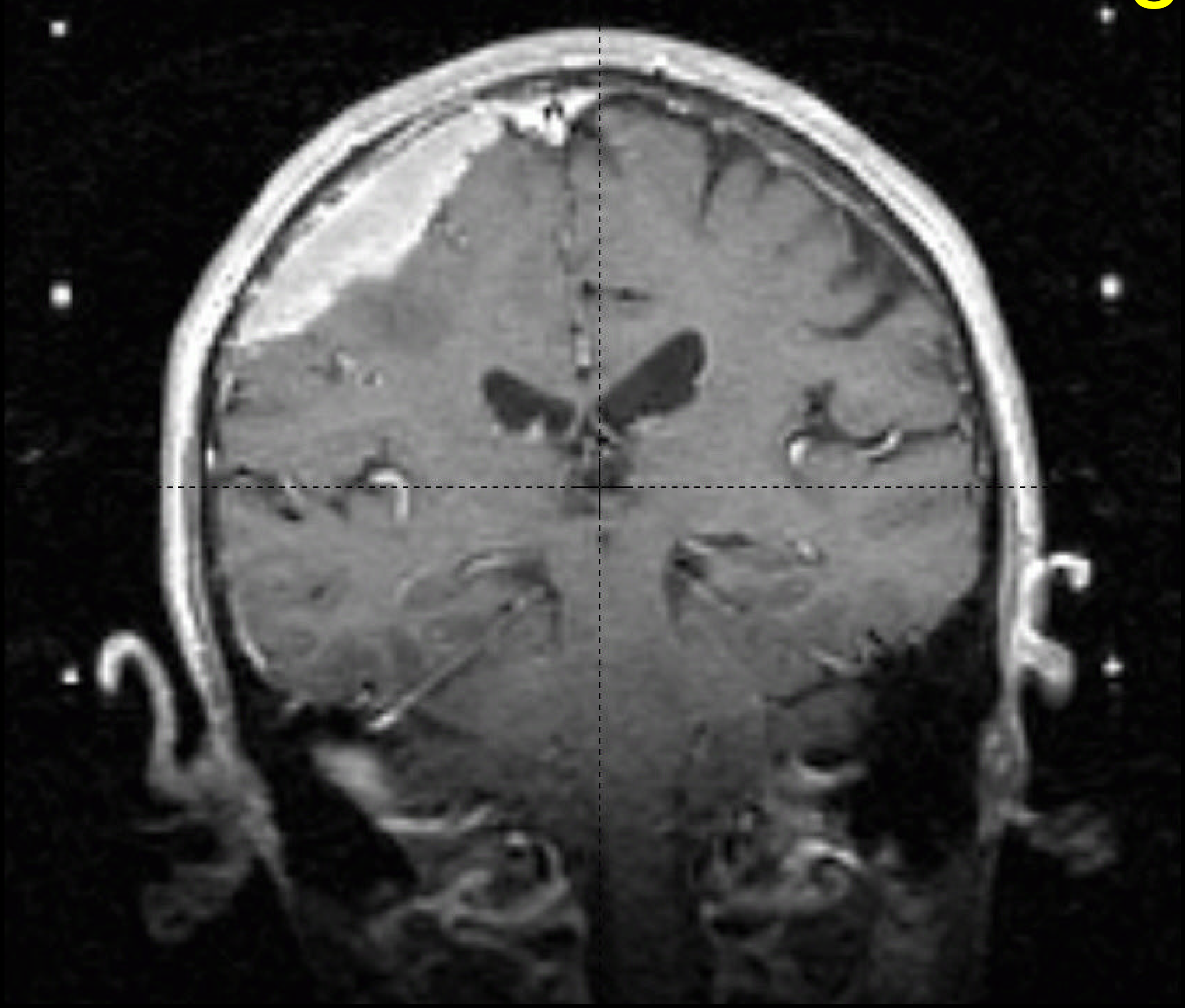
Environment

- All data fitting and optimization models formulated in GAMS
  - Ease of formulation / update
  - Different types of model
- Nonlinear programs solved with CONOPT (generalized reduced gradient)
- LP's and MIP's solved with CPLEX

Patient 1 - Axial Image

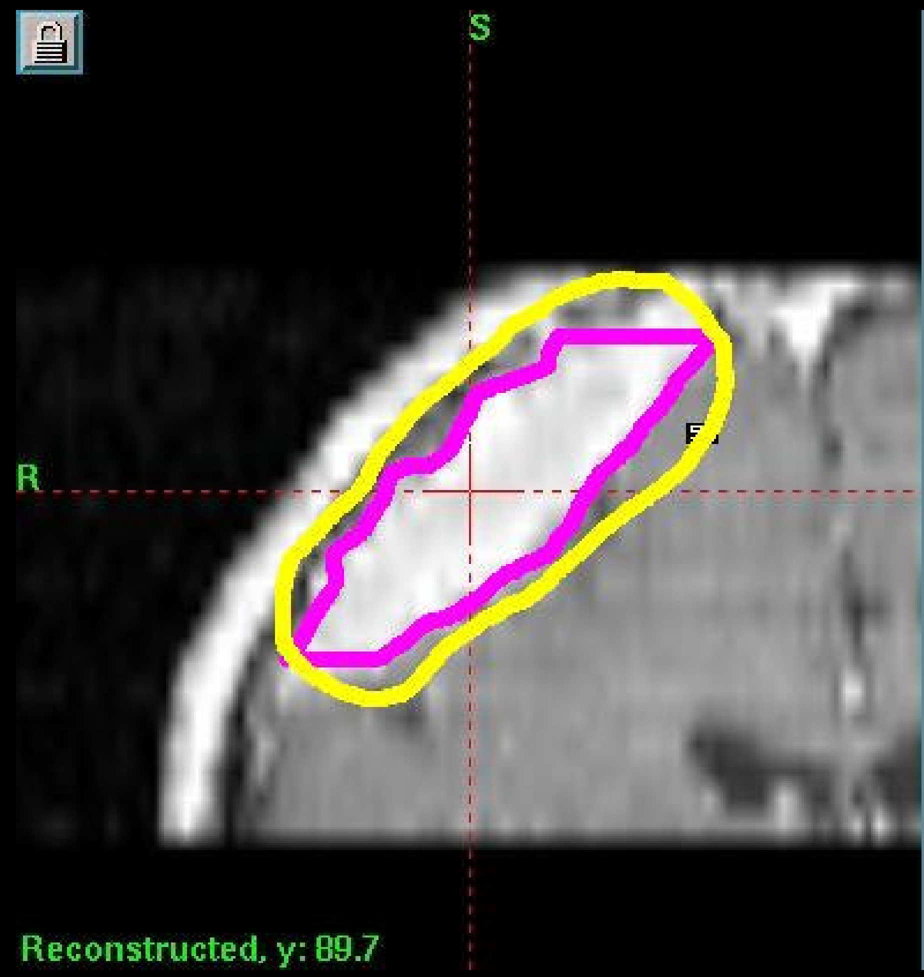


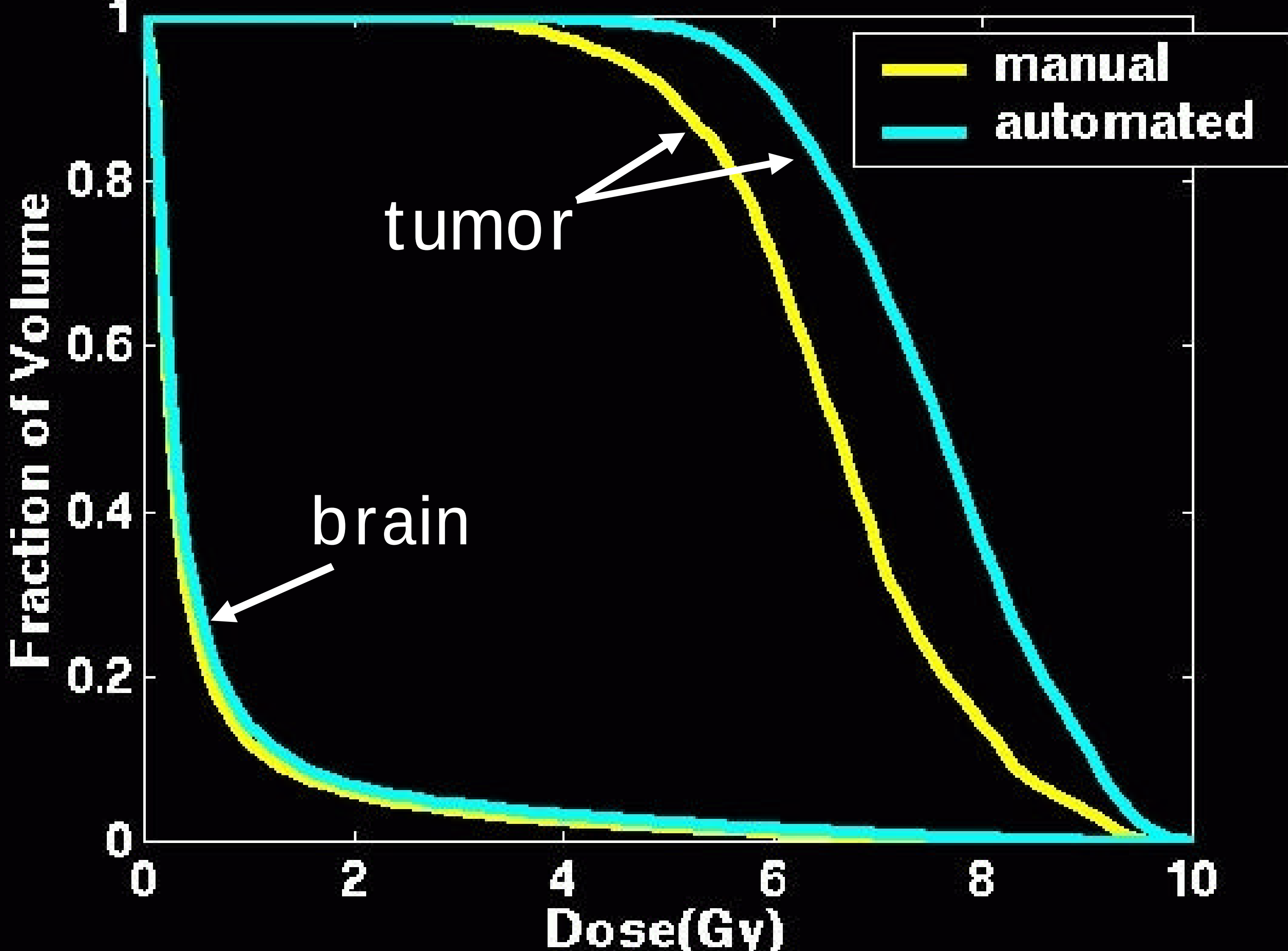
Patient 1 - Coronal Image



manual

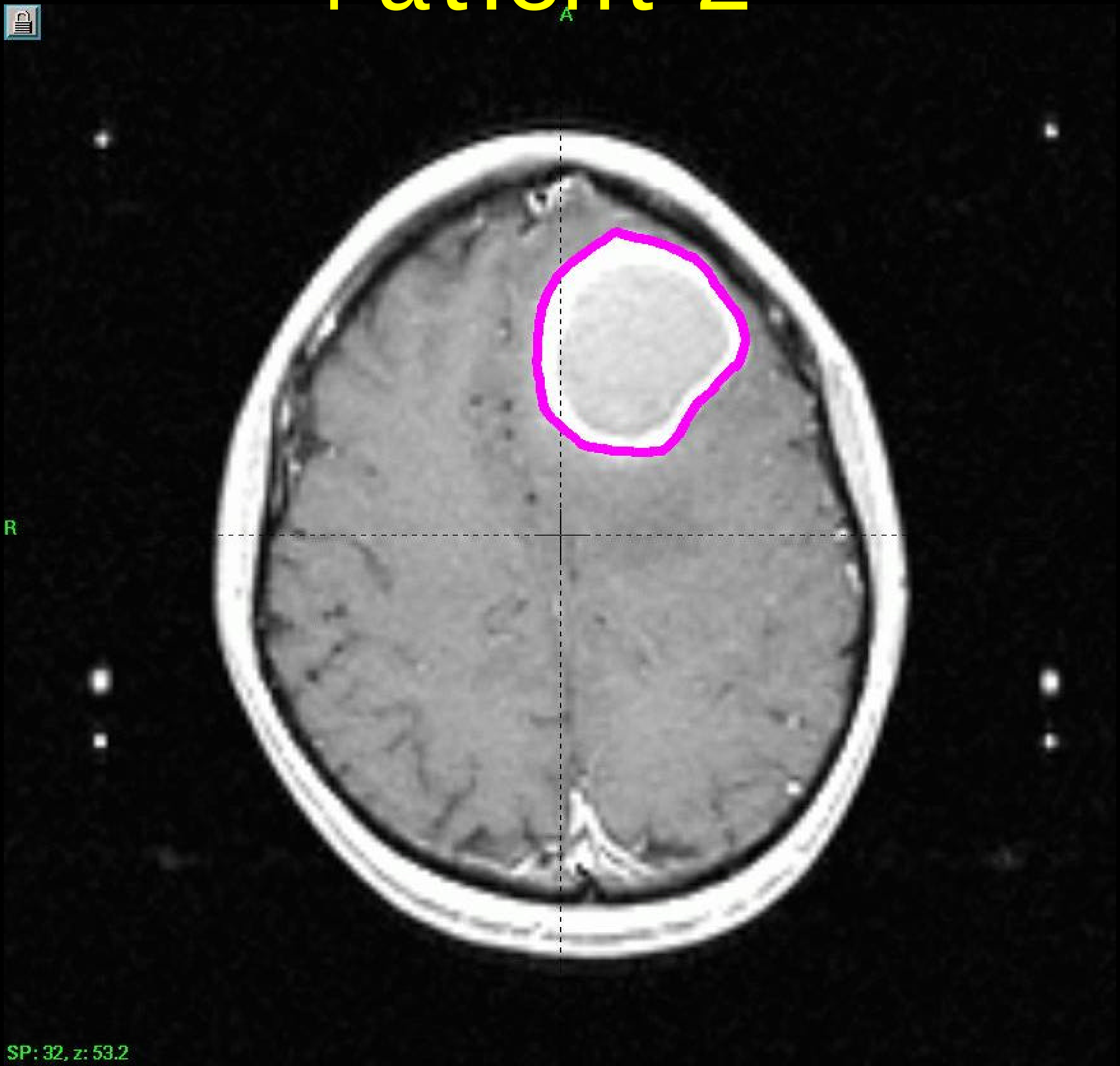
optimized





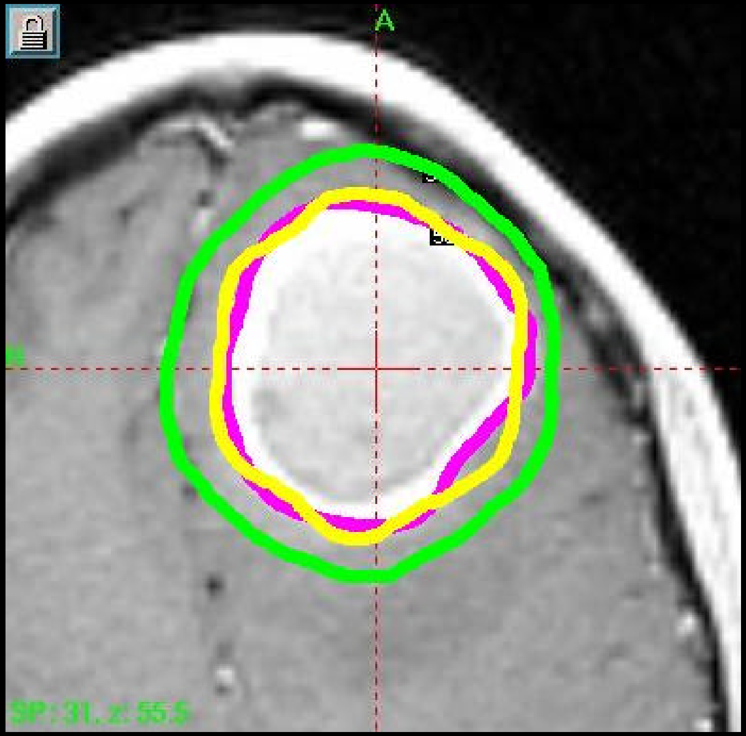
manual
automated
tumor
brain
1
0.8
0.6
0.4
0.2
0
Fraction of Volume
0
2
4
6
8
10
Dose(Gy)

Patient 2

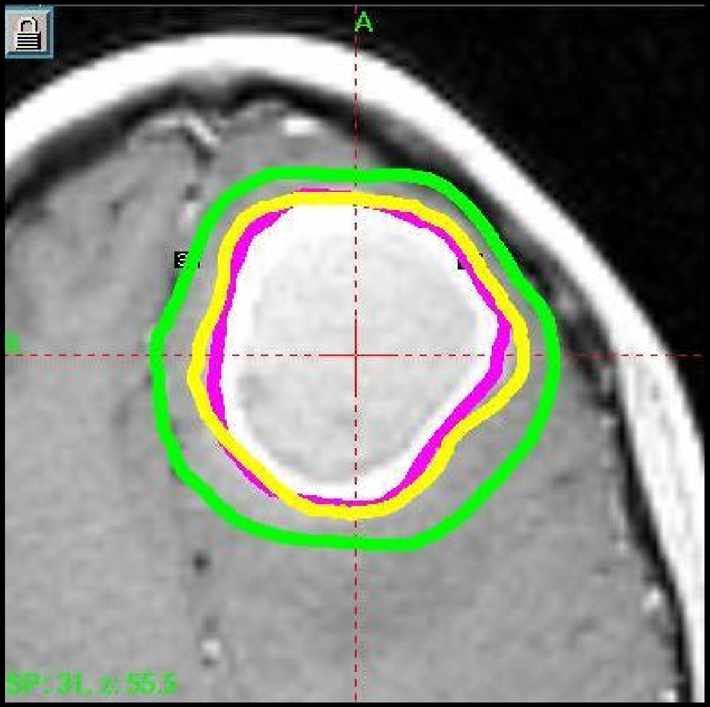


Patient 2 - Axial slice

15 shot manual



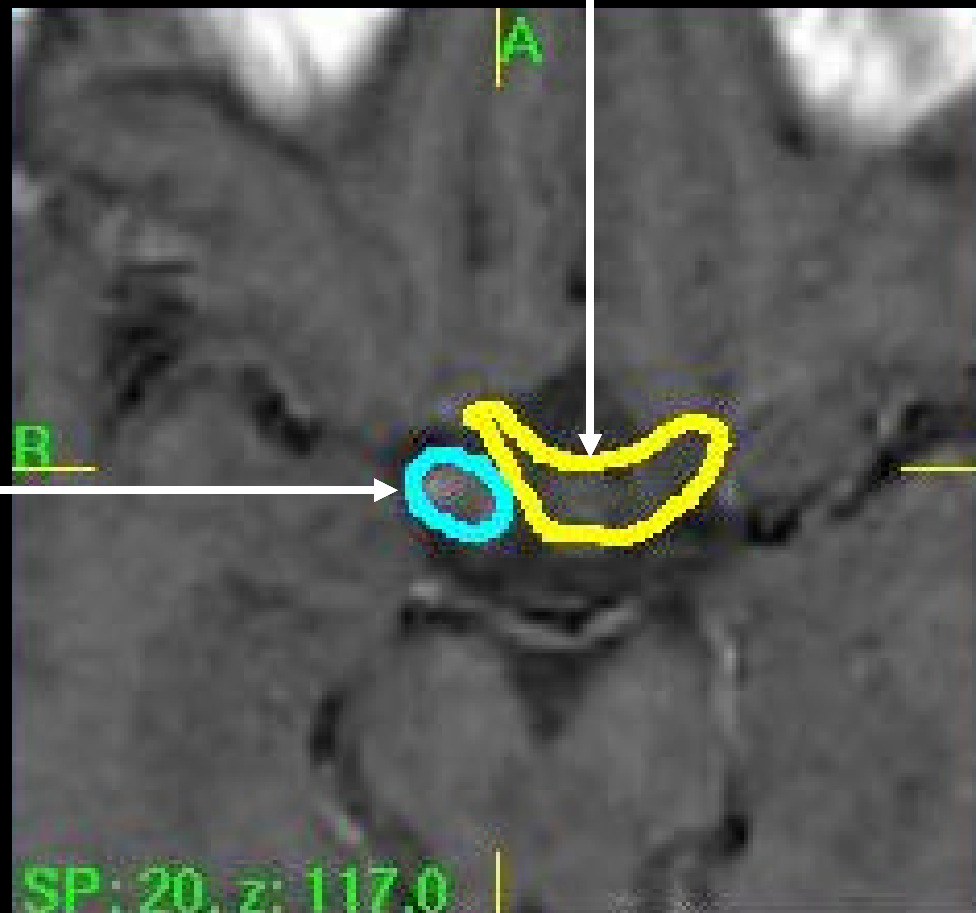
12 shot optimized

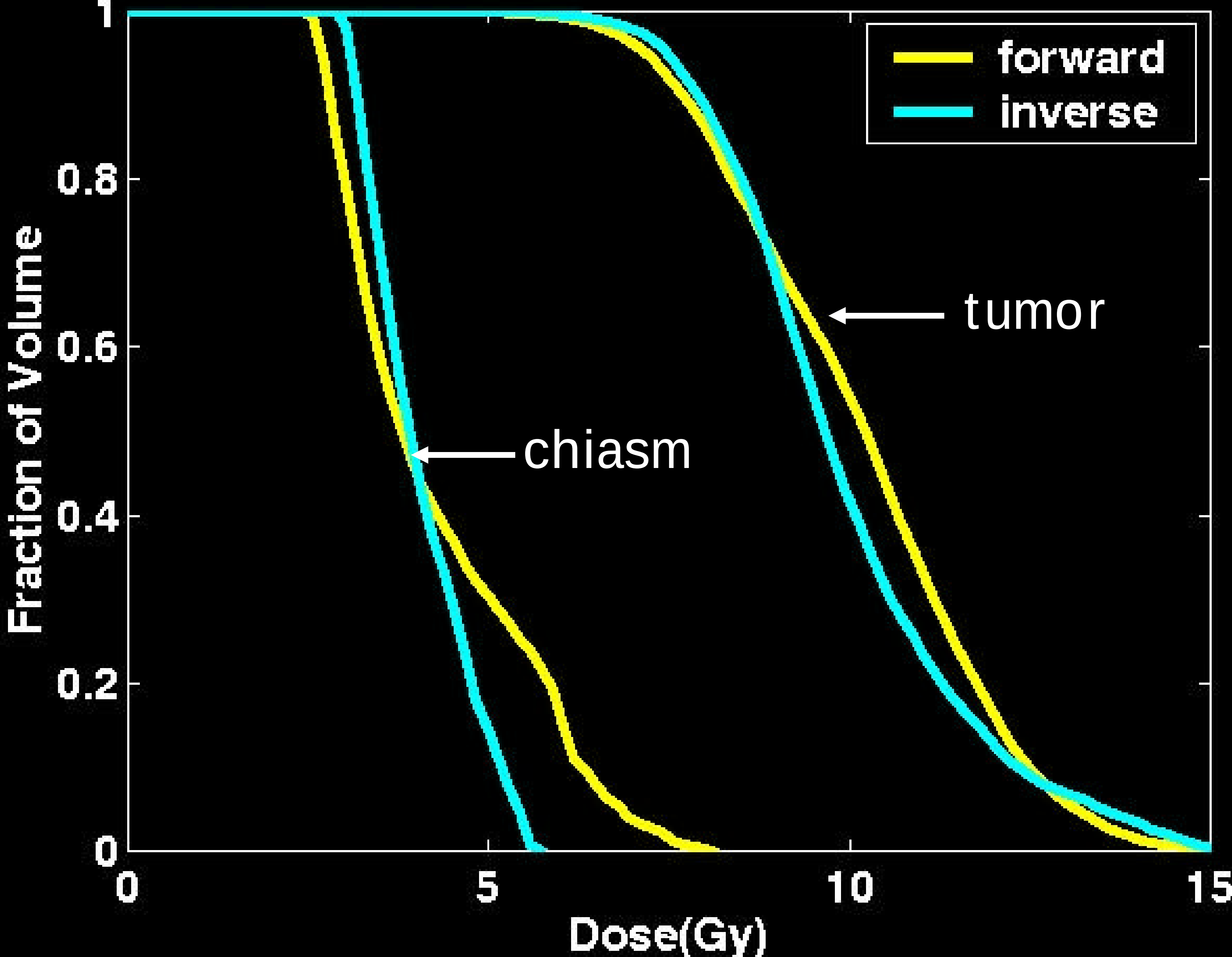


Patient 3

optic chiasm

pituitary adenoma

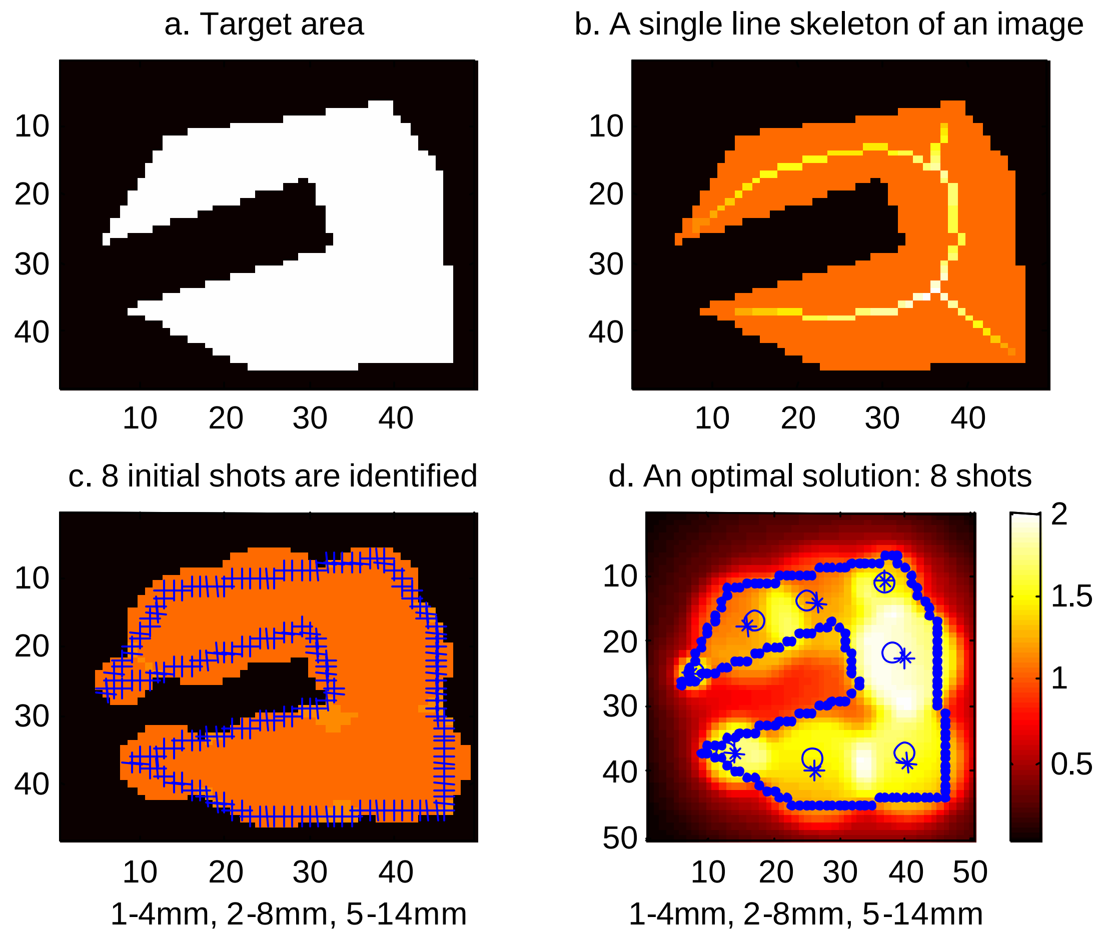



forward
inverse
tumor
chiasm
Fraction of Volume
Dose(Gy)
1
0.8
0.6
0.4
0.2
0
0
5
10
15

Speed

- Speed is quite variable, influenced by:
  - tumor size, number of shots
  - computer speed
  - grid size, quality of initial guess
- In most cases, an optimized plan can be produced in 10 minutes or less on a Sparc Ultra-10 330 MHz processor
- For very large tumor volumes, the process slows considerably and can take more than 45 minutes

Skeleton Starting Points



Run Time Comparison

Average Run Time	Size of Tumor		
	Small	Medium	Large
Random (Std. Dev)	2 min 33 sec (40 sec)	17 min 20 sec (3 min 48 sec)	373 min 2 sec (90 min 8 sec)
SLSD (Std. Dev)	1 min 2 sec (17 sec)	15 min 57 sec (3 min 12 sec)	23 min 54 sec (4 min 54 sec)

DSS: Estimate number of shots

- Motivation:
  - Starting point generation determines reasonable target volume coverage based on target shape
  - Use this procedure to estimate the number of shots for the treatment
- Example,
  - Input:
    - number of different helmet sizes = 2;
    - (4mm, 8mm, 14mm, and 18mm) shot sizes available
  - Output:

Helmet size(mm)	4 & 8	4 & 14	4 & 18	8 & 14	8 & 18	14 & 18
# shots estimated	25	10	9	7	7	7

Conclusions

- An automated treatment planning system for Gamma Knife radiosurgery has been developed using optimization techniques (GAMS, CONOPT and CPLEX)
- The system simultaneously optimizes the shot sizes, locations, and weights
- Automated treatment planning should improve the quality and efficiency of radiosurgery treatments

Conclusions

- Problems solved by models built with multiple optimization solutions
- Constrained nonlinear programming effective tool for model building
- Interplay between OR and MedPhys crucial in generating clinical tool
- Gamma Knife: optimization compromises enable real-time implementation