Decomposition and High Throughput Solution of EP's

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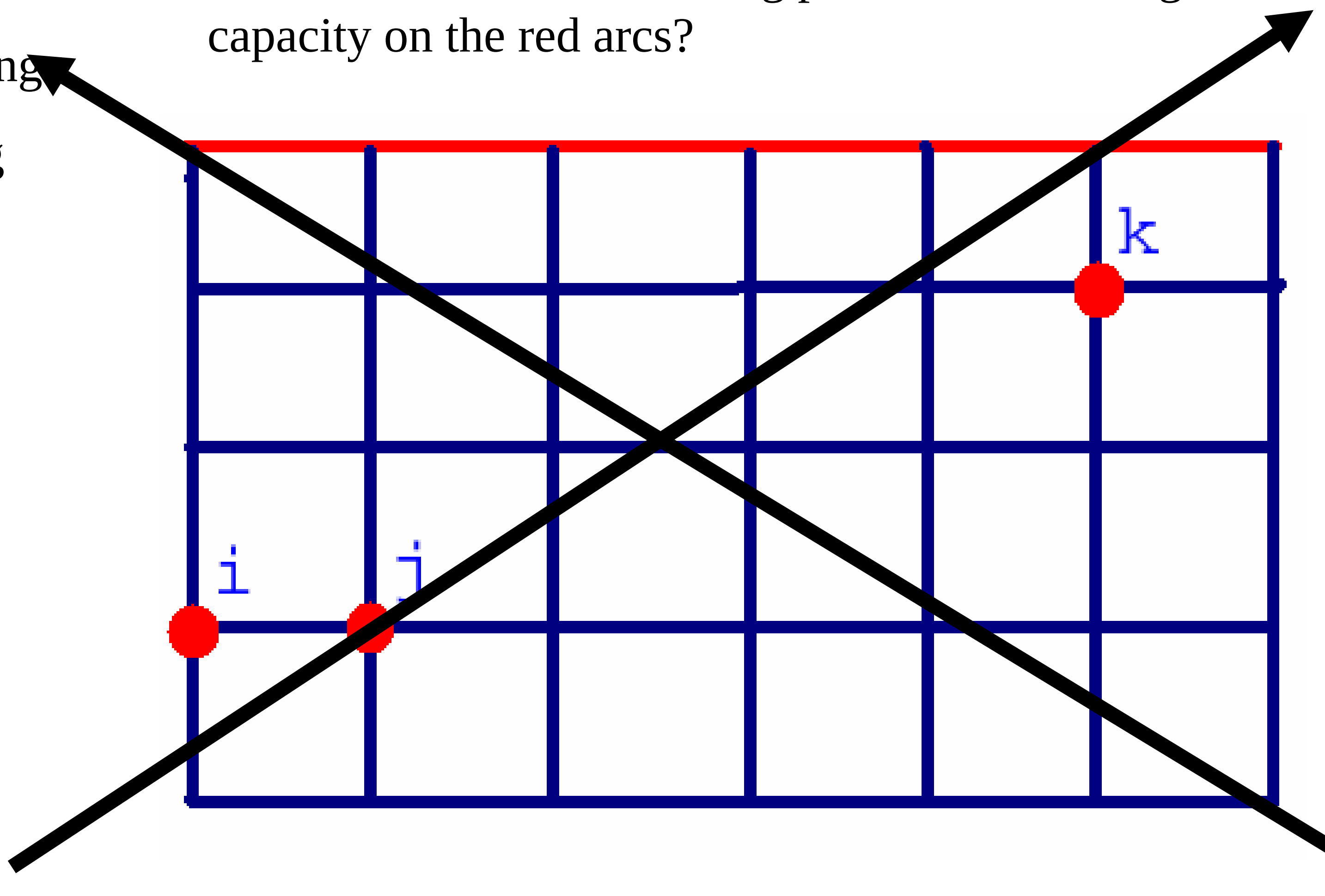
So what, why bother, who cares?

- Motivate why we use grid computation
- Show how we implement it
  - GAMS
  - Condor
- Explain the benefits
- What are the limiting factors?

Walras meets Wardrop

What is the effect on housing prices of increasing capacity on the red arcs?



$$T_{ik} \leq c_{ij}(F_{ij}) + T_{jk} \perp x_{ij}^k \geq 0$$

$$F_{ij} = \sum_k x_{ij}^k$$

$$N_{jk} + \sum_i x_{ij}^k = \sum_i x_{ji}^k$$

x_{ij}^k = people travel (i,j) work at k

T_{ik} = (shortest) time i to k

N_{ik} = number live i work k

$$D_k(w_k) \leq N_{kk} + \sum_i x_{ik}^k \perp w_k \geq 0$$

$$V_{ik}(w_k, p_i, T_{ik}) \leq U \perp N_{ik} \geq 0$$

$$\bar{N} = \sum_{i,k} N_{ik}$$

$w_k =$ labor cost at k

$p_i =$ housing price at i

$N_{ik} =$ number live i work k

$$T_{ik} \le c_{ij}(F_{ij}) + T_{jk} \perp x_{ij}^k \ge 0$$

$$F_{ij} = \sum_k x_{ij}^k \qquad \perp F_{ij}$$

- Square
- Prices found
- Apply shocks

$$N_{jk} + \sum_i x_{ij}^k = \sum_i x_{ji}^k \quad \perp T_{jk}$$

$$D_k(w_k) \le N_{kk} + \sum_i x_{ik}^k \perp w_k \ge 0$$

$$V_{ik}(w_k, p_i, T_{ik}) \le U \perp N_{ik} \ge 0$$

- Tolling

$$\bar{N} = \sum_{i,k} N_{ik} \qquad \perp U$$

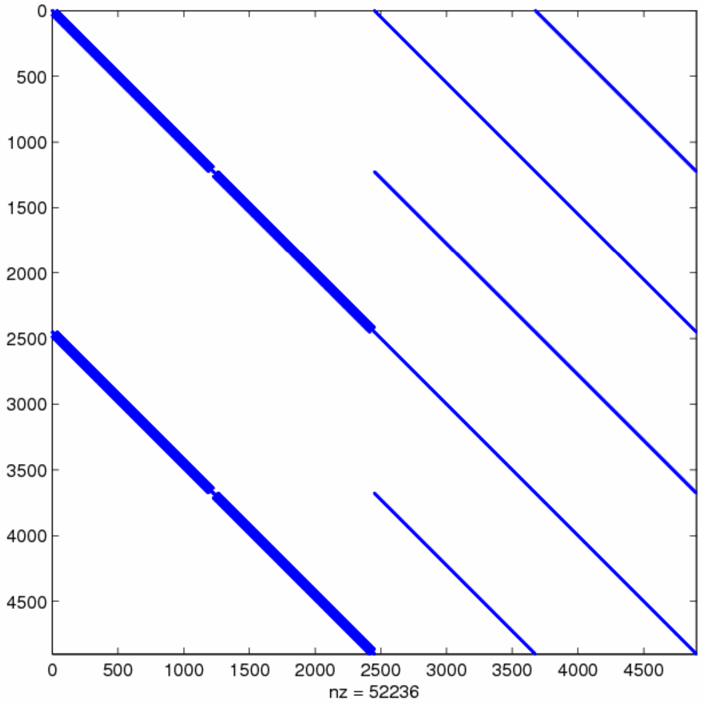
- Different housing types
- Different transportation types

$$\sum_k N_{ik} H_{ik}(w_k, p_i) \le \bar{H}_i \perp p_i \ge 0$$

Stochastic Games

- Two players, infinite (discrete) time
- Prototype (Cournot) example on grid
  - (i,j) = players machines
- Investment and depreciation affects probabilities of changing state
- Variables are investment levels, quantities, prices

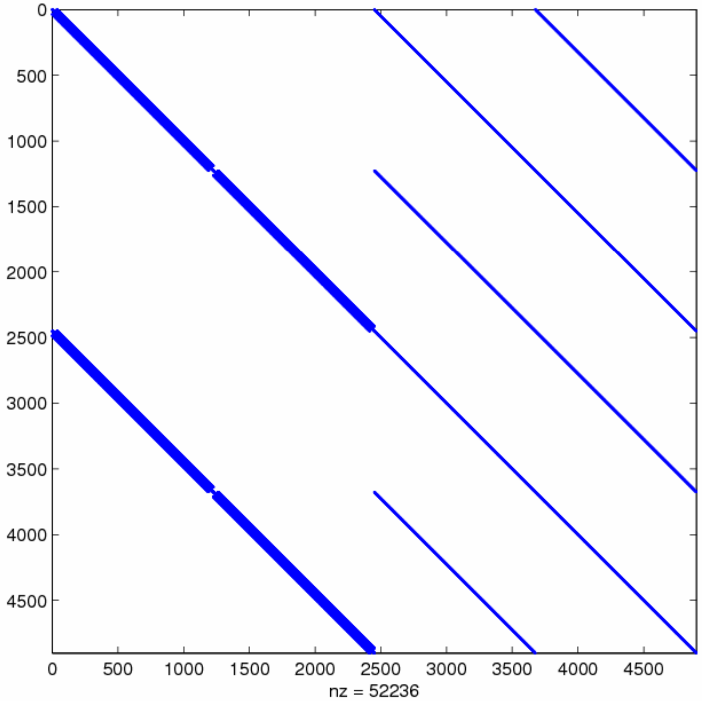
Sparsity pattern



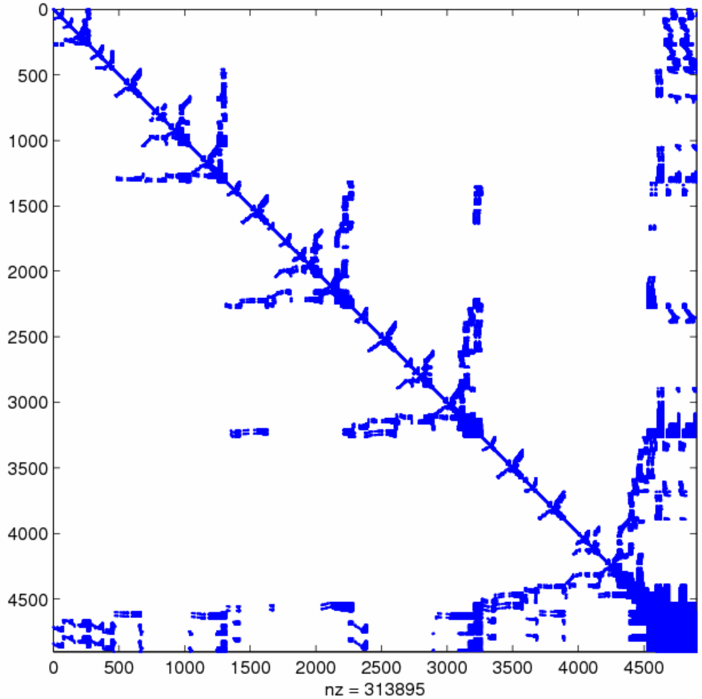
Jacobian

n	MCP size	LU-SOL	UMF PACK
20	3.2K	0.7	0.4
50	20K	8.5	2.8
100	80K	52.3	12.3
200	320K	479.0	56.4

Sparsity pattern



Jacobian



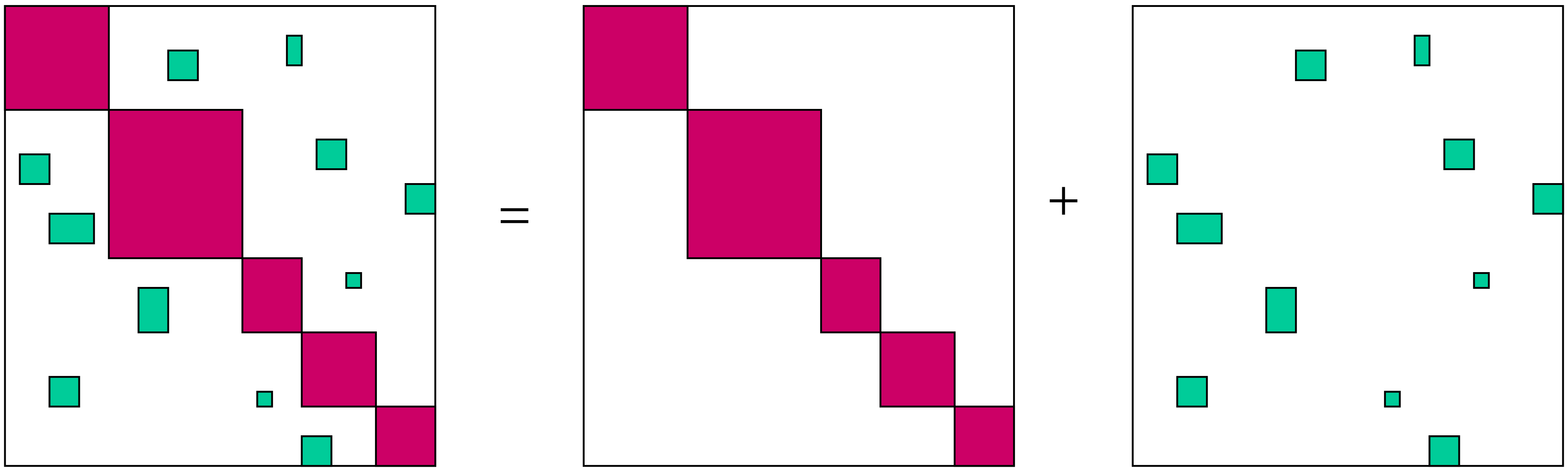
LU factors of Jacobian

A use for grid computation

- Enhance speed (or size) of computation model
  - Linear algebra
    - May not have LU exploitable structure
  - Decomposition approaches
    - Benders, Dantzig-Wolfe, Lagrangian
    - Jacobi, Gauss-Seidel

Trade/Policy Model (MCP)

- Split model (18,000 vars) via region



- Gauss-Seidel, Jacobi, Asynchronous
- 87 regional subprobs, 592 solves

Gauss-Seidel

```
loop(iter$(not done),
  loop(p,
    r(i) = yes$inp(i,p);
    x.fx(i)$(not r(i)) = x.l(i);
    solve trademod using mcp;
    x.lo(i) = 0; x.up(i) = xup(i) );
);
```

Jacobi

```
gemtap.solvelink = 3;
loop(iter$(not done),
  loop(p,
    r(i) = yes$inp(i,p);
    x.fx(i)$(not r(i)) = x.l(i);
    solve gemtap using mcp;
    x.lo(i) = 0; x.up(i) = xup(i);
    h(p) = gemtap.handle );

  repeat; loop(p$h(p),
    if(handlestatus(h(p)),
      gemtap.handle = h(p);  execute_loadhandle gemtap;
      h(p) = 0; ););
  until card(h) = 0;);
);
```

Asynchronous

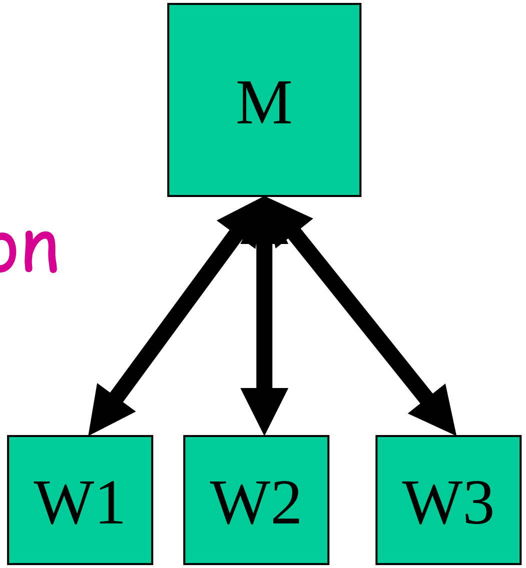
```
gemtap.solvelink = 3;
repeat; loop(p$(h(p)),
  if ( handlestatus(h(p)),
    gemtap.handle = h(p);
    execute_loadhandle gemtap;  h(p) = 0;
    if (sum(k, dev(k)) > tol,
        loop(k$(h(k) eq 0 and dev(k),
            gemtap.number = ord(k);
            r(i) = yes$inp(i,k);
            x.fx(i)$(not r(i)) = x.l(i);
            solve gemtap using mcp;
            x.lo(i) = 0; x.up(i) = xup(i);
            h(k) = gemtap.handle;
);););););
until (card(handle) = 0);
```

Condor as a Grid Computer

- Uses dedicated clusters and cycles from desktop workstations (> 1000 machines available for "ferris")
- Heterogeneous machines, with or without shared file system
- Machines updated regularly
- Fault tolerance
  - Jobs submitted are eventually executed
- Available for download, configurable

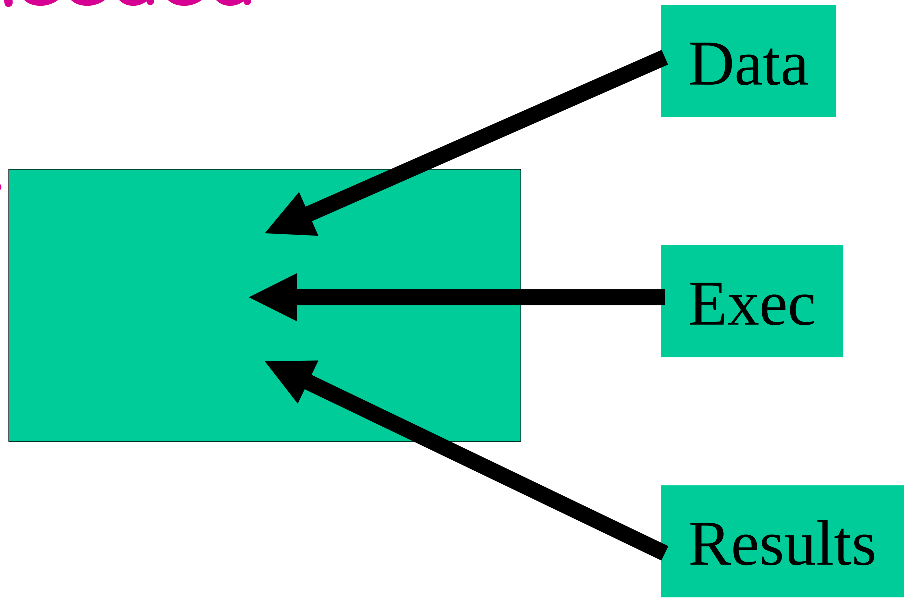
Master-Worker Paradigm

- Master
  - Generates tasks
  - Responsible for synchronization
- Worker
  - Processes individual tasks
  - Reports back results
- Simple! Dynamic! Fault tolerant!
- No worker inter-communication



Worker or task

- Local copy of job needed
  - Zip file, job dir
  - Mimic environment
- Start flag
- End flag
- Trigger file
  - Updates

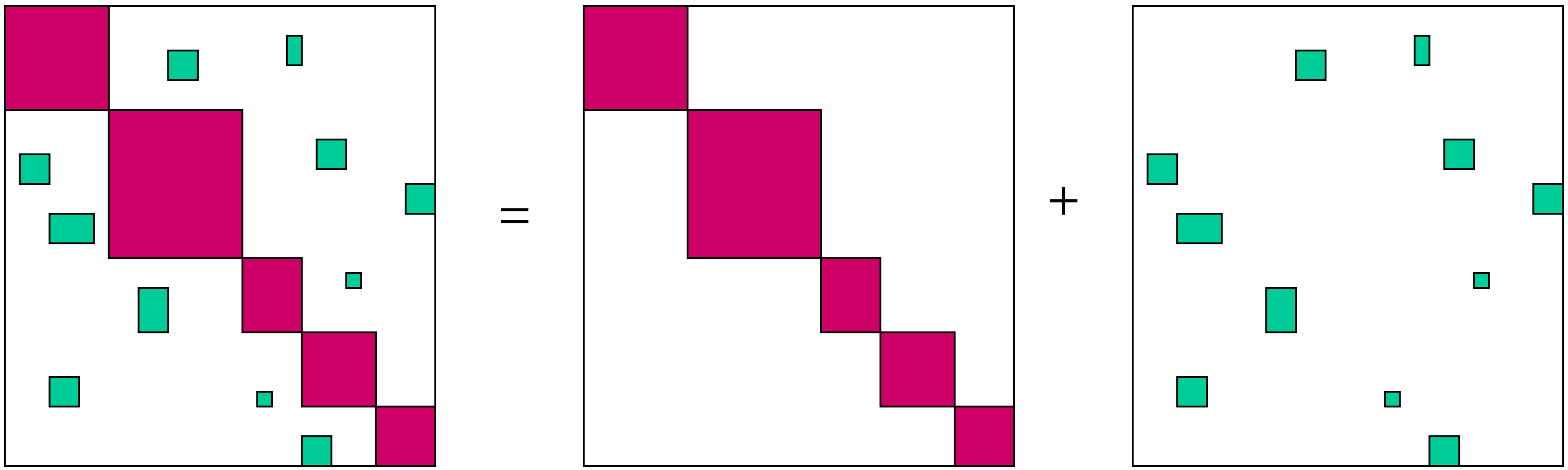


MW-GAMS

- Generate "gams task" worker
- Data common to all tasks is sent to workers only once
- (Try to) retain workers until the whole computation is complete—don't release them after a single task
- Modeler just flips a switch!
  - Usecondor=mw,lnx,win,glow,sun

Trade/Policy Model (MCP)

- Split model (18,000 vars) via region



- Gauss-Seidel, Jacobi, Asynchronous
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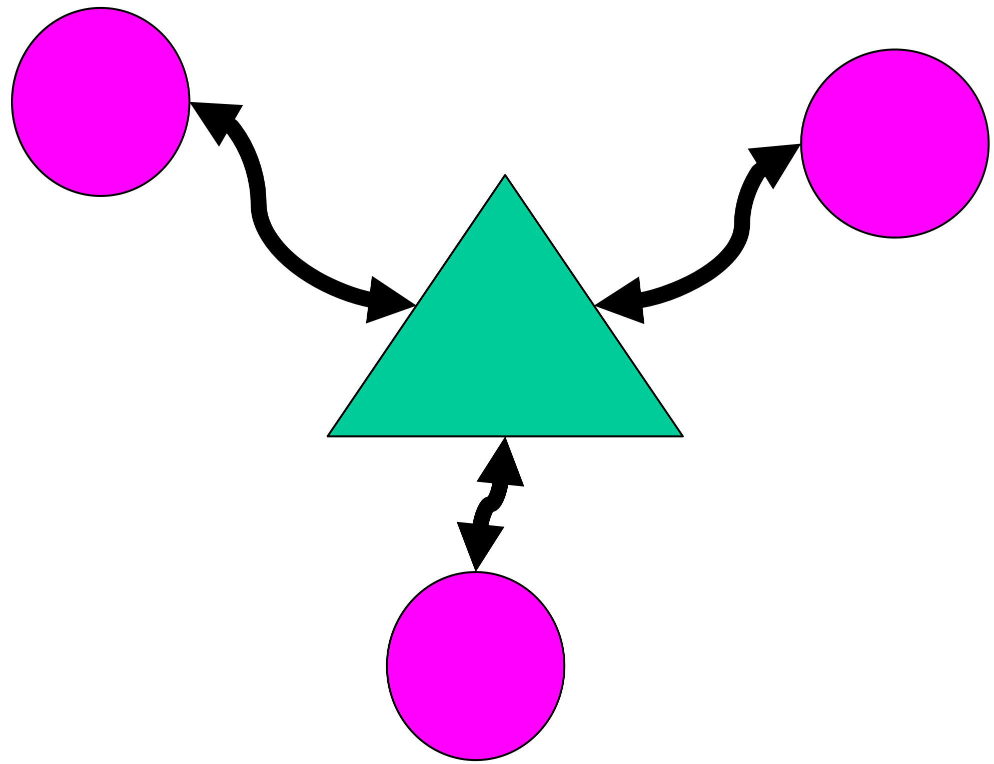
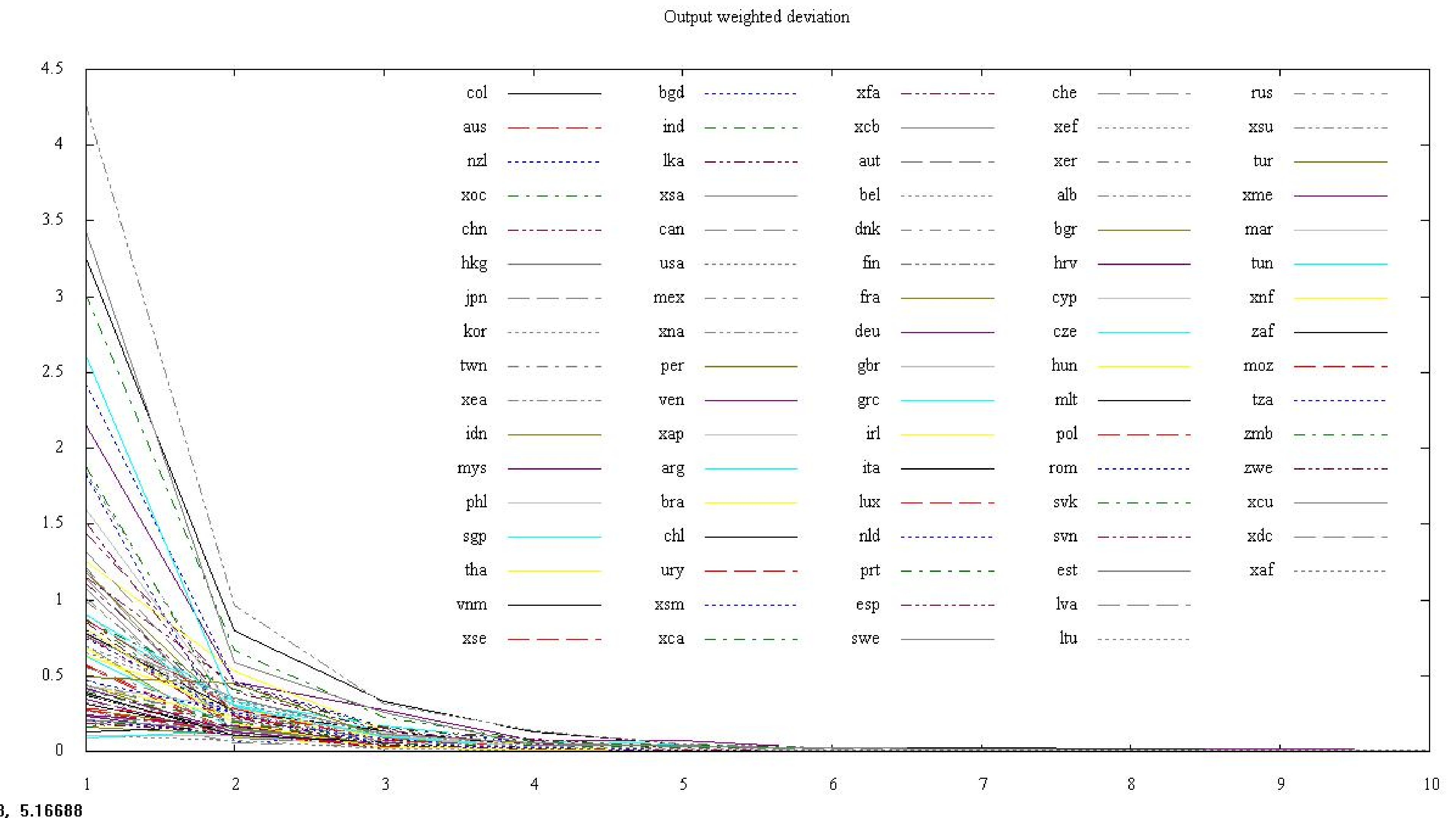
Model knowledge decomposition

- Pink model - open economy (regions)
- Green model - (partial) spatial equilibrium (commodities)
- Links are imports and exports



Calibrate supply and demand functions to points, and communicate functional forms, not points

Deviations by iteration



GAMS/Grid

- Commercial modeling system – abundance of real life models to solve
- Any model types and solvers allowed
  - Scheduling problems
  - Radiotherapy treatment planning
  - World trade (economic) models
  - Sensitivity analysis
  - Cross validation, feature selection
- Little programming required
- Separation of model and solution maintained

Conclusions

- Grid systems available (e.g. Condor, IBM, SUN)
- Grid computing convenient via simple language extensions to modeling languages
- Can experiment with coarse grain parallel approaches for solving difficult problems
- Exploiting underlying structure and model knowledge key for "larger, faster" solution

- Easy, adaptive, improves, need expertise

Shortcomings/Future Work

- Iterative scheme updates small amount of model "data"
- As convergence occurs (prove it!) models become easy to solve (great start point)
- Model regeneration time is longer than solution time!