

# Algebraic Modeling Past, Present and Future

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#### **Model Structure**

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J.H. Duloy, R.D. Norton, CHAC

4 (c) Regional farmer employment accounting rows:

$$-RESr + 3 \sum_{d \in r} \sum_{q} dFLq + \sum_{d \in r} \sum_{t} dFLt = 0, \text{ each } r$$

$$-\begin{bmatrix} \text{Regional farmer} \\ \text{employment} \\ \text{activity} \end{bmatrix} + 3 \begin{bmatrix} \text{Sum over districts} \\ \text{and quarters of} \\ \text{quarterly farmer} \\ \text{employment} \end{bmatrix}^{37}$$

$$+\begin{bmatrix} \text{Sum over districts} \\ \text{and months of} \\ \text{monthly farmer employment} \end{bmatrix}^{37} = 0$$

1 (d) Total employment accounting row in man-years:
$$-12LMAN + \sum_{t} LMANt = 0$$

$$-12\begin{bmatrix} \text{Total employment} \\ \text{in man-years} \end{bmatrix} + \begin{bmatrix} \text{Sum over months of} \\ \text{total employment} \\ \text{in man-months} \end{bmatrix} = 0$$

12 (e) Total monthly employment accounting rows in man-months:
$$-2.2LMANt + \sum_{d} dDLt + \sum_{d} dFLq + \sum_{d} dFLt = 0,$$
each  $t$  and  $q$  such that  $t \in q$ 

$$-2.2\begin{bmatrix} \text{Total} \\ \text{employment} \\ \text{in month } t \end{bmatrix}^{3.8} + \begin{bmatrix} \text{Sum over districts of} \\ \text{day labor employment} \\ \text{in month } t \end{bmatrix}$$

$$\begin{bmatrix} \text{Sum over districts of} \\ \text{quarterly farmer} \\ \text{employment in the} \\ \text{quarter containing} \\ \text{month } t \end{bmatrix}$$

$$\begin{bmatrix} \text{Sum over districts} \\ \text{of monthly farmer} \\ \text{employment} \end{bmatrix} = 0$$

$$3^{7} \text{ In irrigation districts the quarterly contract device is used for farmers, but in non-irrigated districts farmers are assumed to be available on a monthly basis, so that seasonal migration to irrigated areas may occur.$$

38 The activities for hiring farmers and day laborers are stated in units of tens of man-days per month (or quarter), and there are 22 working days per month; hence the

conversion factor of 2.2 is required in the first term of this equation.



# **Model Data**

Table 3

Sequence of standard operations for cotton cultivation (days of unskilled labor, machinery services, and draft animal services required per hectare by month)

Cultivation month and operation	Mechanized		Partially mechanized			Non-mechanized	
	Unskilled labor	Machinery	Unskilled labor	Machinery	Animals	Unskilled labor	Animal
1st Preparatory tasks		0.12		0.12		1.0	2.0
Fallow		0.5		0.5		3.0	6.0
Cross-plowing						2.5	5.0
Harrowing		0.2		0.2		0.5	1.0
Land levelling		0.25		0.25		1.0	2.0
Canal cleaning	1.0		1.0			1.0	
2nd Irrigation ditches	1.0	0.2	1.0	0.2		2.0	2.0
Forming borders		0.2		0.2		2.0	
Linking borders b	1.0		1.0				
Water application	2.0		2.0.			2.0	
Harrowing		0.2		0.2		2.0	4.0
Seeding and fertilization	0.2	0.2	0.2	0.2		4.0	
Maintenance of field works		0.2	0.2			2.0	
3rd Thinning plants	4.0		4.0			4.0	
Cultivation		0.2	2.0		4.0	2.0	4.0
Weeding	6.0		6.0	. •		6.0	
Applications of insecticides (2)							

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L.M. Bassoco, T. Rendón, Data base for CHAC



## **Matrix Generator**

```
CUMPPE
  IF 1X(248), LT 0.5, AND X(243), GT
Y(249)'X(249)
                                                                       COMPIS
  IF (X(249), T]0,5,4ND, X(249), GT, (00) Y(249) 7(249,:) *(1+X(249))
Y(250)1 X(250)
                                                                       COMNET
  IF (x(250),LT,0,5,AND,x(250),GT,,00) Y(250)'Z(250,1)*(1+x(250))
                                                                       CUMDNE
  IF (X(251),LT,0,5,AND,X(251),GT,.00) Y(251) Z(251,1)*(1+X(251))
Y(252) 1 X (252)
                                                                       CUMTWO
  IF (X(252),LT'0,5,AND,X(252),GT.,OO) Y(252)/Z(252,1)
Y(253) (X(253)
                                                                       CUMTER
  IF (x(253),LT]0,5,AND,x(253),GT,.00) Y(253) (Z(253,1)*(1*x(253))
Y(254) 1X(254)
                                                                       COMPOU
  IF (X(254), LT 0.5, AND, X(254), GT., 00) Y(254) 12(254
Y(255) 1Y(206) +Y(267)
                                                                       CUMPIV
Y (256) 1X (256)
                                                                       CCMLCT
  IF (X(256), LT 0,5, AND, X(256), GT, .00) Y(256) (2(256, 1) *(1+X(256))
Y(257) X(257)
                                                                       CUMLEG
  IF (X(257), LT'0,5, AND, X(257), GT, .00) Y(257) Z(257, L) *(1+X(257))
Y(258) (258)
                                                                       CUMBLS
  IF (X(258).LT_0.5.AND.X(258).GT..00) Y(258) Z(258,1) +(1+X(258))
Y(259) 1X(259)
                                                                       CU b=
  IF (X(259),LT'0,5,AND,X(259),GT,,00) Y(259)'Z(259,1)*(1+X(259))
Y(260) | X(260)
  IF (X(260).LT 0.5.AND, X(260).GT., 00) Y(260)!Z(260,1)*(1+X(260))
Y(261) 1Y(63)
                                                                       EXPORT
A(595) x(595)
                                                                       NETDII
  IF (X(262), LT 0,5, AND, X(262), GT, ,00) (262) Z(262, 1) *(1+X(262))
Y(263) X(263)
                                                                       NETDFI
  IF (X(263),LT 0,5,AND,X(263),GT,.00) Y(263) Z(263,1)*(1*X(263))
Y(264) 'X(264)
                                                                       HKKRMT
  IF (x(264),LT_C,5,AND,x(264),GT,,00) Y(264) Z(264,1)*(1-x(264))
Y(265) 1X(265)
                                                                       NETTRN
  IF (X(265),LT_C,5,AND,X(265),GT,,00) Y(265)'Z(265,1)*(1-X(265))
A(599),X(599)
                                                                      DFFCUR
  IF (x(266),LT_C,5,AND,x(266),GT,,00) Y(266) Z(266,1) = (1-x(266))
                                                                      DEFCAP
```



# **Matrix Generator Input**

```
AGGREGAT
                           0.0165
ALA ALG ALV ARO AZU CAR CEG CHV FRI GAR JIT JON MAI MAT MEL P
PLU SAL SAN SOR SOT SOY TRI
    0.0286
     99999
                      1 0
AZU AZU
           -0.25
                                 0.0070
                                            2627020.
JIT JIT
                                 0,1150
           -0.4
                                            174752.
PEP PEP
           -0.6
                                 0.0590
                                            19.
85209.
PLU PLU
           -1800.
                                 0.5770
CCC
CHI
               -0.2
CHV
                      14.459
           0.1500
FOR
               -0.3
SOR
                      285.818
           0.0630
CEG
           0.0930
                      0.665
                                 1.0
ALV
           0.0100
                      226.109
                                 1.0
ALA
           0.0400
                      179.019
                                 1.0
GAR
           0.0990
                      1 427
                                 1.0
MAI
           0.0860
                      77.997
                                 1.0
FEC
               -0.3
FRI
           0.1830
                      33,001
                                 1.0
ARO
           0.1220
                      126,197
                                 1.0
PAP
           0.0930
                      27.138
                                 1.0
GAR
                      0.158
           0.0990
                                 1.0
GRA
               -0.1
MAI
           0.0860
                      132.804
TRI
                      343,979
           0.0800
                                 1.0
FRU
               -2.0
SAN
           0.0780
                      10.850
                                 1.0
           0.0680
MEL
                      6.9350
                                 1.0
OLE
               -1.2
SAL
                      193,910
           0.0830
                                 1,0
JON
                      9.224
           0.2410
                                 1.0
CAR
           0.1550
                      72.490
                                 1.0
SOY
           0.1600
                      57.220
                                 1.0
END
ALA
           .02
                      0:0
```



## **MPS File – Column Section**

```
X.ASGHÇ2
           B,AS,,C2
                            -1,00000
X.ASGHC2
           A, TRA
                             6.98400
                             0,33500
           D. . . GH. N
X.ASGHC3
            R...GHC 3
                             1.00000
X.ASGHC3
                           -1.00000
           B.AS.,C3
X, ASGHC 3
           A.TRA
                            6.98400
X. ASGHAS
           D. . . GH. N
                            0.20600
X.ASGHAS
           R... GHAS
                            1.00000
X, ASGHAS
           B.AS.,AS
                           -1.00000
X. ASGHAS
            A, TRA
                            6,98400
X.ASGHS1
           D.,,GH,p
                            0.15000
X,ASGHS1
           R.,,GHS1
                            1,00000
X.ASGHS1
           B.AS.,S1
                           -1.00000
X.ASGHS1
                            6.98400
X.ASGHCN
           R...GHCN
                            1.00000
X.ASGHCN
           B.AS., CN
                           -1.00000
           A, TRA
X.ASGHCN
                            6,98400
X.ASKSC1
           D. . . KS. N
                            0.26000
X.ASKSC1
           R.,,KSC1
                            1.00000
X.ASKSC1
           B.AS..CI
                           -1.00000
           A, TRA
X.ASKSC1
                            7.56000
X.ASKSC2
           D, , , KS , N
                            0.31000
           R,,,KSC2
X,ASKSC2
                            1,00000
X.ASKSC2
           B,AS,.C2
                           -1,00000
X,ASKSC2
           A.TRA
                            7.56000
X,ASKSC3
           D...KS.N
                            0,33500
X.ASKSC3
           R. . . KSC3
                            1.00000
X,ASKSC3
           B,AS,,C3
                           -1.00000
X.ASKSC3
                            7,56000
X.ASKSAS
           D. . . KS. N
                            0,20600
X.ASKSAS
           R...KSAS
                            1.00000
X.ASKSAS
                           -1.00000
           B. AS. . AS
X.ASKSAS
           A, TRA
                            7,56000
X.ASKS31
                            0,15000
           D.,,KS.P
X.ASKS31
           R. . . KSS1
                            1.00000
X.ASKSS1
           B. AS. . S1
                           -1.00000
X.ASKSS1
                            7,56000
          R,,,KSCN
X.ASKSCN
                            1.00000
X. ASKSCN B. AS. . CN
                           -1.00000
```

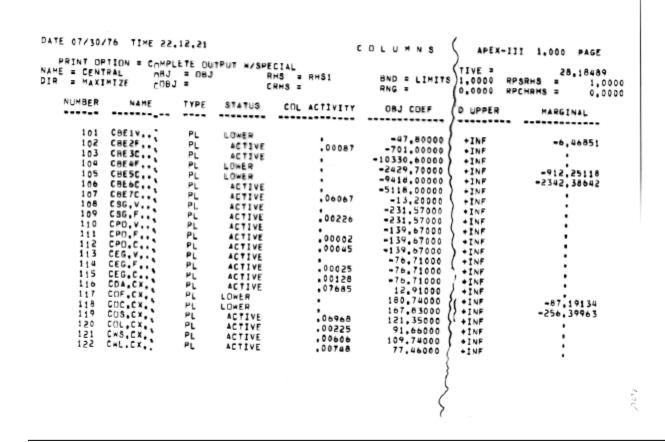


## **MPS** Revision File

```
BRANCH
                             MAJERR
     NEXT
     REVISE
                 REV5
                             TAPE14
                                            **** CARD READ SUMMARY ****
HEADER, CARD NO
                             CNAME
                                              REVA
                             CCOLUMNS
MEADER, CARD
                             e MODIFY
                             CRHS
HEADER, CARD NO
                             @ MODIFY
HEADER, CARD
                             BENDATA
HEADER, CARD
                             SNAME
                                             REV1
HEADER, CARD
                             ecolumns
HEADER, CARD
                             & MODIFY
HEADER, CARD
                             ZENDATA
HEADER. CARD
                       43
                             ENAME
                                             REV2
HEADER, CARD
                             ECOLUMNS
                             e MODIFY
                             CENDATA
                             ENAME
                                             REVA
                      53
                             ERHS
                      54
                             e MODIFY
HEADER, CARD NO
                             CENDATA
MEADER, CARD NO
                             ENAME
                                             REV5
HEADER. CARD
                      7.0
                             ERHS
HEADER, CARD NO
                      71
                                MODIFY
        CARD
                      72
                                   RHS1
                                             CLA.V.01
                                                           5.03328
        CARD
                      73
                                  RHS1
                                             CLA.V.02
                                                            5.03328
        CARD
                      74
                                  RHS1
                                             CLA.V.O3
                                                            5.03328
        CARD NO
                      75
                                  PHS1
                                             CLA.V.OU
                                                           5.03328
        CARD NO
                      76
                                  RHS1
                                             CLA, V. 05
                                                           5.03328
        CARD NO
                      77
                                  RHS1
                                             CLA.V.Ob
                                                           5,03328
        CARD NO
                      78
                                  RHS1
                                             CLA.V.07
                                                           5,03328
        CARD NO
                      79
                                  RHS1
                                             CLA.V.OS
                                                           5.03328
        CARD NO
                      80
                                  RHS1
                                             CLA. V. 09
                                                            5.03328
        CARD NO
                                   RHS1
                                             CLA.V.10
                                                            5.03328
        CARD NO
                                   RHS1
                                             CLA. V. 11
                                                           5,03326
        CARD NO
                      83
                                   RHS1
                                                           5.03328
        CARD NO
                                   RHS1
                                             CLA, V. TO
                                                          60.39936
HEADER, CARD NO.
                             CENDATA
```



# **MPS Output**





PLANNING PROBLEM AND OBJECTIVES INITIALLY OFTEN

UNSTRUCTURED

ILL-DEFINED

CONFLICTING

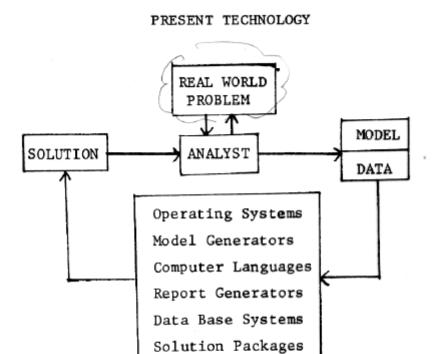
UNCERTAIN

CHANGING

EMOTIONAL

MATHEMATICAL MODEL USED TO RECOGNIZE AND FORMULATE PROBLEMS, DEFINE ISSUES AND EXPLORE SOLUTION SPACE





- Essentially no documentation



MAJOR CONSTRAINTS : COST

SKILLS

TIME

TOOLS

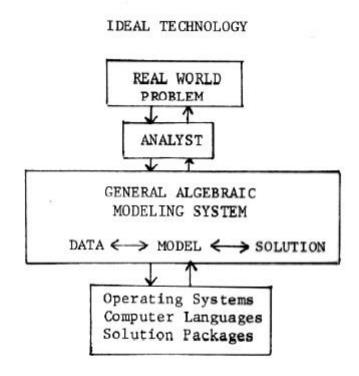
DOCUMENTATION

TRUST

.

.





#### RESULT: - Limited drain of resources

- Same representation of models for humans and machines
- Model representation is also model documentation



#### DEVELOPMENT OF GAMS

#### Phase 1 (1978)

- The system can be used to represent and analyze any algebraic model (be it linear or nonlinear)
- The system can perform algebraic manipulations on all data
- The system can generate and solve linear programs automatically
- The system can generate reports on data and solutions via simple 'display' statements



#### DEVELOPMENT OF GAMS

Phase 2 (1979)

- The system can generate and solve nonlinear programs
- The system will provide links to special-purpose algorithms for econometric problems, network problems, etc.
- Appropriate extensions to the language will be made as the need arises



#### DEVELOPMENT OF GAMS

#### Phase 3 (?)

- Automatic structure recognition
- Internal generation of exact pointderivatives
- Improved data-base design with e.g. unit analysis, and links to existing data bases
- Availability of GAMS on different machines
- World-wide availability of the system so that it can be used as a market for testing models and algorithms



# **Change in Focus**

# **Computation**Past

- Algorithm limits applications
- Problem representation is low priority
- Large costly projects
- Long development times
- Centralized expert groups
- High computational cost, mainframes
- Users left out

#### **Model** Present

- Modeling skill limits applications
- Algebraic model representation
- Smaller projects
- Rapid development
- Decentralized modeling teams
- Low computational cost, workstations
- Machine independence
- Users involved

# Application Future

- Domain expertise limits application
- Off-the-shelf graphical user interfaces
- Links to other types of models
- Models embedded in business applications
- New computing environments
- Internet/web
- Users hardly aware of model



# **Typical GAMS User**

 Successful PhD level professional in any field often outside of traditional OR

10 years of experience

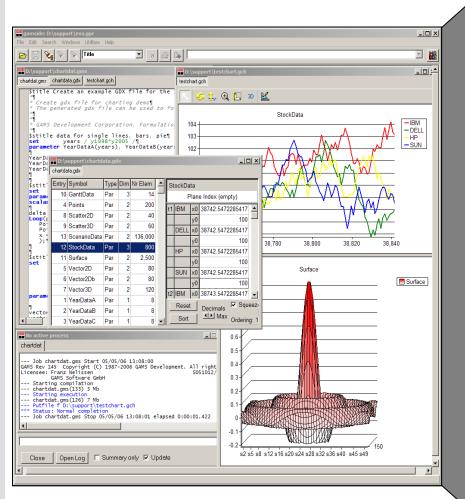
Wants to add value to his work by using Optimization



## Who is the *User* of a Model?

- (Academic) Researcher
  - One time use (Research Paper)
- Domain&Model Expert
  - Model Results used for Consulting
- Black Box User
  - Model integrated in (Optimization) Application
- Each Category has its own needs
  - Development & Deployment





#### General Algebraic Modeling System

- Roots: World Bank, 1976
- Went commercial in 1987
- GAMS Development Corp.
- GAMS Software GmbH
- Broad academic & commercial user community and network

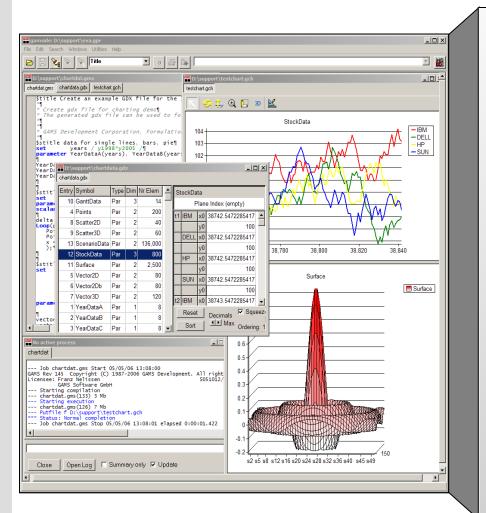




## **GAMS Software GmbH**

- Founded 1995
- Tool Provider: Software Sales, Support, and Development, Research, Technical Consulting
- Offices at Cologne and Braunschweig
- GAMS used in 41 European Countries
- Transnational Organizations: EC, FAO, NATO, OECD, UN, WTO
- · Germany:
  - More than 50% of the DAX 30 enterprises
  - The top five consulting companies
  - More than 300 academic institutes
  - 100 different universities and research institutes

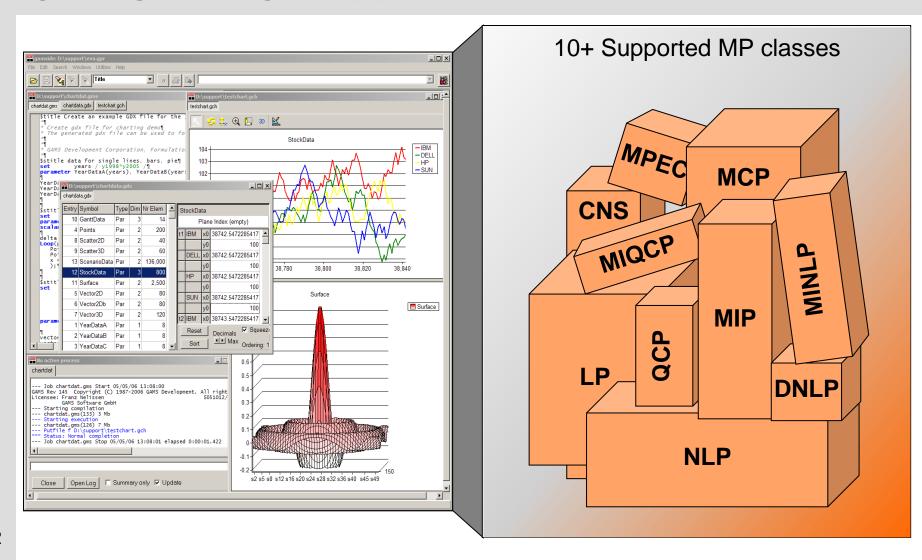




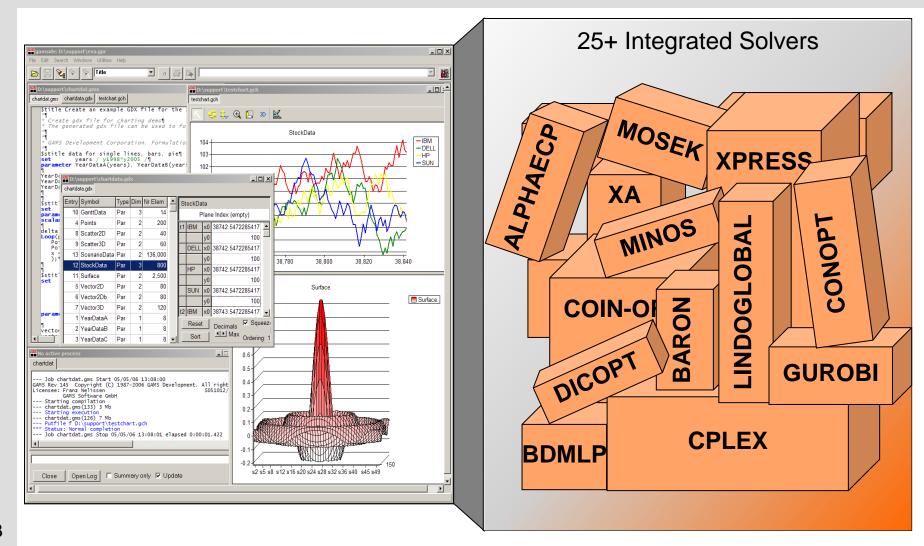
#### General Algebraic Modeling System

- Algebraic Modeling Language
- 25+ Integrated Solvers
- 10+ Supported MP classes
- 10+ Supported Platforms
- Connectivity- & Productivity Tools
  - IDE
  - Model Libraries
  - GDX, Interfaces & Tools
  - Grid Computing
  - Benchmarking
  - Compression & Encryption
  - Deployment System
  - ...











# **GAMS' Fundamental concepts**

- Platform independence
- Open architecture and interfaces to other systems
- Different layers with separation of
  - model and data
  - model and solution methods
  - model and operating system
  - model and interface



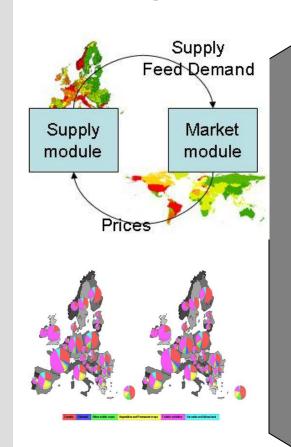
#### **Market Demands**

- Minimize Risks for new Clients / Management:
  - Deliver (expected) Results
  - Support Rapid Prototyping
  - Increase Productivity
  - Do not lock user into a certain environment
- Provide cutting edge technology: Models and users must benefit from:
  - Changing environments (Hardware, OS, GUI)
  - Enhanced / new modeling & solver technology
  - Improved / new interfaces to other systems
- Protect user investments: Deployed models often have 15+ years lifecycle



## Dissemination of OR into other fields I

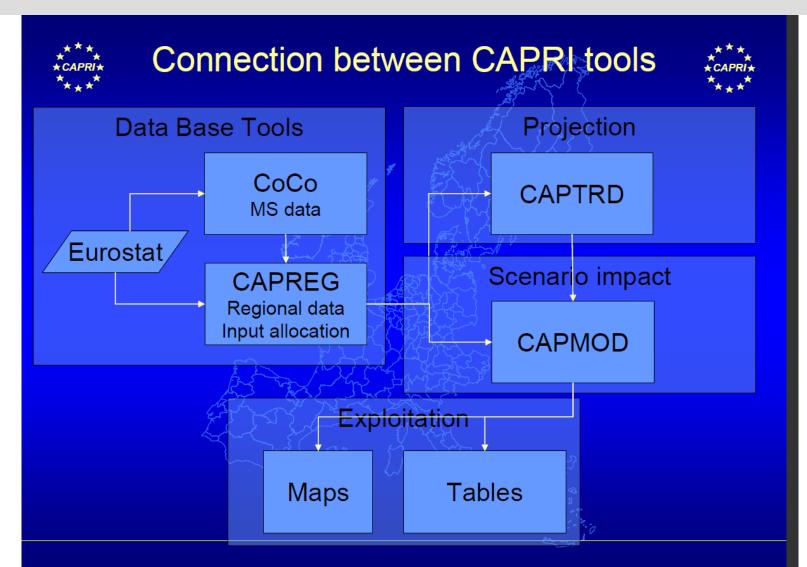
#### **Common Agricultural Policy Regional Impact Analysis (CAPRI)**



- A global agricultural sector model system with focus on 27 countries of the European Union and Norway
- Covers agricultural production in 18 trade blocks and about 250 regions
- Evaluates regional and aggregate impacts of trade policies on production, income, markets, trade and environment
- Used by network of ~ 37 research institutions and EU Commission services

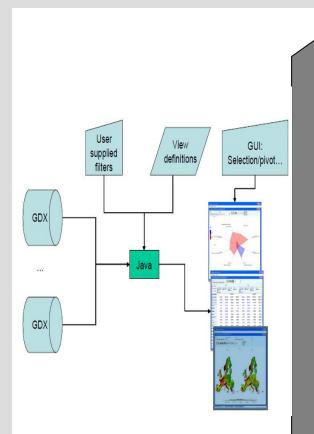


## **CAPRI**





# **CAPRI – Some Technical Aspects**



- System bridged several generations of GAMS, solvers, hardware, platforms, and GUIs.
- System includes both large scale
  - Non linear Problems
  - Mixed Complementarity Problems
- Distributed Model Code, Databases and User Communities
- Java based GUI (Reports / GIS)
- Coordinated by the Institute for Food and Resource Economics, University of Bonn (Wolfgang Britz)



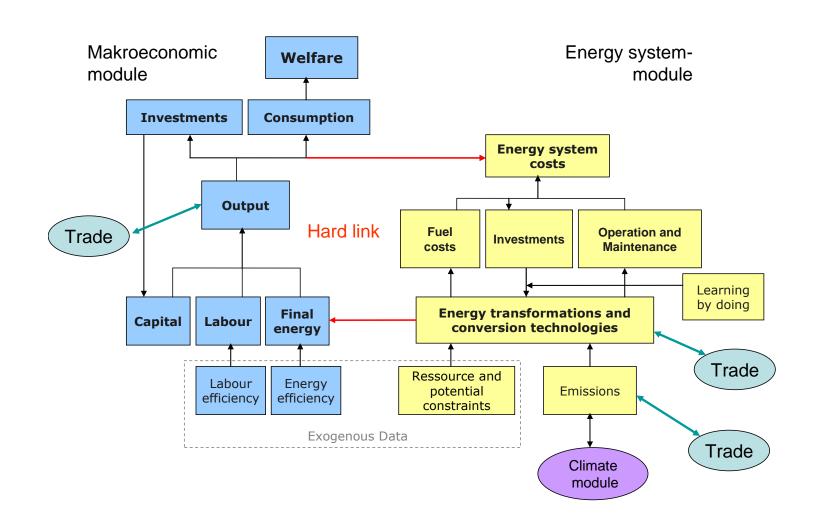
## Dissemination of OR into other fields II

#### **REMIND-R** by Potsdam Institute for Climate Impact Research (PIK)

- A global energy economy elimate model in a multi-regional setting
- 11 world regions and 7 types of traded energy (incl. emission rights)
- Climate policy analysis:
  - Business as usual
  - Different climate policies
- Combines complex optimization and simulation models
- Developed by group of experts from different fields
- Model documentation, code and data available
- Coordinated by Marian Leimbach (PIK)



## **REMIND-R**



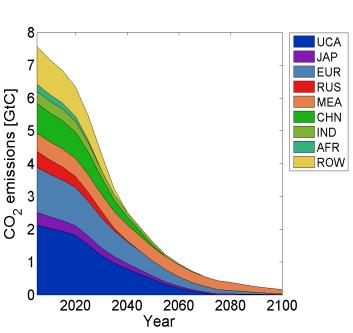


# CO<sub>2</sub> - Emissions

#### Reference scenario

#### 25 UCA JAP 20 RUS $\mathrm{CO}_2$ emissions [GtC] MEA CHN IND 15 AFR ROW 10 2020 2040 2060 2080 2100 Year

#### Policy scenario

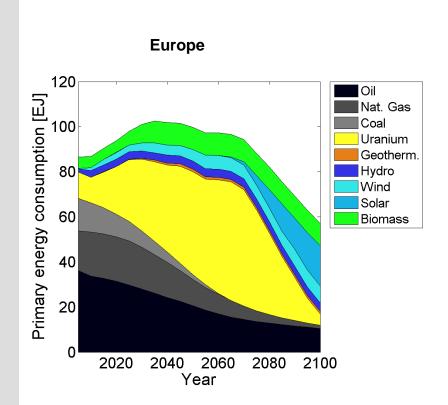


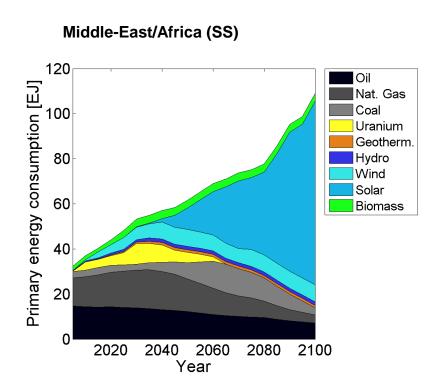
Reaching a 2°C target requires a fast and drastic decrease of emissions of all regions:

- 50% until 2035
- 78% until 2050



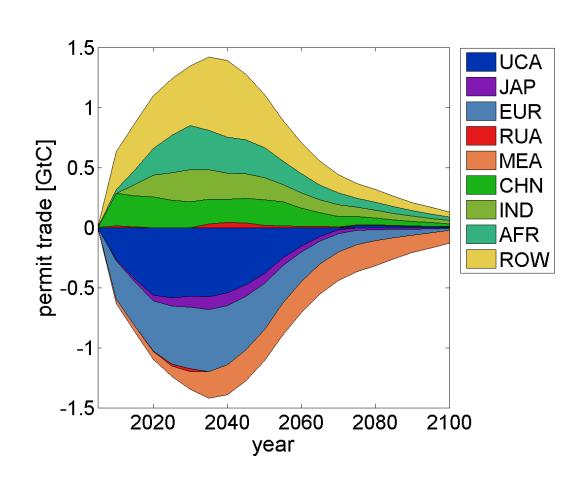
# Primary energy production (regional)







# **Trading of Emission Rights**





## **R&D at GAMS**

- 30% of revenue invested in R&D
- Product Development
- Research with academic & commercial partners
  - Emerging technologies (5-15 years)
    - Computing (grid, cloud, parallel, ...)
    - Mathematics/Algorithms
    - Modeling
  - Dissemination of technology from academia/research labs to clients



## **R&D at GAMS**

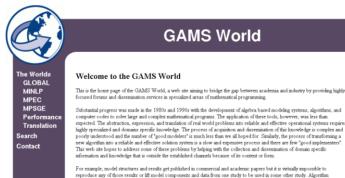
- GAMS is a platform for
  - implementing research ideas quickly
  - assuring quality to achieve commercial strength software

 deploying at minimal cost and risk to the user

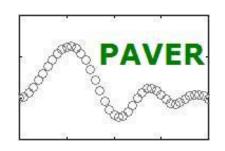


# **Global Optimization**

- Various research codes for Global Optimization: BARON, LGO, OQNLP, LindoGlobal
- Deterministic GO codes need more than function and gradient values (provide algebra to the solver)
- Establish quality assurance procedures and test model libraries
- Performance testing (set expectations right)



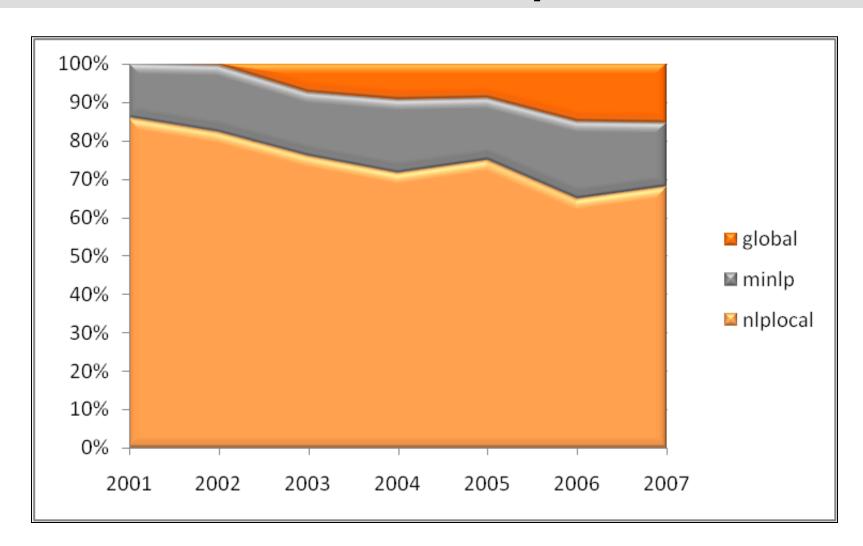
implementers face a similar dilemma when trying to get their hands on real world data models and data to test and refine their systems. This web site offers a few, well focused and maintained services to help with the dissemination of problems and solution:



Performance Analysis and Visualization for Effortless Reproducibility



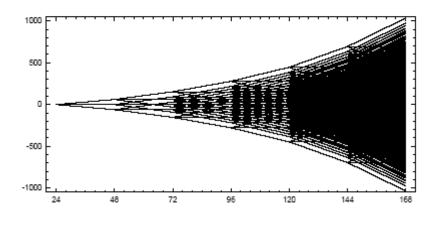
# **Relative Sales of Global Optimization**

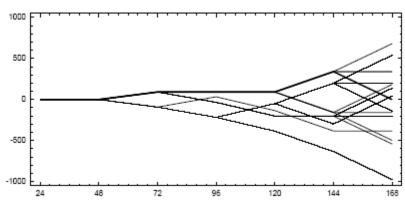




## **ScenRed**

- Prof. Römisch et. al. at HU Berlin
- Stochastic Optimization
- Scenario reduction aims to reduce the number of scenarios and to maintain the probability information as good as possible







## **ScenRed**

- ScenRed helps reducing the size of stochastic linear programs so they can be solved by today's LP technology.
- Seamless integration into GAMS



## **Extended Mathematical Programming (EMP)**

- Extended Nonlinear Programs
- Embedded Complementarity Systems
- Bilevel Programs
- Disjunctive Programs
- ...
  - Breakouts of traditional MP classes
  - No conventional syntax
  - Limited support with common model representation
  - Incomplete/experimental solution approaches
  - Lack of reliable/any software



## **Hierarchical Models**

Bilevel Program:

$$\min_{x,y} f(x,y)$$
s.t.  $g(x,y) \le 0$ ,
 $y \text{ solves } \min_{s} v(x,s) \text{ s.t. } h(x,s) \le 0$ 

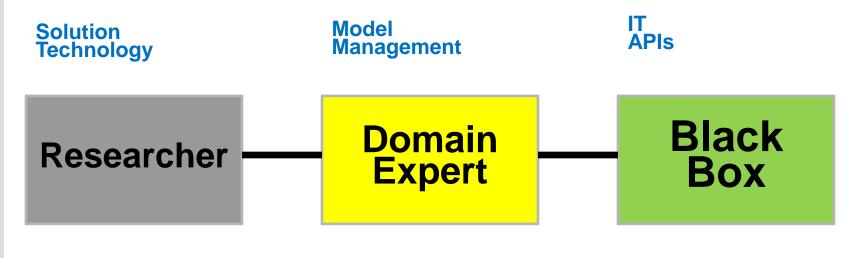
Additional Information:

\$onecho > %emp.info%
Bilevel x min v h
\$offecho

 EMP Tool automatically creates an MPEC by expressing the lower level optimization problem through its optimality conditions



#### **Model Users & Model Use**



Model used to write Research Paper/Thesis

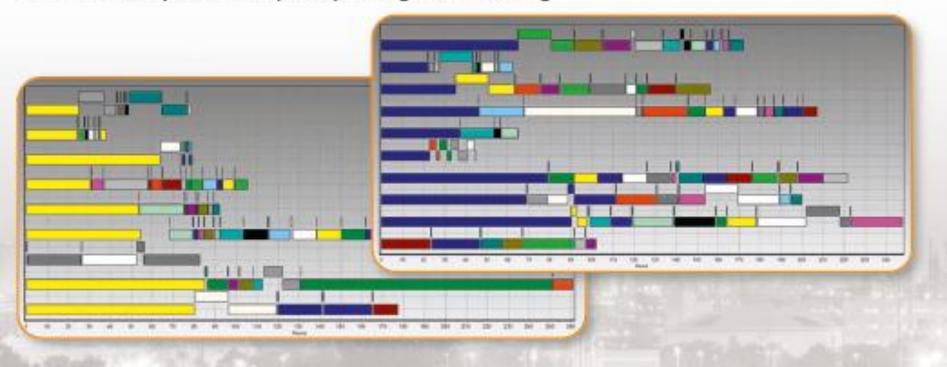
Combining models and algorithms to find answers for problems in the application domain

"Innocent" user of sophisticated optimization based approaches

#### Scheduling and Planning at BASF

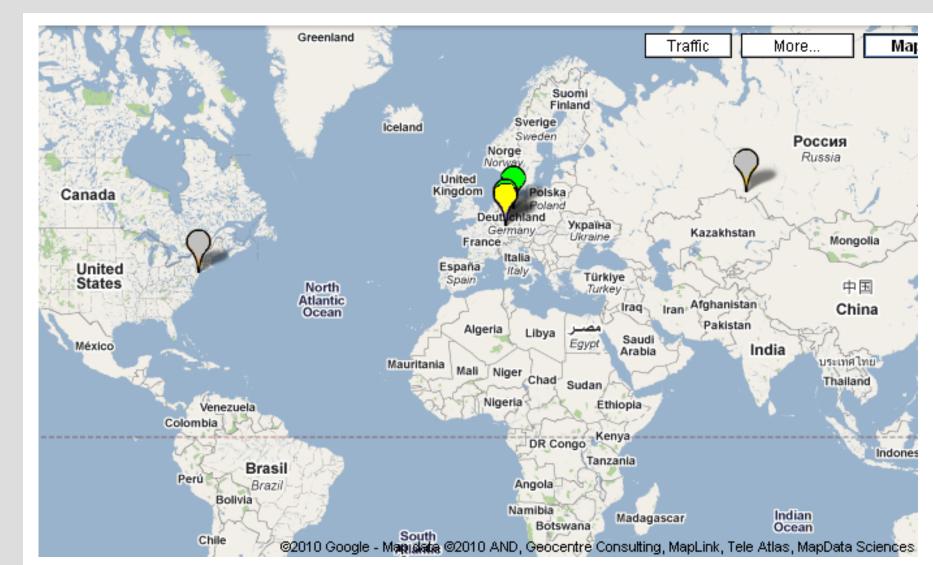
Close cooperation between logistics, information services and the scientific computing group of BASF, Prof. Dr. C. A. Floudas (Princeton University), Dr. A. V. Eremeev and Dr. P. A. Borisovski (Omsk Branch of Sobolev Institute of Mathematics SB RAS), SAP AG, and Mathesis GmbH led to a number of successfully deployed applications based on exact and hybrid optimization techniques. One of the results is a novel modeling approach of batch and continuous plants:

- State-task network formulation resulting in mixed-integer linear program
- Unit-specific, event-specific continuous-time formulations
- Hybrid methods and decomposition schemes to handle large instances
- Tight lower bounds derived from auxiliary models
- Implementation in GAMS with parallel GAMS/CPLEX
- New interfacing technology and integration approaches to connect to SAP-APO
- Used on a daily basis to improve planning and scheduling



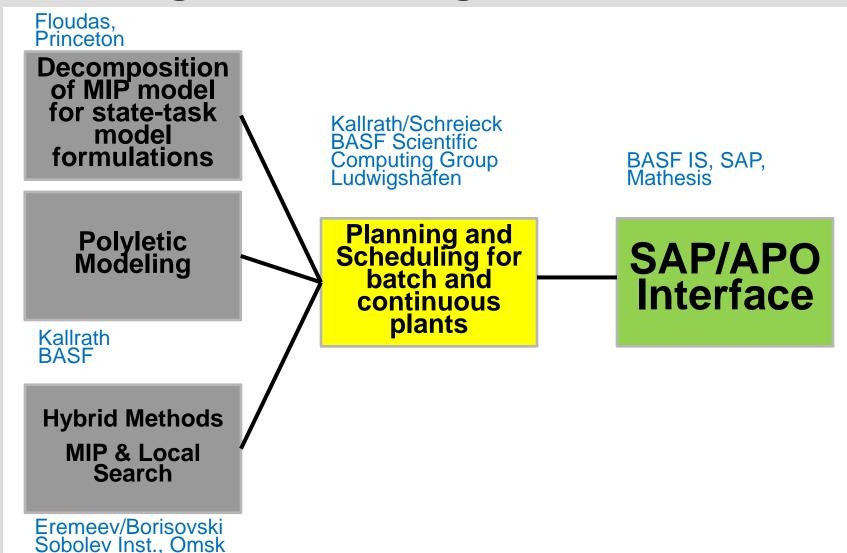


## **Distributed Models – Distributed Modelers**





# Planning & Scheduling at BASF





# Thank you!

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