Interactions between Modeling Systems and Advanced Solvers

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Agenda

- General Algebraic Modeling System
- Current State of AMLs
- Extending Algebraic Modeling
- Implementation

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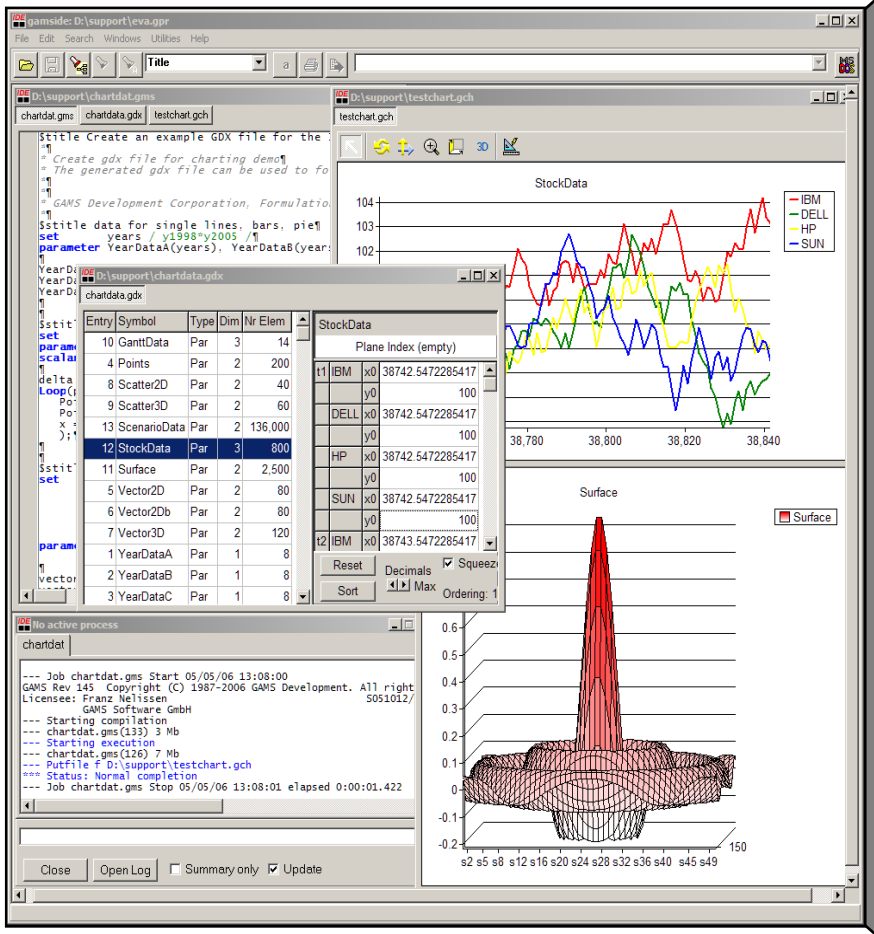
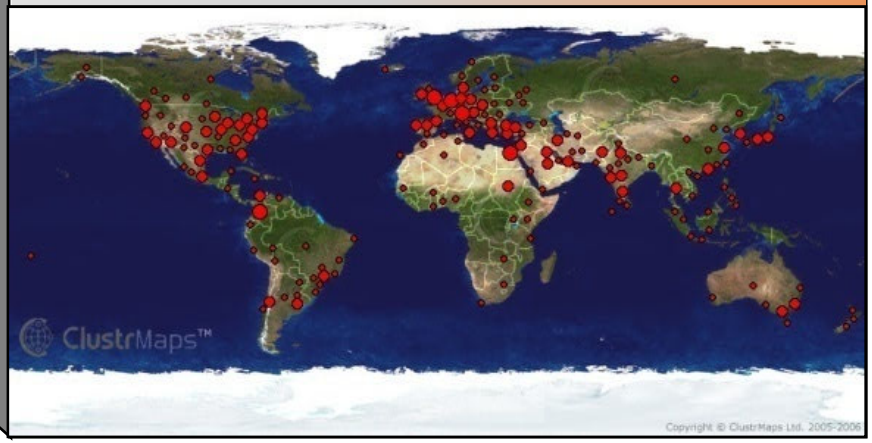
GAMS at a Glance



General **A**lgebraic **M**odeling **S**ystem

- Roots: World Bank, 1976
- Went commercial in 1987
- GAMS Development Corp. (DC)
- GAMS Software GmbH (Cologne)

- Broad academic & commercial user community and network



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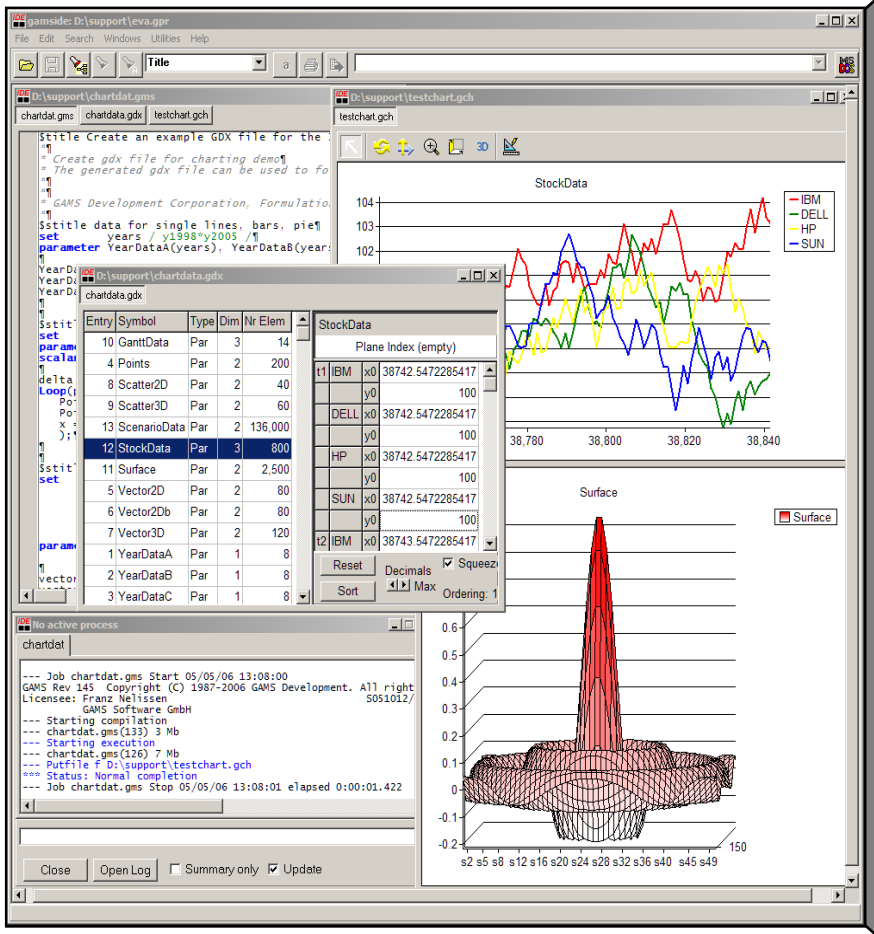

General **A**lgebraic **M**odeling **S**ystem

- Algebraic Modeling Language
- 25+ Integrated Solvers
- 10+ Supported Model Types
- 10+ Supported Platforms
- Connectivity- & Productivity Tools
  - IDE
  - Model Libraries
  - GDX, Interfaces & Tools
  - Grid Computing
  - Benchmarking
  - Compression & Encryption
  - Deployment System
  - …

Recent Enhancements

- New Solvers
  - Coin-OR Solver (Glpk, Cbc, Ipopt, Bonmin)
  - AlphaECP
  - LINDOglobal
- New Platforms: Solaris on Sparc64 and MacOS on Intel
- GAMS supports CPLEX 11 features
  - Improved Mixed Integer Programming Performance
  - Enhanced Parallel MIP
  - Multiple MIP Solutions
  - Performance Tuning Tool
- Extended Mathematical Programming (EMP)

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Algebraic Modeling Languages (AML)

- Traditional but fundamental view of AMLs

- **Key concept:** Different layers with separation of
  - model and data
  - model and solution methods
  - model and operating system
  - model and interface

Current state: Model-Side

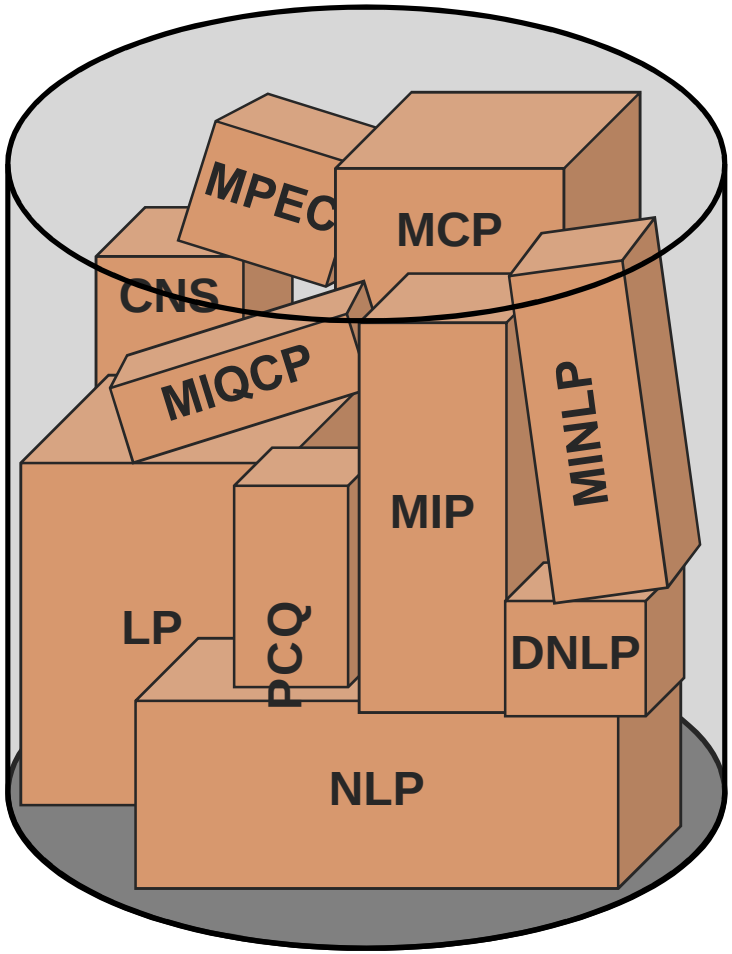
- Traditional problem format

$$\min_{x} c(x) \quad s.t.\ A_1(x) \le b_1,\ A_2(x) = b_2$$

- Interactions between models possible
  - Series of models
  - Scenario analyses
  - Iterative sequential feedback
  - Decomposition

Current state: Solver-Side

AMLs support a wide collection of established mathematical programming classes through solver clusters



Breakouts of traditional MP classes

New trends in research broaden algebraic modeling

- Global Optimization
- Solvers that are based on automated symbolic reformulation of model types
- Hybrid tools that make use of traditional model representation plus additional information as
  - logical constructs (indicators, disjunctions)
  - constraint modifications
    - activation and deactivation
    - softening and tightening
    - probability

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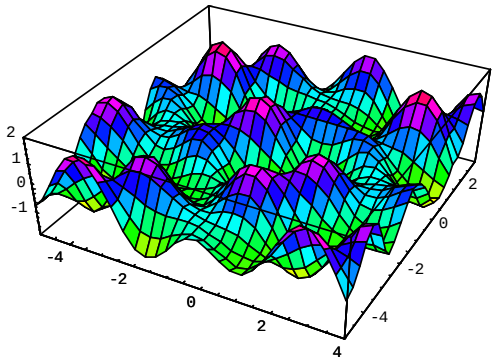
Global Optimization

- Practical optimization problems are often nonlinear and non-convex, with discrete variables
- They may contain disconnected feasible regions with multiple local optima



à Find the best of all

<u>AMLs perfect platform to promote GO</u>

- Experience with (local) nonlinear optimization
- Separation of model and solution technology
- Established Quality Assurance
- Mathematical algebra is required (not black box)
    - Baron, LINDOglobal

Reformulation-based Solvers

- GAMS/NLPEC
  - solves MPECs as NLPs
  - 20+ different reformulation strategies
- GAMS/DECIS
  - solves two-stage stochastic linear programs with recourse
  - two-stage decomposition (Benders)
  - stores only one instance of the problem and generates scenario sub-problems as needed
  - solution Strategies (Universe problem/Importance sampling)
- GAMS/PATHNLP
  - solves NLPs as MCPs
  - internal reformulation via KKT conditions
  - requires 1^{st} and 2^{nd} order derivatives

Hybrid Approaches

- Logical Mixed Integer Programming (LogMIP)
  - Reformulation and logic-based methods on Generalized Disjunctive Programs (GDP)
- Indicator constraints (CPLEX)
  - Alternative to conventional BigM formulations
- Extended Nonlinear Programming (ENLP)
  - Softening and tightening constraints
- …

Automatic Reformulation Framework

à Need of a framework for automated mathematical programming reformulations that

- integrates the different hybrid approaches
- makes GAMS ready for new cutting-edge approaches
- provides new facilities for seamless integration of new model types (Conic Programming, SDP,…)
- automates symbolic reformulations to avoid error-prone and time-consuming manual algebra (re)writing
- makes additional information consistently available

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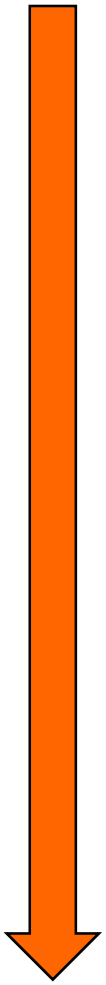
“Evolution in the GAMS way”



- committed to backward compatibility
- try as research code

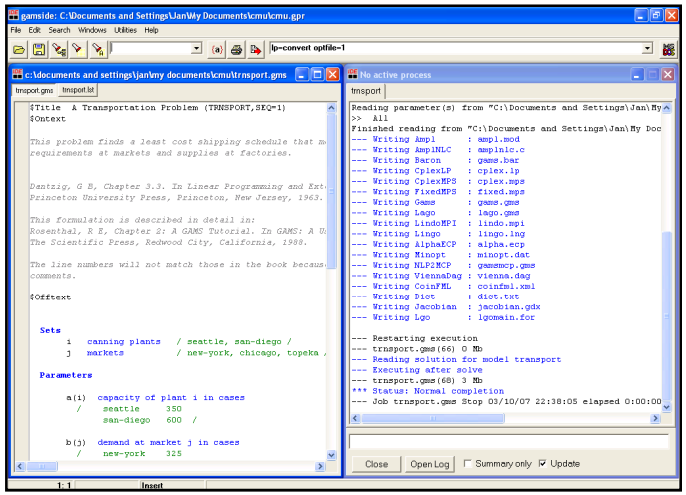
- analyze the big picture
- find a generalization

- implement as sub-language
- does it proof itself?

- generally accepted notation?
- integrate in GAMS language
- work with solver developers

GAMS/Convert

- Model translation tools
  - GAMS à other formats/languages
  - Algebraic information still available



- GAMS
  - Creates scalar “standardized”model

- NLP2MCP
  - Converts model into a scalar MCP model

- CHull
  - Creates the convex hull of a (nonlinear) disjunctive program

NLP2MCP

- Why convert to MCP
  - Second order information implicitly available
  - New model types cannot be formulated as (N)LP
    - Bi-level, embedded problems
  - Exploit multiplier information

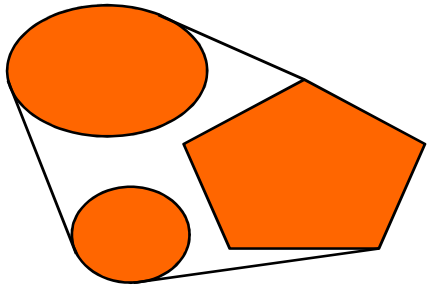
- Likely that MCP solver will find a solution
  - Solution is only guaranteed to be feasible for the original problem
  - In the convex case, every KKT point corresponds to a global solution of the NLP

CHull

- Convex Hull reformulation of linear and nonlinear models with disjunctions
- User provides disjunction information

```
file dj2 / '%gams.scrdir%loginfo.scr' /;  dj.nd=0; dj.nw=0; dj.lw=0;
put dj2 '* convex hull for example 1';
loop(lt(j,jj),
   put / 'disj ' y.tn(j,jj) ' ' seq.tn(j,jj) ' else ' seq.tn(jj,j));
putclose;
```

- Result is a scalar GAMS model representing the Convex Hull



Extended Nonlinear Programming

Soft penalization of constraints

- Model

$$\begin{array}{ll} \min\limits_{x_1,x_2,x_3} & \exp(x_1) \\ \text{s.t.} & \log(x_1) = 1 \\ & x_2^2 \leq 2 \\ & x_1/x_2 = \log(x_3), 3x_1 + x_2 \leq 5, x_1 \geq 0, x_2 \geq 0 \end{array}$$

- Additional information

```
$onecho > %gams.scrdir%empinfo.scr
e1 sqr  5
e2 MaxZ 2
$offecho
```

è

$$\begin{array}{ll} \min\limits_{x_1,x_2,x_3} & \exp(x_1) + 5\,\|\log(x_1) - 1\|^2 + 2\max(x_2^2 - 2, 0) \\ \text{s.t.} & x_1/x_2 = \log(x_3), 3x_1 + x_2 \leq 5, x_1 \geq 0, x_2 \geq 0 \end{array}$$

GAMS “Solver” EMP

- Reformulates model based on user-provided information
  - CHull
  - ENLP
  - EMCP (ENLP plus NLP2MCP)
  - …
- Facilitates to only write out the reformulated model
- Passes the generated model to an appropriate solver
- Reads solution back into original space

Conclusion

- Continuously bridge the gap between academia and industry
- Incorporate cutting edge approaches
- Be able to solve new model classes
  - using existing methods
  - make it easy for solver developers to provide new algorithms

Thanks for your time!

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Convex Hull (old format)

```
* this writes the loginfo file for the CHull
file dj / '%gams.scrdir%loginfo.scr' /;  dj.nd=0; dj.nw=0; dj.lw=0;
put dj '0'/ card(lt);
loop(lt(j,jj),
   put / '2' / '* 2 ' j.tl ' ' jj.tl ' 1' / '1' / 'seq 2 ' j.tl ' ' jj.tl / 0
             / '* 2 ' j.tl ' ' jj.tl ' 1' / '1' / 'seq 2 ' jj.tl ' ' j.tl / 0 );
putclose;

option minlp=convert; m.optfile=1;
$echo chull u1ch.gms > convert.opt
solve m us minlp min t;
```

GDP Example

$$\min Z = T$$

$$s.t. \quad T \geq x_1 + 8$$

$$T \geq x_2 + 5$$

$$T \geq x_3 + 6$$

$$\begin{bmatrix} Y_1 \\ x_1 - x_3 + 5 \leq 0 \end{bmatrix} \vee \begin{bmatrix} \neg Y_1 \\ x_3 - x_1 + 2 \leq 0 \end{bmatrix}$$

$$\begin{bmatrix} Y_2 \\ x_2 - x_3 + 1 \leq 0 \end{bmatrix} \vee \begin{bmatrix} \neg Y_2 \\ x_3 - x_2 + 6 \leq 0 \end{bmatrix}$$

$$\begin{bmatrix} Y_3 \\ x_1 - x_2 + 5 \leq 0 \end{bmatrix} \vee \begin{bmatrix} \neg Y_3 \\ x_2 - x_1 \leq 0 \end{bmatrix}$$

$$T, x_1, x_2, x_3 \geq 0$$

$$Y_k \in \{true, false\}, k = 1,2,3.$$

Stage / Job	1	2	3
A	5	-	3
B	-	3	2
C	2	4	-

```
$onecho  > %gams.scrdir%logdisj.scr
Disjunction d(j,jj);
d(j,jj) with lt(j,jj) is
if pr(j,jj)
   then seq(j,jj);
   else seq(jj,j);
endif;
$offecho
```

Raman & Grossmann (1994)