Stochastic Optimization: Recent Enhancements in Algebraic Modeling Systems

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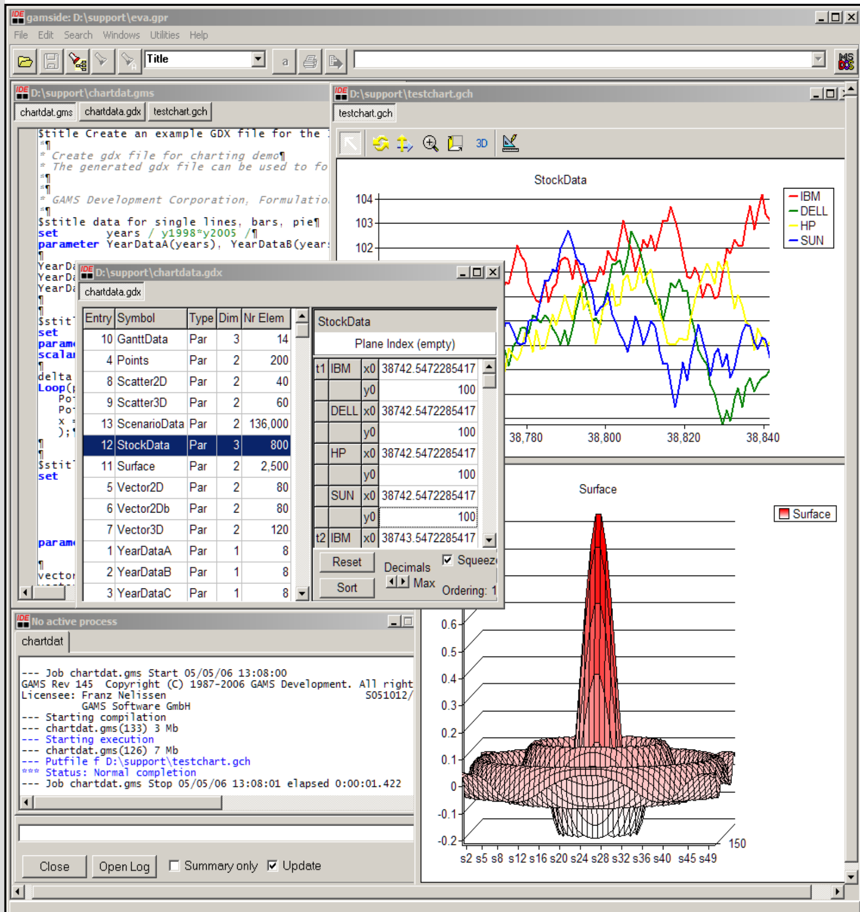
GAMS at a Glance



General **A**lgebraic **M**odeling **S**ystem:

Algebraic Modeling Language, Integrated Solver, Model Libraries, Connectivity- & Productivity Tools

Design Principles:

- Balanced mix of declarative and procedural elements
- Open architecture and interfaces to other systems
- Different layers with separation of:
  - model and data
  - model and solution methods
  - model and operating system
  - model and interface

AML and Stochastic Programming (SP)

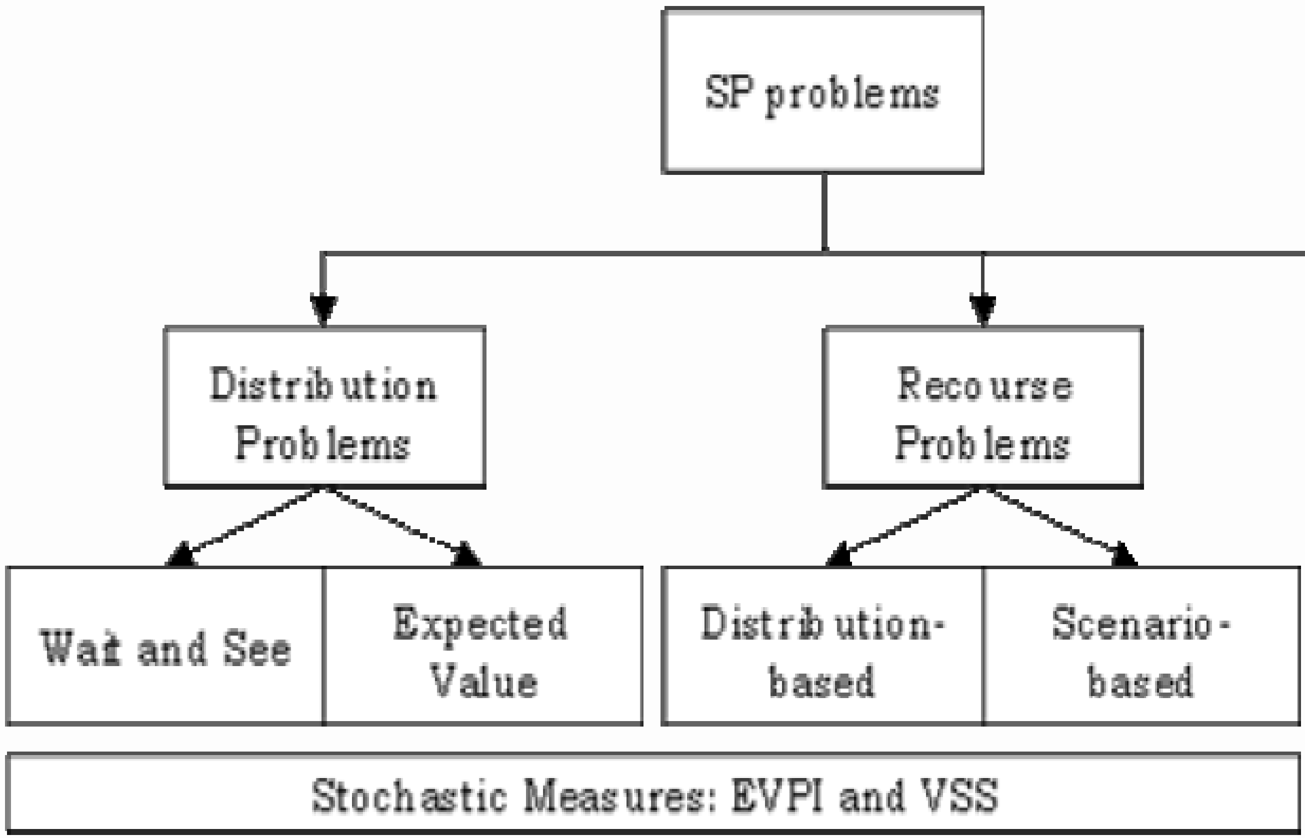
- Algebraic Modeling Languages/Systems good way to represent optimization problems
  - Algebra is a universal language
  - Hassle free use of optimization solvers
  - Simple connection to data sources (DB, Spreadsheets, …) and analytic engines (GIS, Charting, …)
- Large number of (deterministic) models in production
  - Opportunity for *seamless* introduction of new technology like Global Optimization, Stochastic Programming, …
  - AML potential framework for SP

Stochastic Programming Claims and ‘*Facts*’

- Lots of application areas (Finance, Energy, Telecommunication)
- Mature field (Dantzig ’55)
- Variety of SP problem classes with specialized solution algorithms (e.g. Bender’s Decomposition)

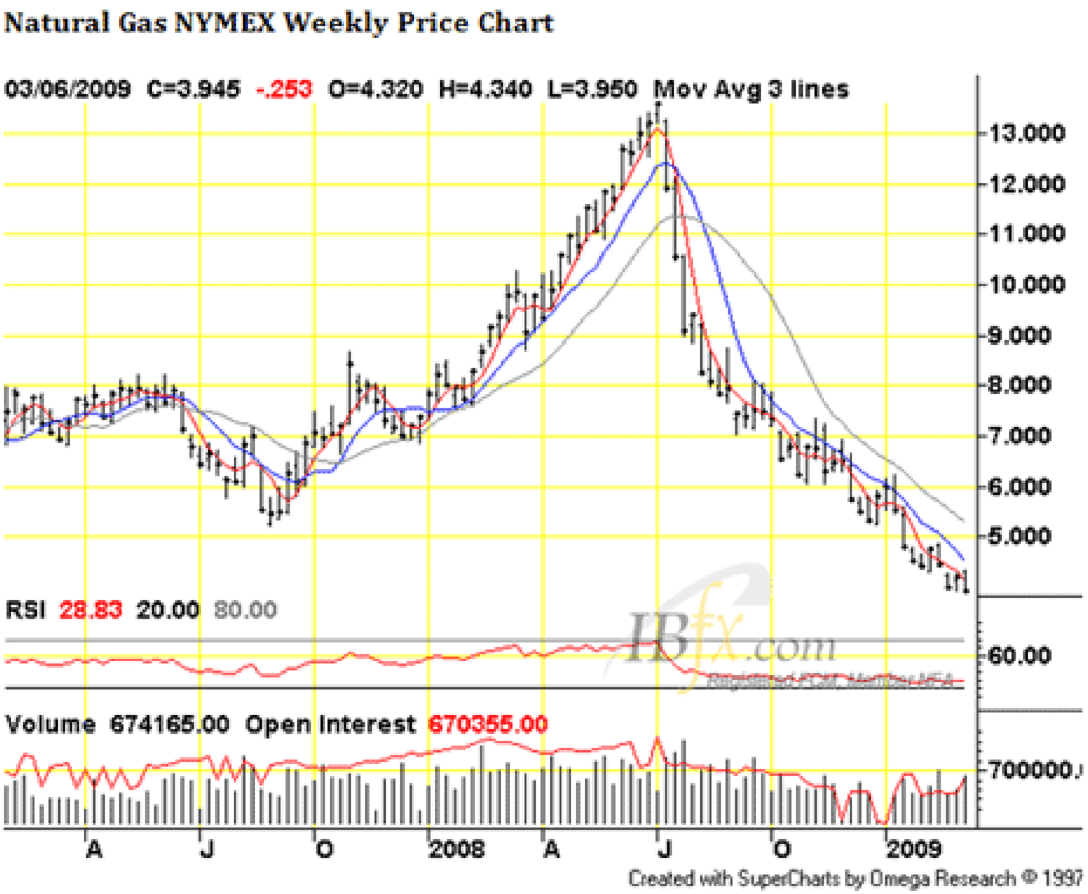
- Compared to deterministic mathematical programming (MP) small fraction
  - Only 0.2% of NEOS submission to SP solvers
- No/few commercially supported solvers for SP
- Various frustrations with industrial SP projects

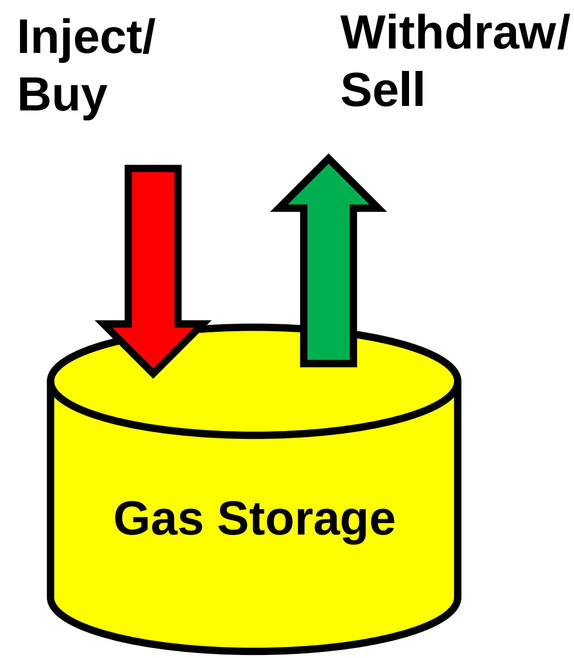
Some Stochastic Programming Classes



Source: G. Mitra

Example Model: Gas Price Model





⇒ **gas1.gms**
(deterministic model)

n-Stage Stochastic Programs

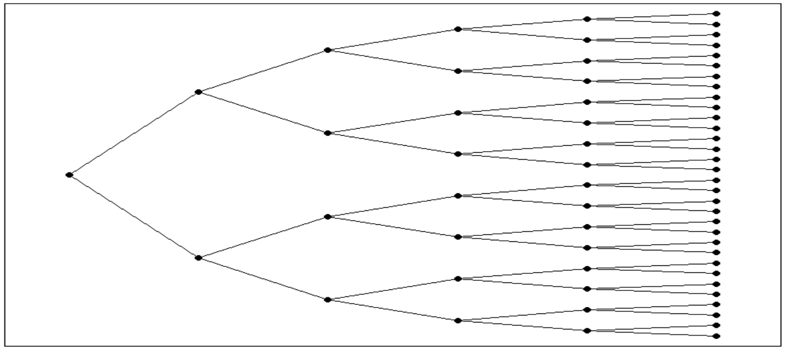
- Construct Scenario Tree:
  - Start with today's price and use a (discrete) distribution
  - Realizations: up, down
- Deterministic Equivalent (DE) (block structure)
  - Here and Now (HN, Mitra)
  - Nested Bender's Decomposition (OSLSE, FortSP, AIMMS)
  - In practice Barrier method

$$Z_{HN} = \min_{x_1} \left\{ c_1 x_1 + E_{\xi_2}\left[\min_{x_2} c_2 x_2 + E_{\xi_3|\xi_2}\left[\min_{x_3} c_3 x_3 + \ldots + E_{\xi_T|\xi_{T-1}|\ldots|\xi_2} \min_{x_T} c_T x_T \right]\right]\right\}$$

subject to:

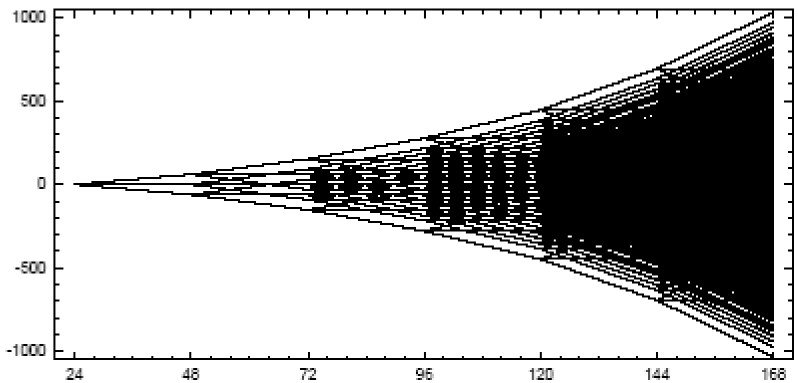
$$\begin{array}{lllll} A_{11}x_1 & & & & = b_1 \\ A_{21}x_1 + & A_{22}x_2 & & & = b_2 \\ A_{31}x_1 + & A_{32}x_2 + & A_{33}x_3 & & = b_3 \\ \vdots & & & \ddots & \vdots \\ A_{T1}x_1 + & A_{T2}x_2 + & A_{T3}x_3 + & \ldots & + A_{TT}x_T = b_T \end{array}$$

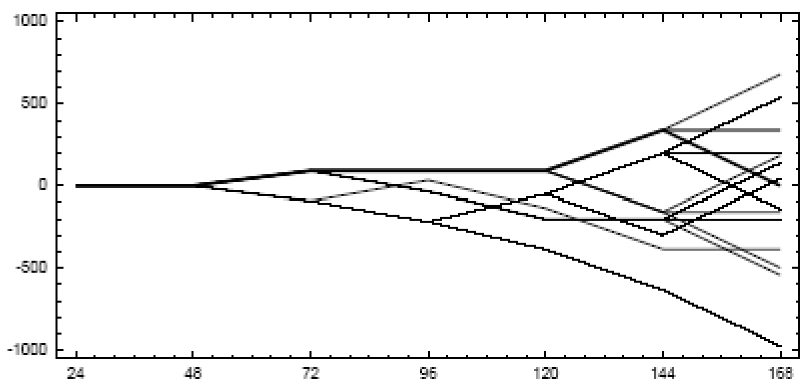
$$\ell_t \le x_t \le u_t;$$



gas2.gms
(n-stage SP, distribution)

ScenRed (Römisch et. al., HU Berlin)



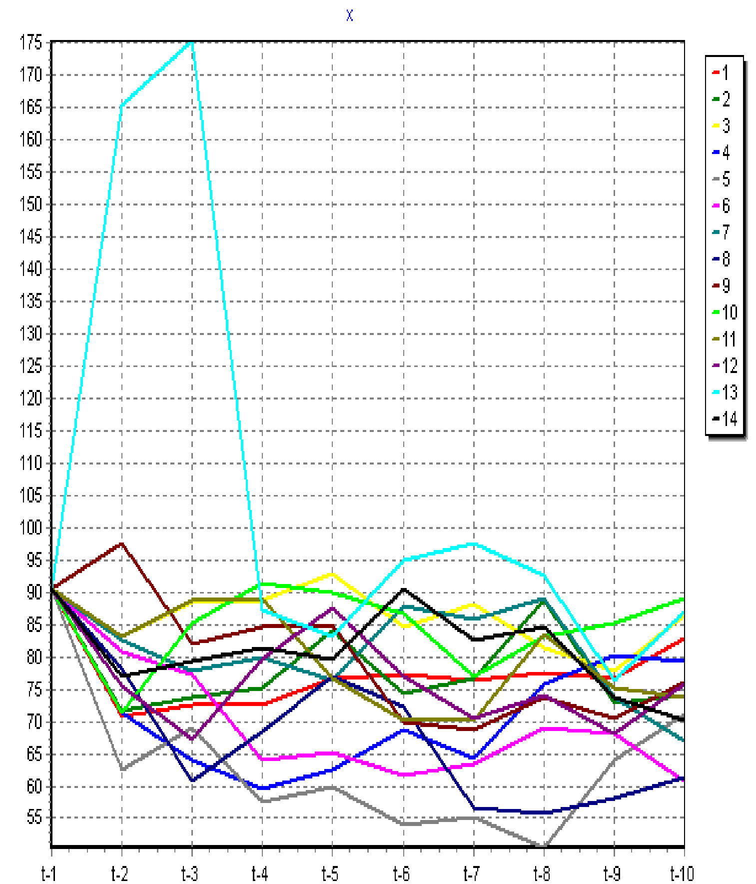


- Find good approximation of original scenario tree of significant smaller size.
- Available since 2002
- Integrated in GAMS system
- No extra cost

⟹ **gas2.gms**
(n-stage SP, distribution)
plus ScenRed

Scenario based Stochastic Programs

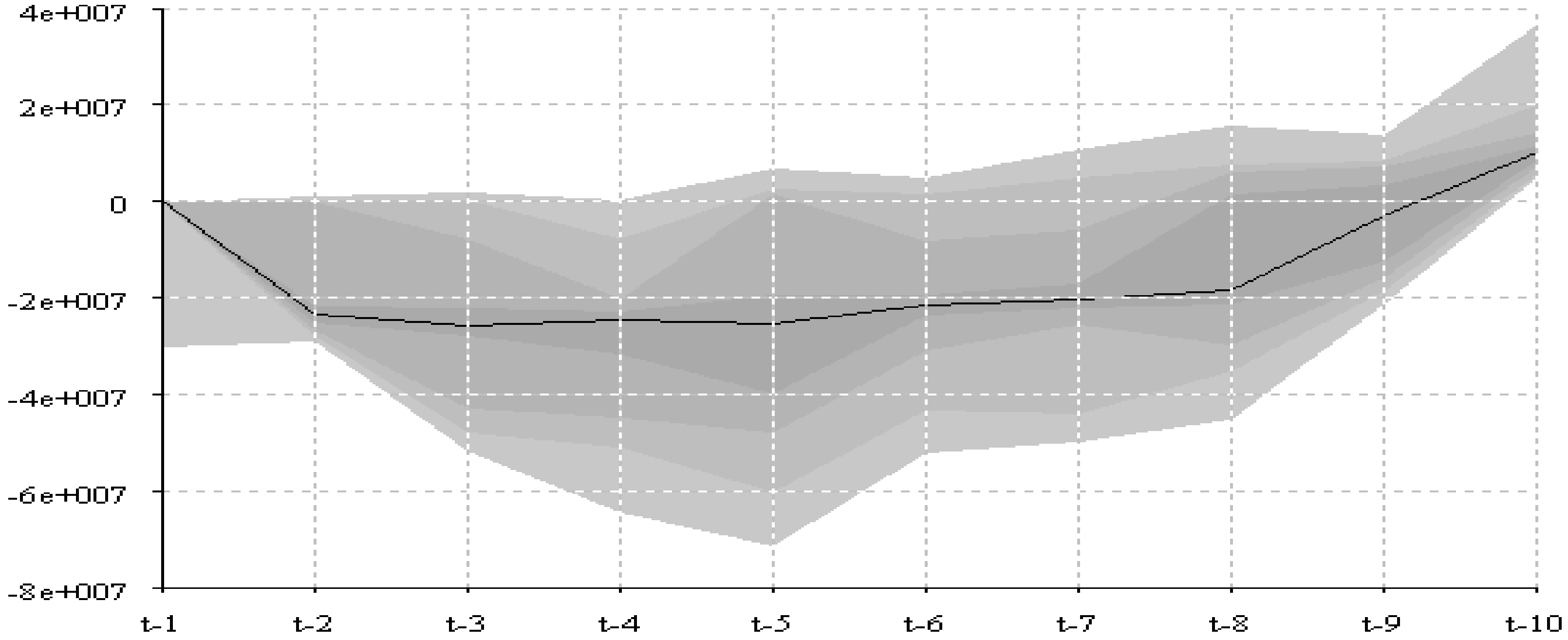
- Random variables with distributions versus independent scenarios:
- Wait-and-see (WS)
  – solve scenarios independently (grid computing)
- Expected value problem (EV)
  – Calculate EV of random variables and solve
- Expectation of EV problem (EEV)
  – Fix decisions of EV problem and evaluate all scenarios



⟹ **gas3.gms (Scenario based: WS, EEV)**

Value of Stochastic Solution/Visualization

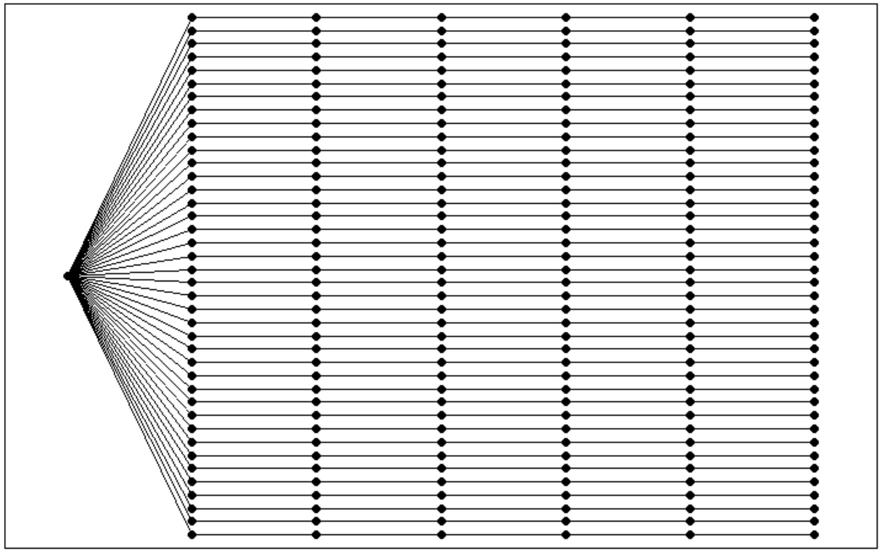
- WS ≥ EEV (maximization!)
- Visualize results!
  – e.g. fan plot (Tom Rutherford, ETH Zürich)



⟹ **gas3.gms (Scenario based: fan plot)**

2-Stage Stochastic Programs

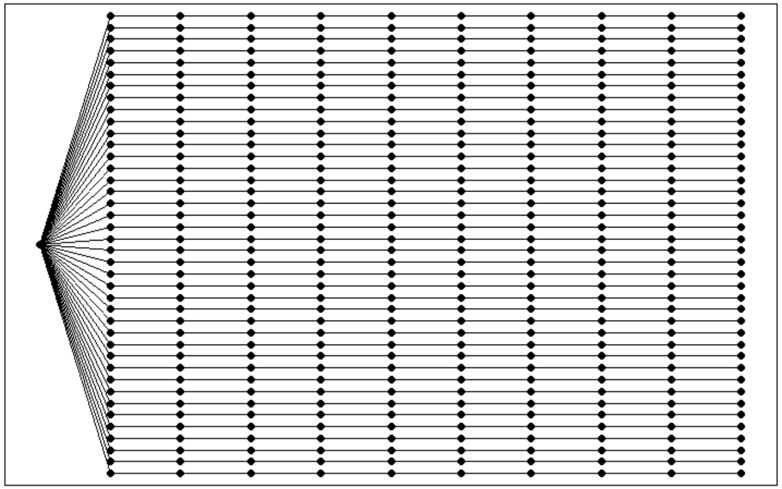
- SP Solver DECIS (Gerd Infanger, Stanford, USA)
  - Stores only one instance of the problem and generates scenario sub-problems as needed
  - Solution Strategies
    - Universe problem (all scenarios)
    - Sampling: Crude Monte Carlo/ Importance sampling

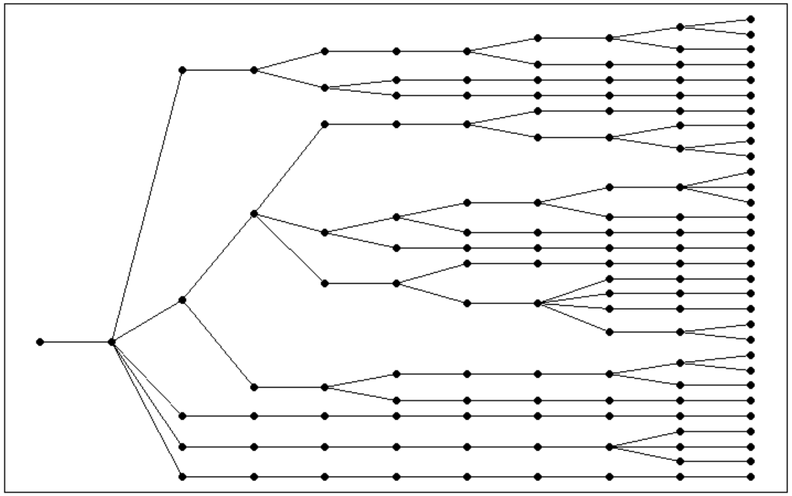


⟹ **gas4.gms (2-stage, DECIS)**

Tree Generation: ScenRed2

- Construct a true scenario tree from independent scenarios:





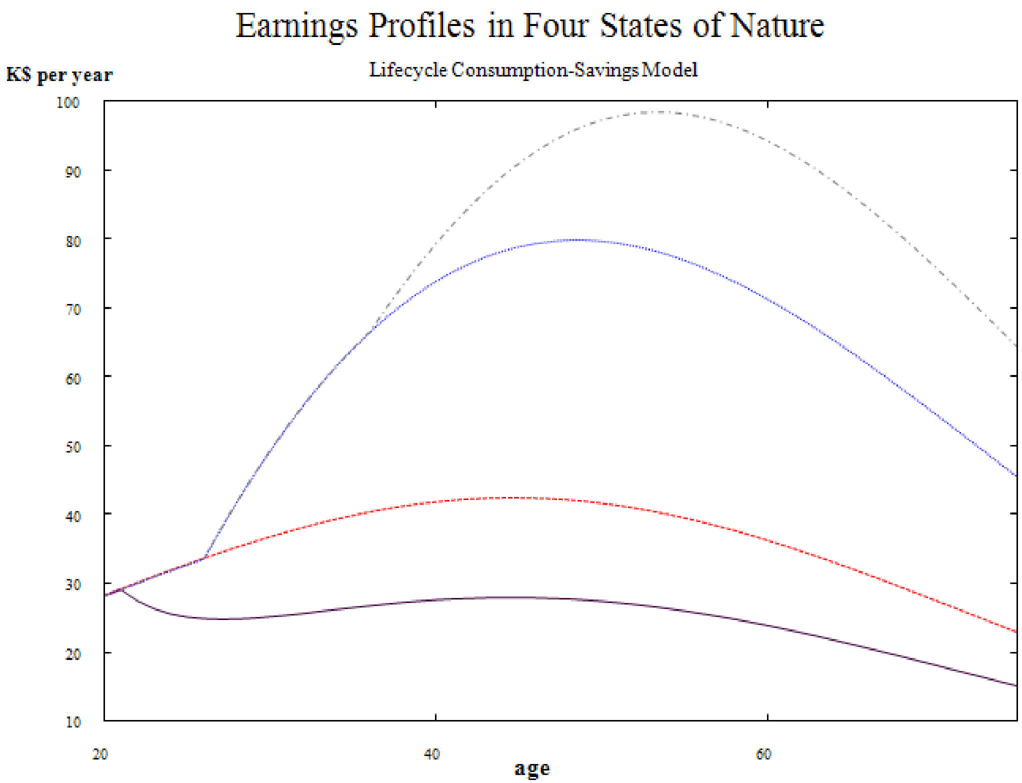
- Reconstruct underlying distribution from a set of scenarios
- Here-and-now (HN): WS $\geq$ HN $\geq$ EEV
- Value of stochastic solution VSS = HN-EEV
- Expected value of perfect information EVPI = WS-HN

⟹ **gas5.gms (n-stage, ScenRed2)**

The Rich World of Stochastic Programming

- n-stage stochastic linear programming (SLP) just one option
- SP models from application areas exist (finance)

- Economic modeling
  - mixed comple-mentarity problems
  - scenario trees with few branches



Conclusion

- Stochastic Programming still challenging and developing field
  - GUPOR: *Uncertainty: An OR Frontier* (Greenberg, 2006)
- Lack of solution technology limits the dissemination of SP
- There is more to SP than n-stage SLP
- Representation of results
- Collection of comprehensive & reproducible examples could help to *spread the word*

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