



Stochastic Optimization: Recent Enhancements in Algebraic Modeling Systems

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GAMS at a Glance

The screenshot displays the GAMS software environment. At the top, a menu bar includes File, Edit, Search, Windows, Utilities, and Help. Below it, a toolbar contains various icons for file operations and editing. The main workspace is divided into several panes:

- Code Editor:** Contains GAMS code for creating an example GDX file. The code includes comments and commands like `set`, `parameter`, `YearData`, and `StockData`.
- Data Table:** A table with columns 'Entry', 'Symbol', 'Type', 'Dim', and 'Nr Elem'. It lists various data entries such as '10 GanttData', '4 Points', '8 Scatter2D', '12 StockData', '11 Surface', and '3 YearDataC'.
- StockData Chart:** A line chart titled 'StockData' showing the performance of four companies: IBM (red), DELL (green), HP (yellow), and SUN (blue) from 1998 to 2005. The y-axis ranges from 102 to 104.
- Surface Plot:** A 3D surface plot titled 'Surface' showing a sharp peak. The x-axis is labeled with 's2', 's5', 's8', 's12', 's16', 's20', 's24', 's28', 's32', 's36', 's40', 's45', 's49'. The y-axis ranges from -0.2 to 0.6.
- Log Window:** At the bottom, a log window shows the execution status of the job 'chartdat.gms', including start and stop times and elapsed time.

General Algebraic Modeling System:
 Algebraic Modeling Language,
 Integrated Solver, Model
 Libraries, Connectivity- &
 Productivity Tools

Design Principles:

- Balanced mix of declarative and procedural elements
- Open architecture and interfaces to other systems
- Different layers with separation of:
 - model and data
 - model and solution methods
 - model and operating system
 - model and interface



AML and Stochastic Programming (SP)

- Algebraic Modeling Languages/Systems good way to represent optimization problems
 - Algebra is a universal language
 - Hassle free use of optimization solvers
 - Simple connection to data sources (DB, Spreadsheets, ...) and analytic engines (GIS, Charting, ...)
- Large number of (deterministic) models in production
 - Opportunity for *seamless* introduction of new technology like Global Optimization, Stochastic Programming, ...
 - AML potential framework for SP

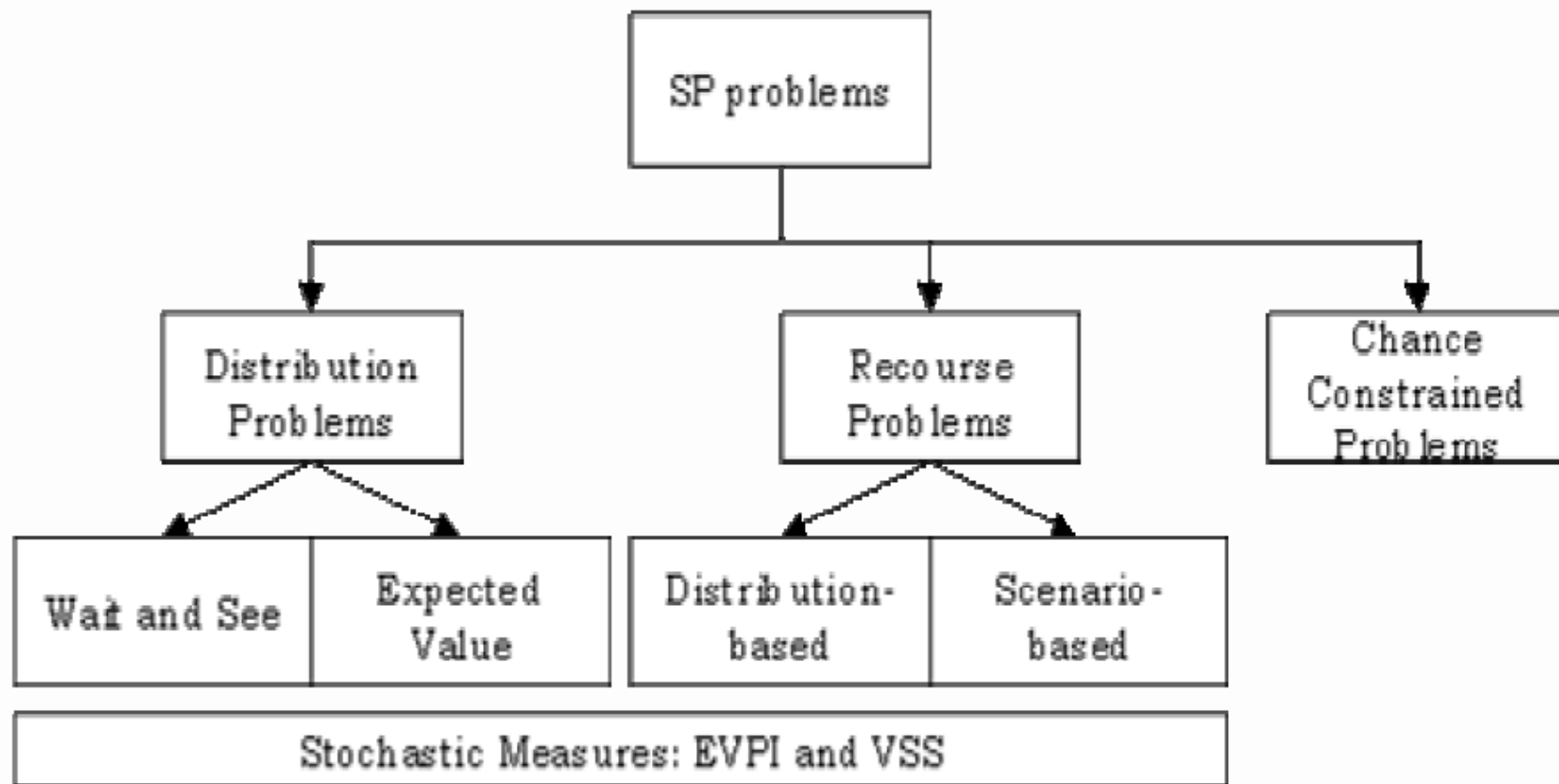


Stochastic Programming Claims and '*Facts*'

- Lots of application areas (Finance, Energy, Telecommunication)
- Mature field (Dantzig '55)
- Variety of SP problem classes with specialized solution algorithms (e.g. Bender's Decomposition)
- Compared to deterministic mathematical programming (MP) small fraction
 - Only 0.2% of NEOS submission to SP solvers
- No/few commercially supported solvers for SP
- Various frustrations with industrial SP projects



Some Stochastic Programming Classes



Source: G. Mitra



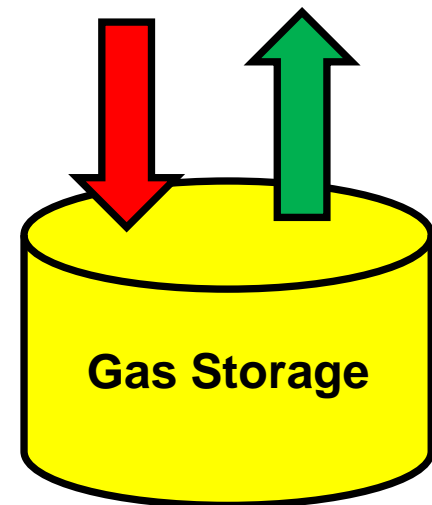
Example Model: Gas Price Model

Natural Gas NYMEX Weekly Price Chart



Inject/
Buy

Withdraw/
Sell



⇒ gas1.gms
(deterministic model)



n-Stage Stochastic Programs

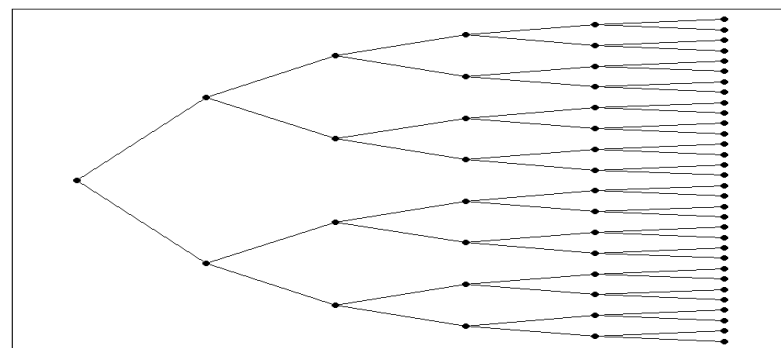
- Construct Scenario Tree:
 - Start with today's price and use a (discrete) distribution
 - Realizations: up, down
- Deterministic Equivalent (DE) (block structure)
 - Here and Now (HN, Mitra)
 - Nested Bender's Decomposition (OSLSE, FortSP, AIMMS)
 - In practice Barrier method

$$Z_{HN} = \min_{x_1} \left\{ c_1 x_1 + E_{\xi_2} \left[\min_{x_2} c_2 x_2 + E_{\xi_3 | \xi_2} \left[\min_{x_3} c_3 x_3 + \dots + E_{\xi_T | \xi_{T-1}, \dots, \xi_2} \min_{x_T} c_T x_T \right] \right] \right\}$$

subject to:

$$\begin{aligned} A_{11}x_1 &= b_1 \\ A_{21}x_1 + A_{22}x_2 &= b_2 \\ A_{31}x_1 + A_{32}x_2 + A_{33}x_3 &= b_3 \\ \vdots &\vdots \\ A_{T1}x_1 + A_{T2}x_2 + A_{T3}x_3 + \dots + A_{TT}x_T &= b_T \end{aligned}$$

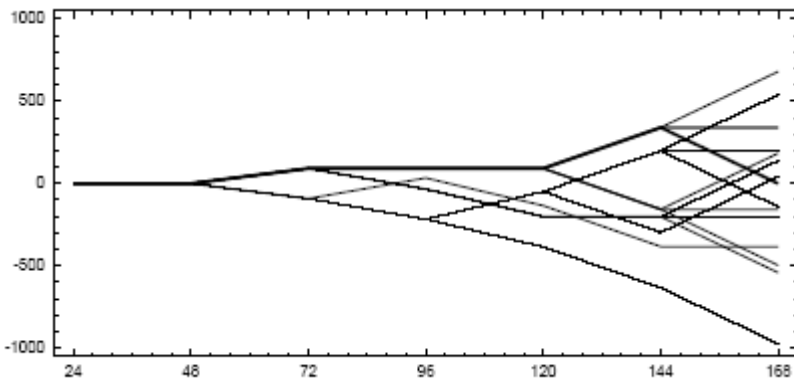
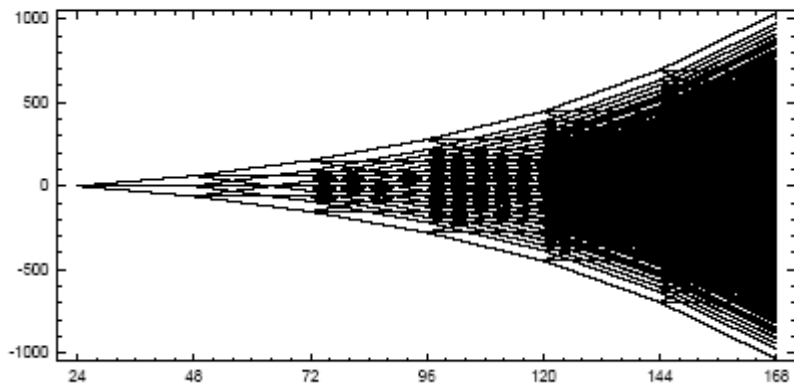
$$\ell_t \leq x_t \leq u_t;$$



⇒ **gas2.gms**
(n-stage SP, distribution)



ScenRed (Römisch et. al., HU Berlin)



- Find good approximation of original scenario tree of significant smaller size.
- Available since 2002
- Integrated in GAMS system
- No extra cost

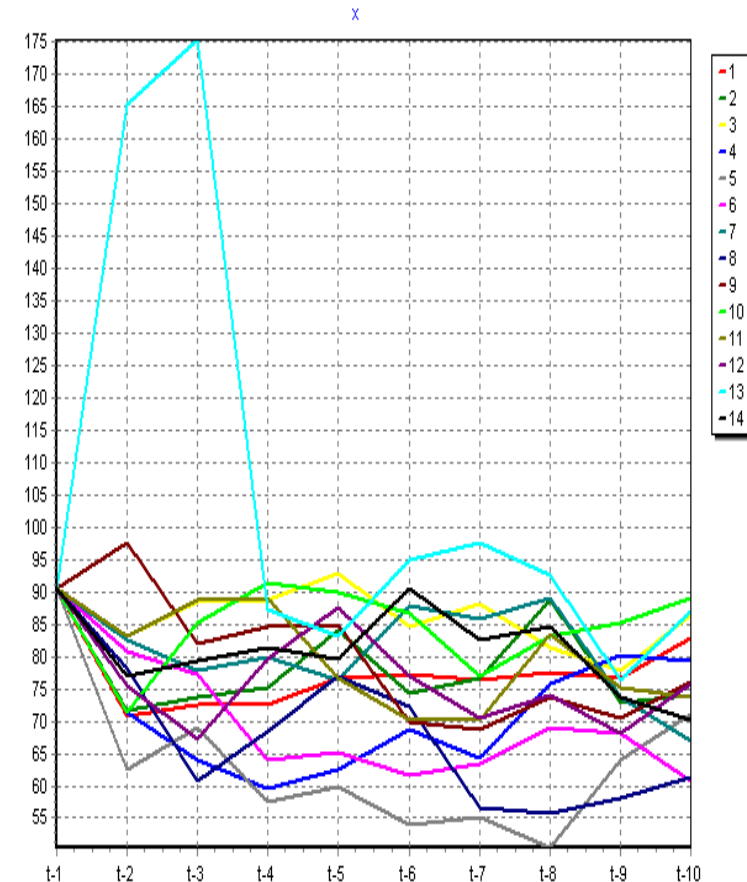
gas2.gms
⇒ (n-stage SP, distribution)
plus ScenRed



Scenario based Stochastic Programs

- Random variables with distributions versus independent scenarios:
- Wait-and-see (WS)
 - solve scenarios independently (grid computing)
- Expected value problem (EV)
 - Calculate EV of random variables and solve
- Expectation of EV problem (EEV)
 - Fix decisions of EV problem and evaluate all scenarios

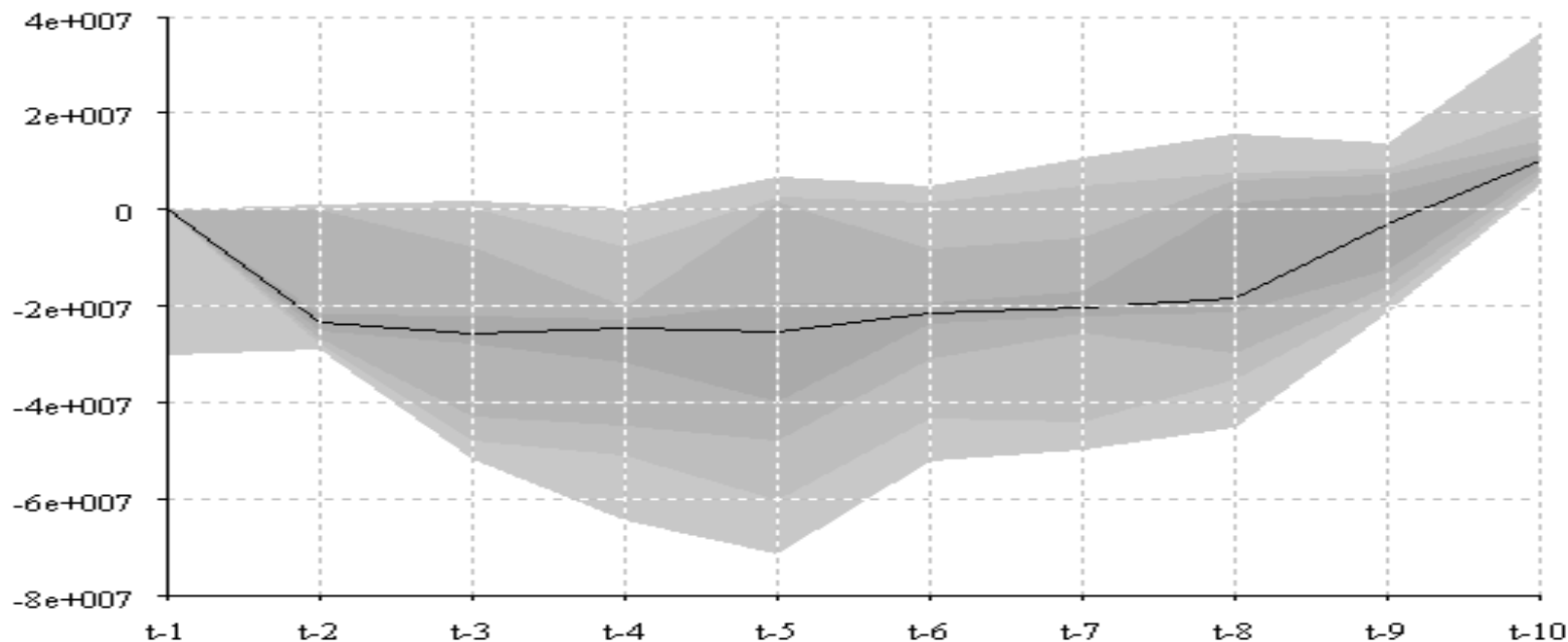
⇒ gas3.gms (Scenario based: WS, EEV)





Value of Stochastic Solution/Visualization

- $WS \geq EEV$ (maximization!)
- Visualize results!
 - e.g. fan plot (Tom Rutherford, ETH Zürich)

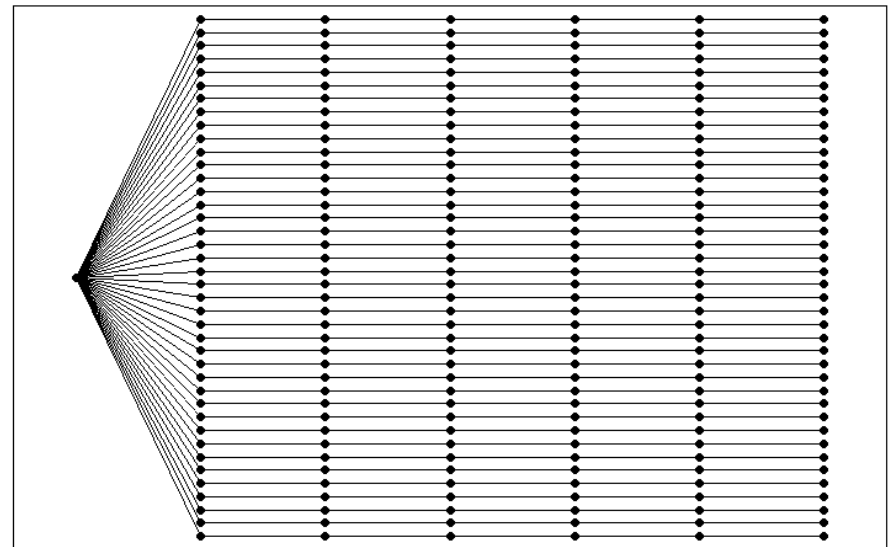


⇒ gas3.gms (Scenario based: fan plot)



2-Stage Stochastic Programs

- SP Solver DECIS (Gerd Infanger, Stanford, USA)
 - Stores only one instance of the problem and generates scenario sub-problems as needed
 - Solution Strategies
 - Universe problem (all scenarios)
 - Sampling: Crude Monte Carlo/Importance sampling

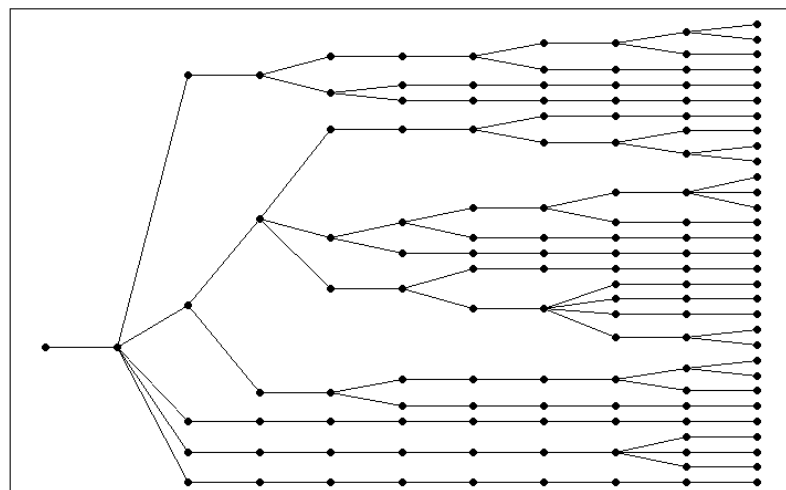
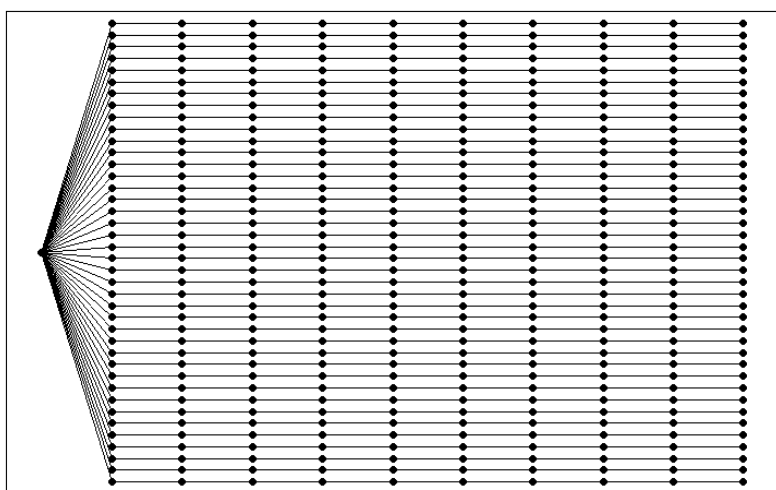


⇒ gas4.gms (2-stage, DECIS)



Tree Generation: ScenRed2

- Construct a true scenario tree from independent scenarios:



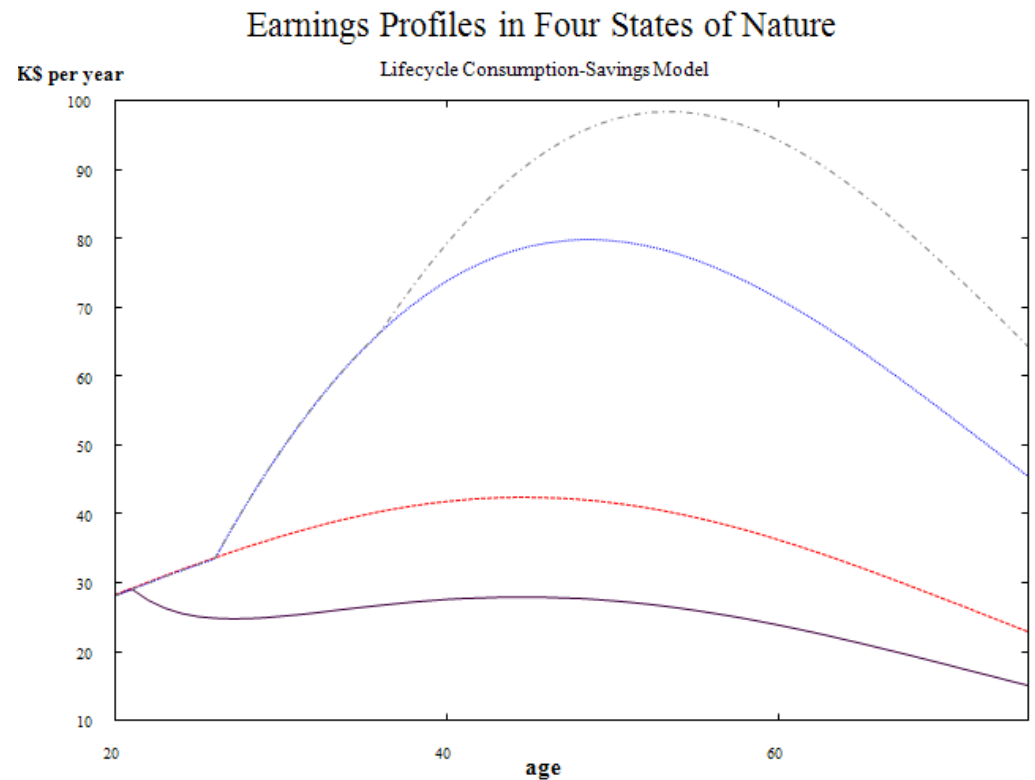
- Reconstruct underlying distribution from a set of scenarios
- Here-and-now (HN): $WS \geq HN \geq EEV$
- Value of stochastic solution $VSS = HN - EEV$
- Expected value of perfect information $EVPI = WS - HN$

⇒ gas5.gms (n-stage, ScenRed2)



The Rich World of Stochastic Programming

- n-stage stochastic linear programming (SLP) just one option
- SP models from application areas exist (finance)
- Economic modeling
 - mixed complementarity problems
 - scenario trees with few branches





Conclusion

- Stochastic Programming still challenging and developing field
 - GUPOR: *Uncertainty: An OR Frontier* (Greenberg, 2006)
- Lack of solution technology limits the dissemination of SP
- There is more to SP than n-stage SLP
- Representation of results
- Collection of comprehensive & reproducible examples could help to *spread the word*



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