MProbe:
*M*athematical *P*rogram *Probe*

John W. Chinneck
Systems & Computer Eng.
Carleton University
Ottawa, Canada

GO Questions

- What are reasonable bounds on the variables?
- What shapes do the constraints and objective have (convex, concave, both, almost linear, etc.)? Which are good candidates for linear approximation?
- What shape does the feasible region have? (convex, non-convex?)
- Are any constraints redundant?
- What is a good near-feasible starting point?
- Etc. etc. …

Needed: a tool for *Analysis* of GO models

- MProbe is a tool for analysis of math programs of all types (linear, nonlinear, integer).
  - Special strengths in analyzing nonlinear models
  - Works with functions of many variables
- Provides a suite of analysis tools: no solver included
- ***NEW:*** MProbe now reads GAMS models

Outline

- Overview of MProbe tools relevant to GO
- Demonstration of software

Main elements of MProbe

- Variables Workshop
  - Shift, tighten variable bounds
- Constraints and Objectives Workshops
  - Analyze shape, effectiveness, redundancy, set up convex enclosures for tighter sampling
- Constrained Region Workshop
  - Analyze shape of feasible region
- Points Workshop
  - Exchange points with solvers, look for near-feasible points, etc.

Orientation

- Most properties (shape etc.) estimated by *sampling* the functions in the model
  - Sample inside box created by variable bounds
  - Sample inside a general convex enclosure
- Best results when sampling region tightly surrounds region of interest (e.g. feasible region)
- Numerous methods for tightening bounds, or finding a small surrounding sampling enclosure

The Variables Workshop

- Main information:
  - Variable names, types, bounds
- Main actions:
  - Navigate between functions containing specific variable and back
  - Change bounds manually
  - Tighten bounds automatically

Tightening Bounds

- If using AMPL: request AMPL bound tightening prior to reading model
- Manual adjustment of bounds
- “tighten current bounds” sequence
- Constraint Consensus bound tightening
- Max/min sampled values from convex enclosure

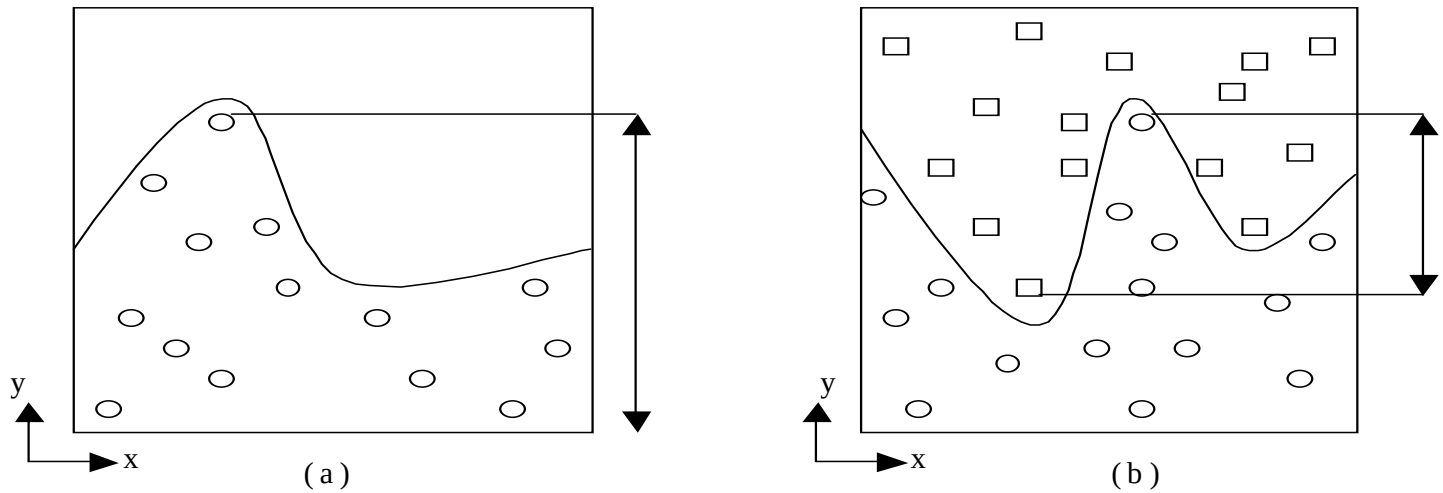
“tighten current bounds on all variables” sequence

- launches 4 methods in sequence:
  1. Linear interval analysis
  2. Nonlinear interval analysis
  3. Find a nucleus box
  4. Range cutting

1. Linear Interval Analysis

- Applies to the subset of linear constraints
- as in presolve, bound changes percolate
- E.g.:
  - constraint $2x_1 - 5x_2 \leq 10$ when $-10 \leq x_1, x_2 \leq 10$
  - Tighten x_2 lower bound by applying the constraint when x_1 is at it's lower bound: $2(-10) - 5x_2 \leq 10 \Rightarrow x_2 \geq -6$.
  - Conclusion: true bounds are $-6 \leq x_2 \leq 10$.

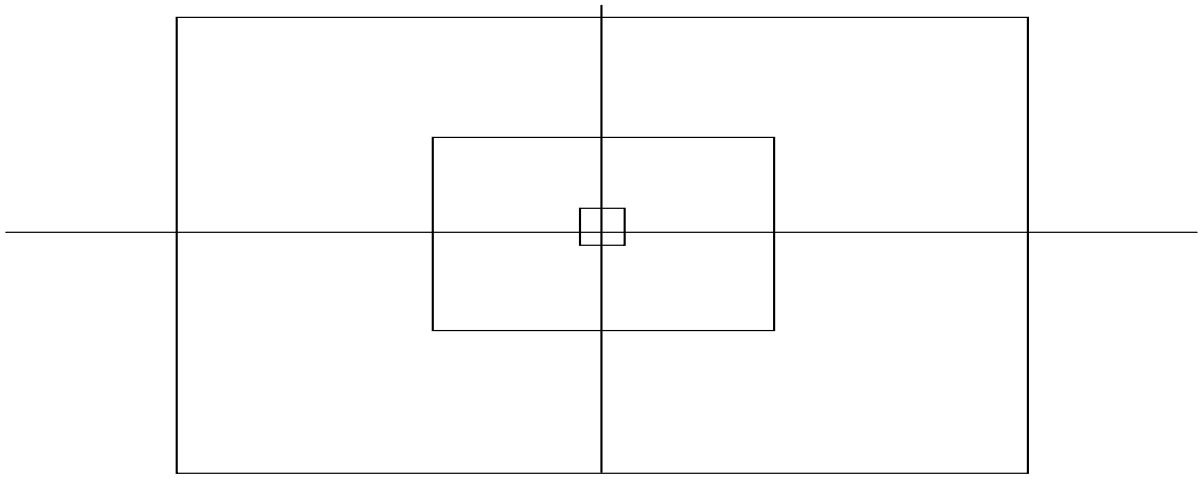
2. Nonlinear Interval Sampling



a) inequality b) equality

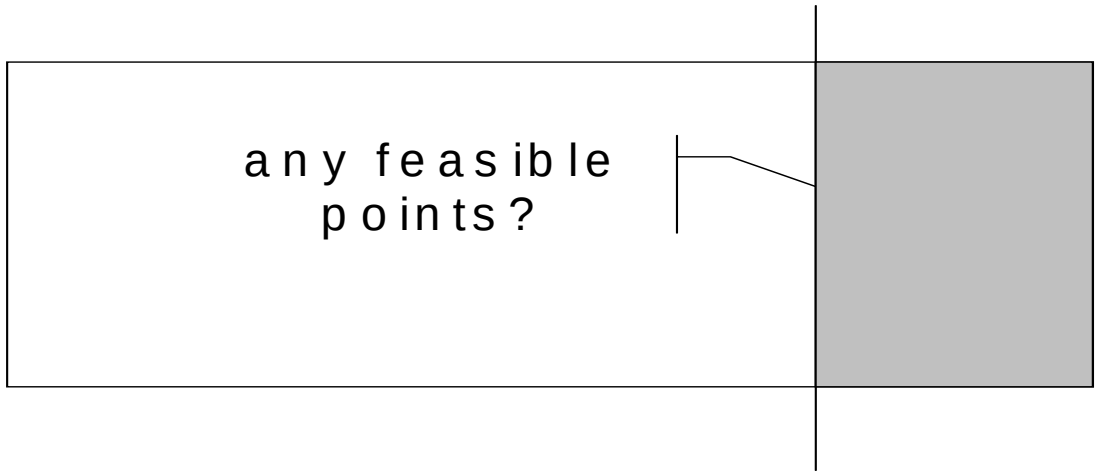
- Apply to each NL constraint in turn
- overtightens
- non-overlap? return the gap itself

3. Get a Nucleus



- Use for unbounded variables
- Look at nonlinear constraints involving the variable that were never satisfied during interval sampling
- try gradually larger boxes centred at origin. Stop when next box shows no feasible points

4. Nonlinear Range Cutting



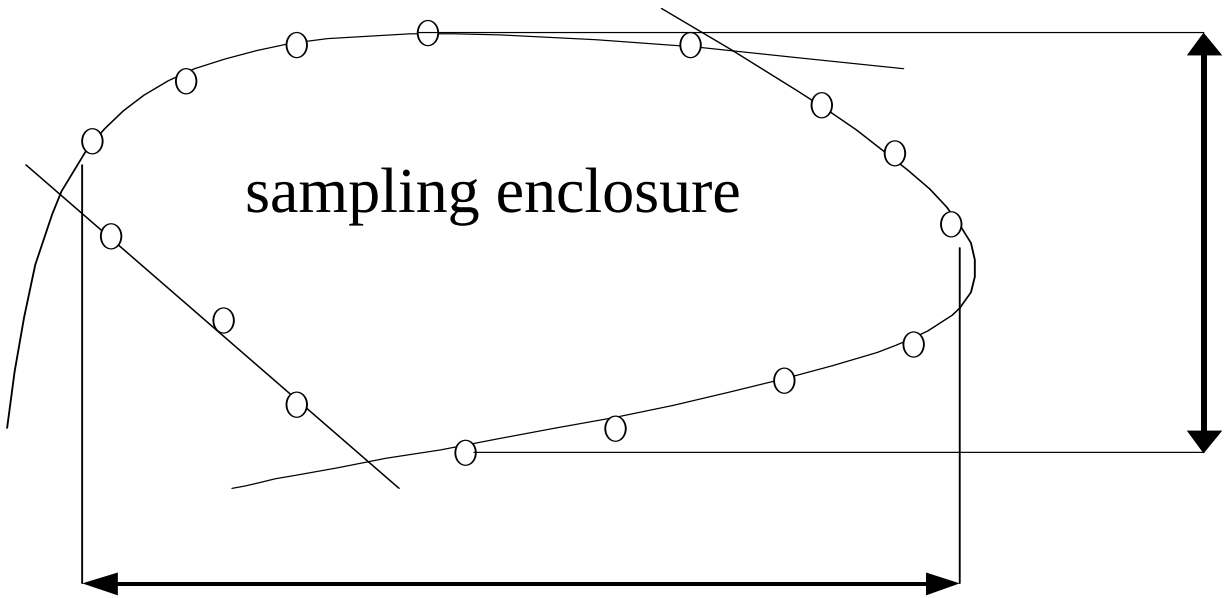
- Accept cut if at least one constraint never satisfied when sampling in the zone
- equality constraints: “satisfied” if find one pt $\leq$ rhs and one pt $\geq$ rhs

Constraint Consensus bound tightening

- “Constraint Consensus” heuristic moves quickly from an initial point that is far from feasibility to a point that is “close” to feasibility (user adjustable)
- General idea: combine gradients of violated constraints to determine direction and distance for an updating move
- Use a number of random initial points
- Shrink bounds to encompass the cloud of resulting approximately feasible points

“replace current bounds with max/min sampled values”

- Apply after sampling inside a convex enclosure
- Use hit points to (over)tighten the variable bounds



Constraints Workshop

Main Information:

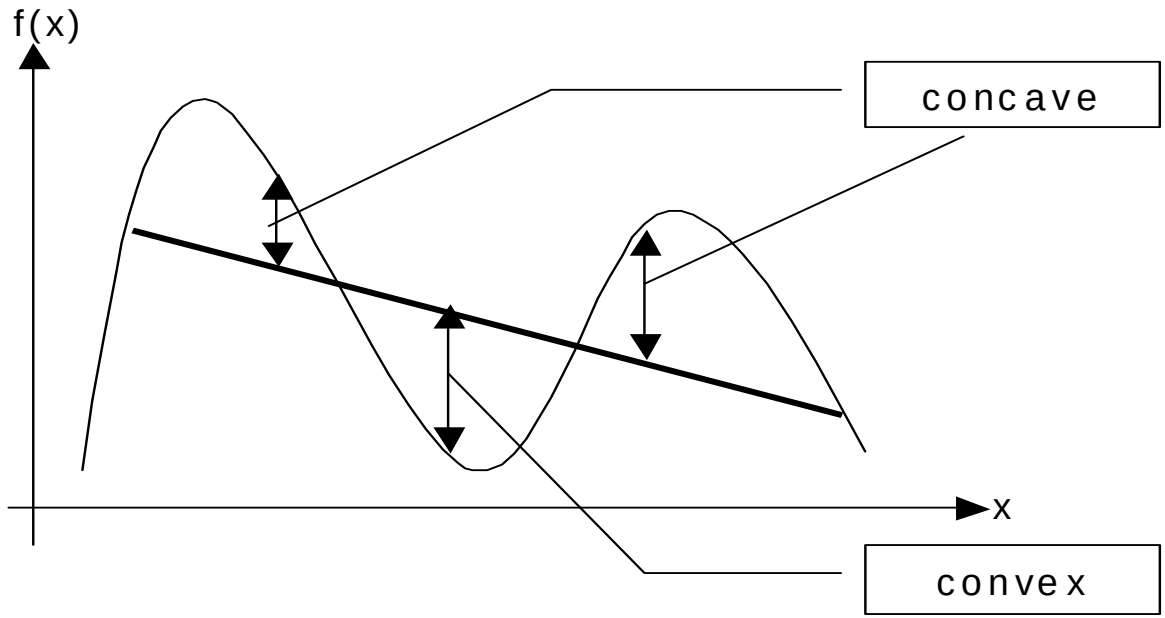
- Constraint names, types, algebraic shape, numbers/types of variables

Main Actions:

- Analyzing constraints for empirical shape, range of values, “slope” effectiveness, region effect, “surface” fraction
- “Profiling” functions
- Setting up convex sampling enclosures

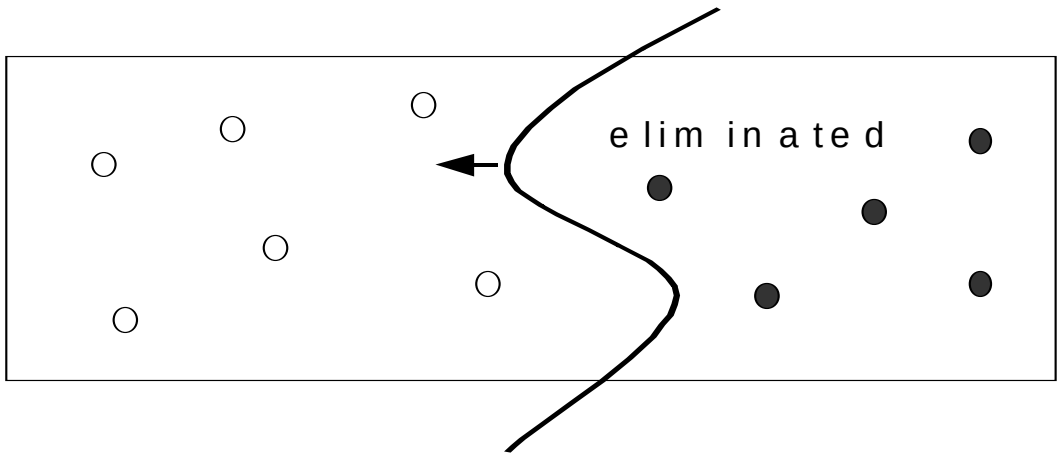
Empirical Function Shape

- Empirical Shape: convex, *almost* convex, concave, *almost* concave, both, linear, *almost* linear, both, etc.
- Can return difference histogram

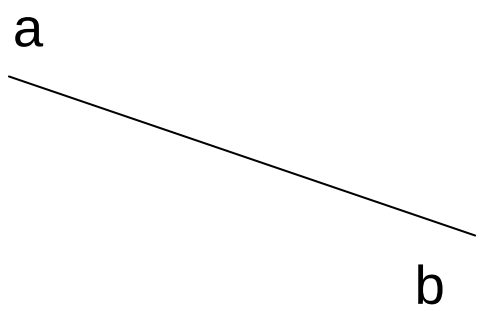


Constraint Effectiveness

- What fraction of the sampling enclosure is eliminated by the constraint?



Function “slope”

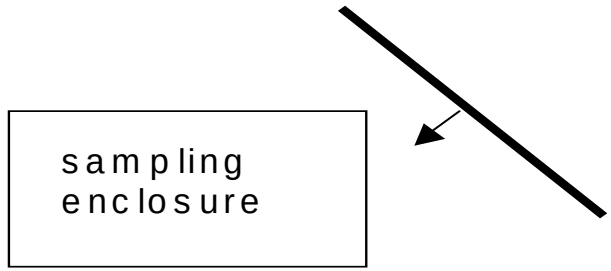


|f(a) – f(b)| / (length a to b)

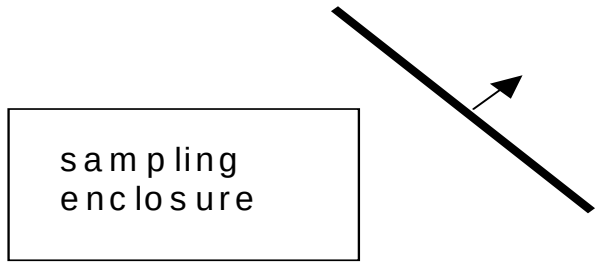
Multidimensional idea of “steepness”

Constraint & Bound Interactions

- Simple Constraint Redundancy (0% effective)

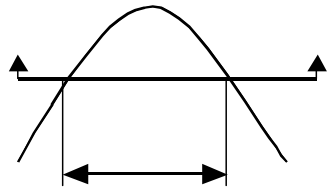


- Simple Feasibility Test (100% effective)

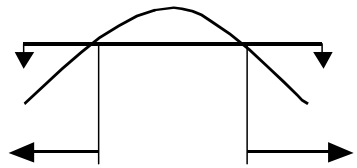


Region, Optimum Effects

- Constraint region effect



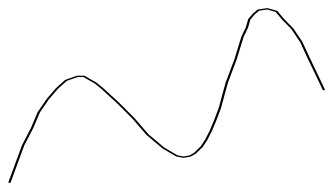
convex region effect



nonconvex region effect

- Optimum effect

Max: global opt possible
Min: local opt likely



Max or Min: local opt likely

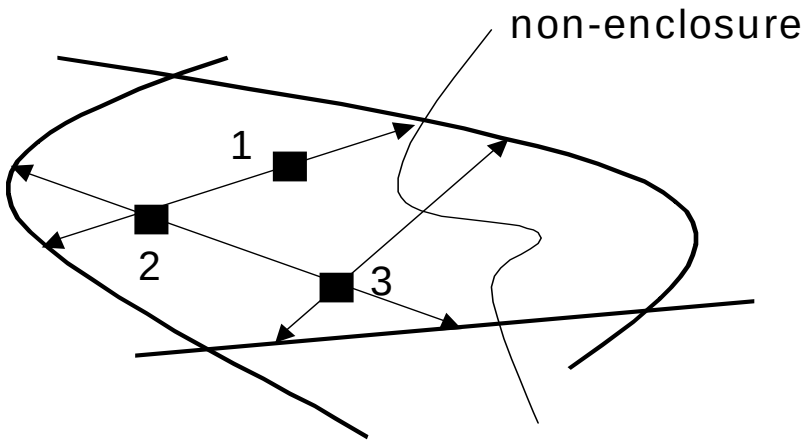
Other Info from Sampling

- Function value statistics
  - histogram, max, min, etc.
- Objective function best sampled value and point (not nec. feasible)
- variables min and max sampled values
- Line segment length
  - effect on conclusions

General Convex Enclosures

- *Goal*: bound region of interest more tightly
- Procedure:
  1. analyze constraint region effects by box sampling
  2. create tighter sampling enclosure using inequalities that have convex region effects and all variable bounds
- Sample via hit-and-run methods

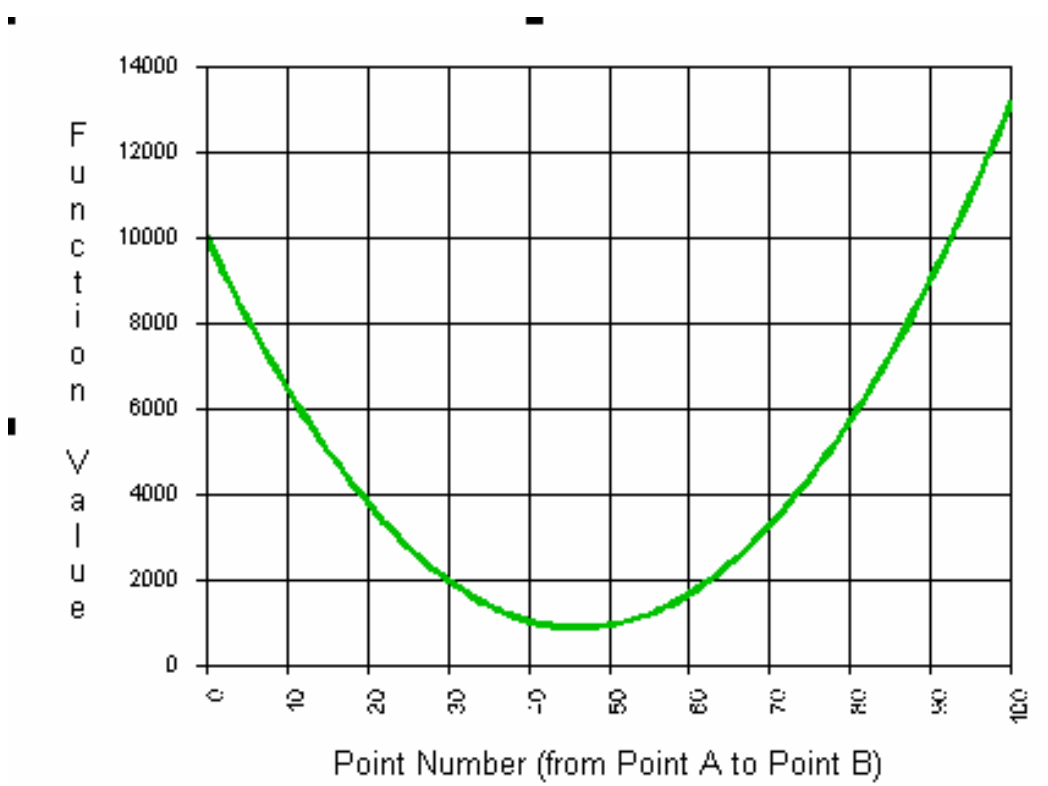
Hit-and-Run Sampling



- hit constraints are *necessary;* unhit constraints are *redundant* (relative to enclosure)
- estimate fraction of enclosure surface area
- non-enclosure constraints sampled as usual (shape, effectiveness, etc.)

Function Profile

- 2 dimensional plot between 2 endpoints in n-space
- End-points selectable and configurable in multiple ways



Objectives Workshop

- Similar to Constraints Workshop
- Shows best value obtained for objective during sampling (not necessarily feasible)

Constrained Region Workshop

- Combines information about individual constraints into estimates about the constrained region:
  - Estimated shape (convex? nonconvex?)
  - Estimated feasibility status
  - Estimated redundancy
- Lists constraints that contribute to each status

Points Workshop

Main Information

- Display interesting points:
  - At specific positions between variable bounds
  - Closest to feasibility
  - Best found and best found feasible for objectives
  - User-defined points
- Display information about a point
  - Distance from feasibility, violated constraints, etc.

Main Actions

- Random sample for interesting points
- Set small box around a point
- Look at objective flatness around a point
- Read/write/make points (exchange with solver)
- Find a near-feasible point

Small Box around a Point

- Create a small box by shrinking bounds to e.g. 10% of each current edge dimension
- Look at objective “flatness” in the box
  - Histogram of objective “slope” in the box.
  - Useful in determining why solver stops at a point

Find a Near-Feasible Point

- Use Constraint Consensus method to find a feasible or near-feasible point
- Good initial point for solver

Other Features

- Trace file
  - Simple text file capture of important results
- Help system
- Reads AMPL, GAMS, MPS format
- Exchange points with solver via simple text file format

Conclusions

- Useful tool for global optimizers
- Many tools are heuristic and based on random sampling
  - Tools don't *always* work, but *often* do.
  - Can be slow for very large or very complex models
  - Performance depends on characteristics of the model
- Download: www.sce.carleton.ca/faculty/chinneck/mprobe.html