Grid Computing in Finance using an Algebraic Modeling System

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Agenda

- Mathematical Optimization in Finance
- Grid Computing
- Sun’s Network.com

GAMS Development / GAMS Software

- Roots: **Research project** World Bank 1976
- Pioneer in **Algebraic Modeling Systems** used for economic modeling
- Went **commercial** in 1987
- **Offices** in Washington, D.C and Cologne

- Professional **software tool provider, not a consulting company**
- Operating in a **segmented niche market**
- Broad **academic** & **commercial** user base and network

General Algebraic Modeling System

Agenda

- **Mathematical Optimization in Finance**
- Grid Computing
- Sun’s Network.com

Mathematical Optimization in Finance

Very active research field with significant contributions and important practical applications

Some of the reasons:

- Continual stream of challenging problems with obvious impact of uncertainty
- High availability of data
- Validation potential – benchmarking
- Very competitive and liquid markets

Many instruments, tools and strategies

Portfolio Optimization Models

- *Seminal Developments: Mean-Variance Portfolio Optimization*
- Scenario Optimization
- Stochastic Programming

The Mean-Variance Model

Markowitz (1952), Nobel prize 1990

Given

Some investments x_i with historical data

- **Rewards = Expected returns** of investments: μ_i (**Mean** of historical returns)
- Risk: **Variance** of investments $Q_{i,j}$

Goal

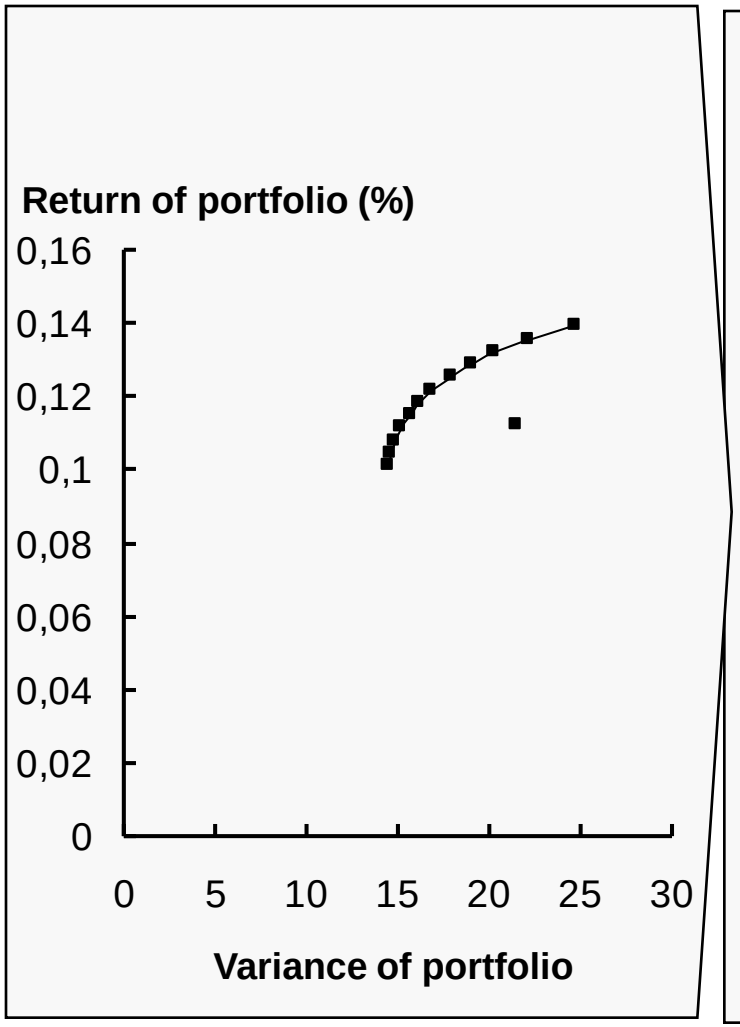
Balance risk *r* of portfolio against expected **returns** of portfolio

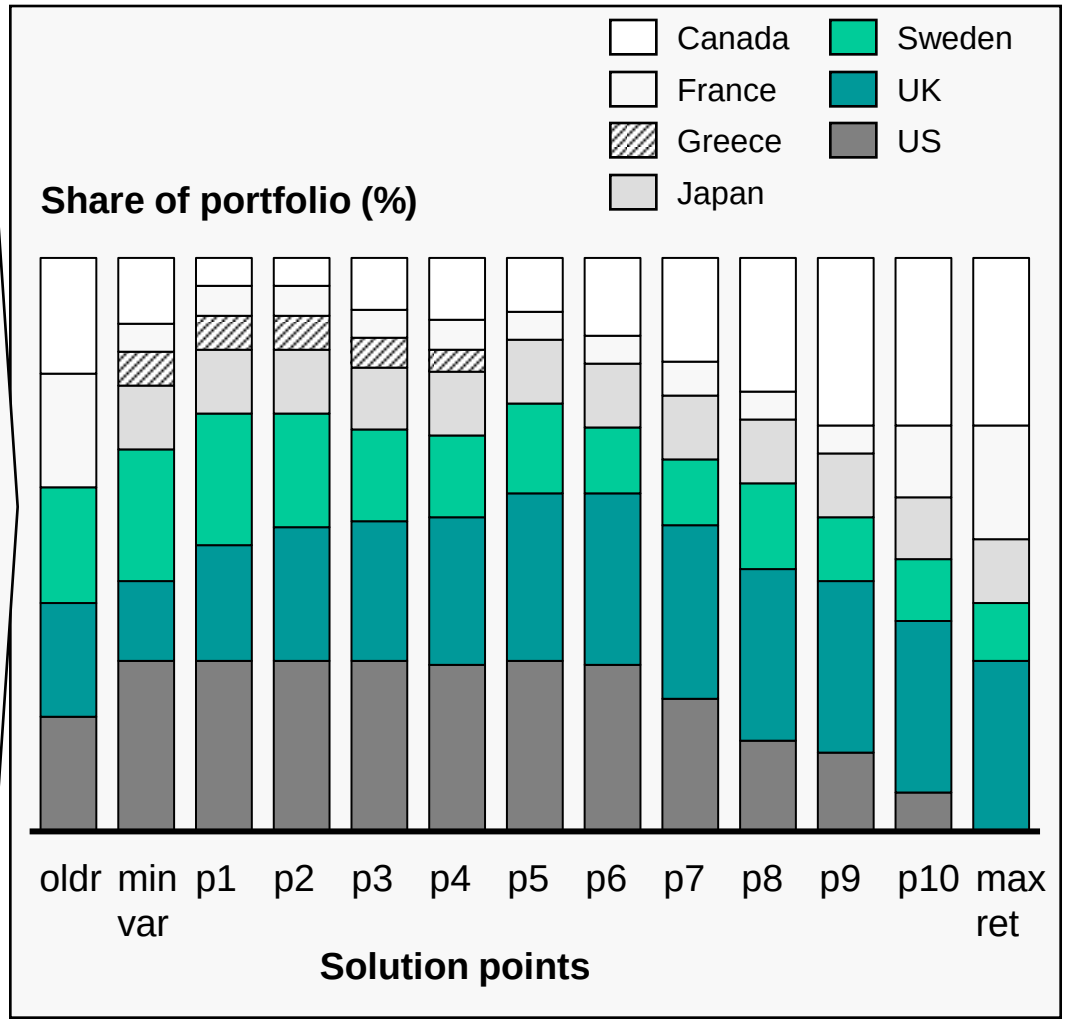
Minimize variance *v* of portfolio for a given target return *r*

Algebra

Variance of Portfolio	$Min \sum_{i=1}^{I}\sum_{j=1}^{J} x_i\, Q_{i,j}\, x_j$
Target return	$s.t. \quad \sum_{i=1}^{I} \mu_i x_i \geq r$
Budget constraint	$\sum_{i=1}^{I} x_i = 1$
No short sales	$x_i \geq 0$

Efficient Frontier and Portfolios





Business Rules

- Institutional or legal requirements: Describe the way the institution is operating
- Additional constraints, which have to be satisfied

- Not defined by modeling experts
- Independent of risk model
- Basel II

Simple Business Rules

Do not change the model type:

- Short selling

- Risk free borrowing

- Upper or lower bounds on certain instruments

More Complex Business Rules

Require introduction of integer (binary) variables:

- **Cardinality Constraint**: Restrict number of investments y_i in portfolio
- **Threshold Constraint**: Investments x_i can only be purchased at certain minimum $l_{l,i}$ or maximum $l_{u,l}$
- more trading restrictions …

Scenario Optimization Models

Scenarios capture complex interactions between multiple risk factors

- Different methods for risk measurement:
  - Mean Absolute Deviation Models
  - Index Tracking Models
  - Expected Utility Models
  - VAR Models (linear Version: CVAR)
- Models are solved over all scenarios

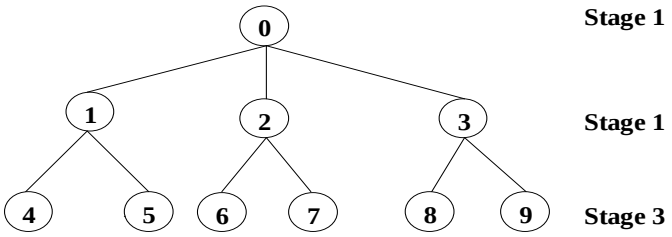
Modeling Issues:

- Linear Models, but business rules may introduce binary variables
- Lots of independent model runs, which can be handled in parallel

Stochastic Programming (SP)

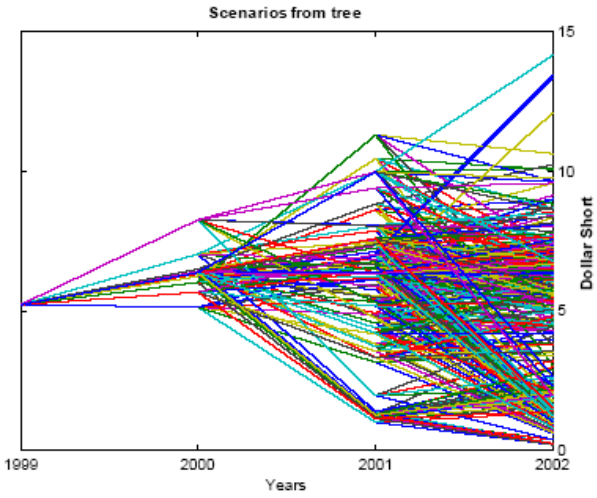
Stochastic Programming models allow **Sequence of Decisions:**

- **Scenarios**: Complete set of possible discrete realizations of the uncertain parameters with probabilities
- **Stages**: Decisions points. First stage decisions now, second stage decision (depending of the outcome of the first stage decision) after a certain period and so on
- **Recourse**: Decision variables can adept to the different out comes of the random parameters at each stage



More Complex Scenario Trees

Figure 1: US dollar short rate scenarios



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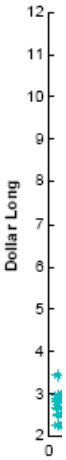

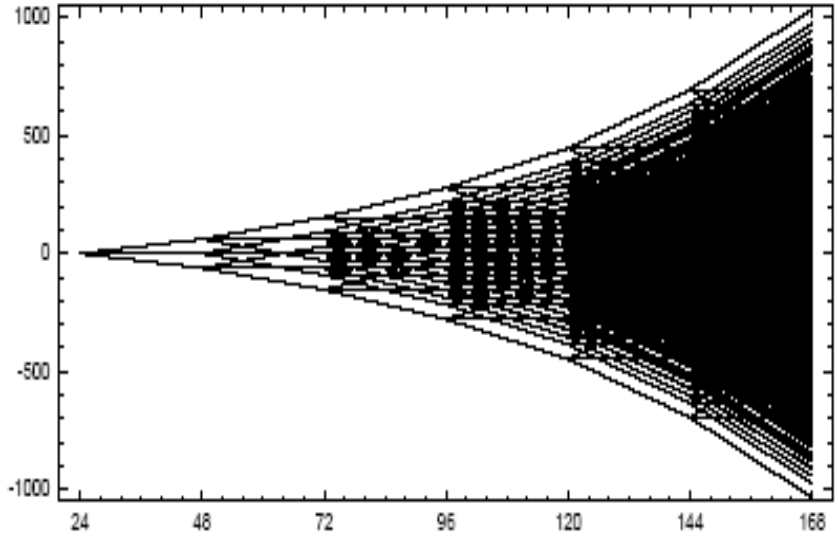

Original load scenario tree

Challenges

Deterministic equivalent: Includes all scenarios and stages
→ Size of model explodes
- Generation difficult
- Solution may not be possible
- Interpretation and validation of results

→ Less applications than one may expect

But: Number of uncertain parameters is small:
- Efficient representation of the uncertain data within the Algebraic Modeling System?
- Scenarios may only differ slightly
- Problems are structured → Specialized Algorithmen available (Decomposition)?

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What is Grid Computing?

A pool of connected computers managed and available as a common computing resource

- Effective sharing of CPU power
- Massive parallel task execution
- Scheduler handles management tasks
- E.g. Condor, Sun Grid Engine, Globus
- Can be rented or owned in common
- Licensing & security issues

Advantages of Grid Computing

- Solve a certain number of scenarios faster, e.g:
  - sequential: 50 hours
  - parallel (200 CPUs): ~15 minutes

- Get better results by running more scenarios*:

#SIM	VaR error	CVaR error
1000	5.42%	6.74%
20,000	1.21%	1.49%

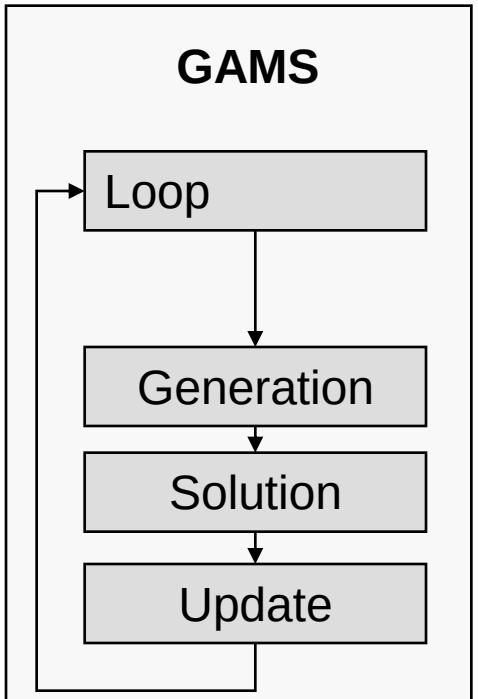
* http://www.tc.cornell.edu/NR/shared/Presentations/24Feb04.Garp.pdf

GAMS & Grid Computing

- **Scalable**:
  - support of massive grids, **but also**
  - multi-cpu / multiple cores desktop machines
  - “1 CPU - Grid”
- Platform **independent**
- Only **minor changes** to model required
- **Separation** of model and solution method
  → Model stays **maintainable**

Simple Serial Solve Loop

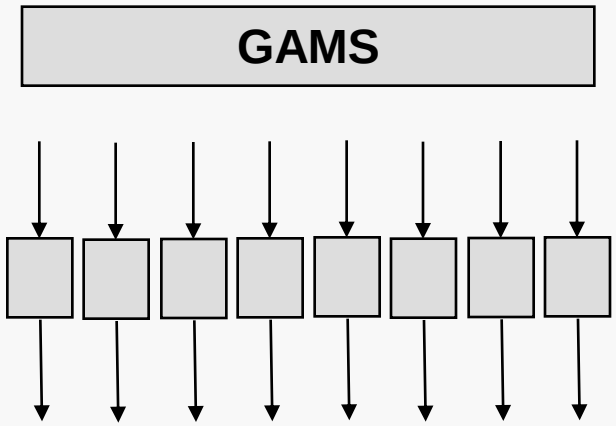
```
Loop(p(pp),
    ret.fx = rmin +(rmax-rmin)
      /(card(pp)+1)*ord(pp) ;
    Solve minvar min var using miqcp;
    xres(i,p)           = x.l(i);
    report(p,i,'inc') = xi.l(i);
    report(p,i,'dec') = xd.l(i)
  );
```



How do we get to parallel and distributed computing?

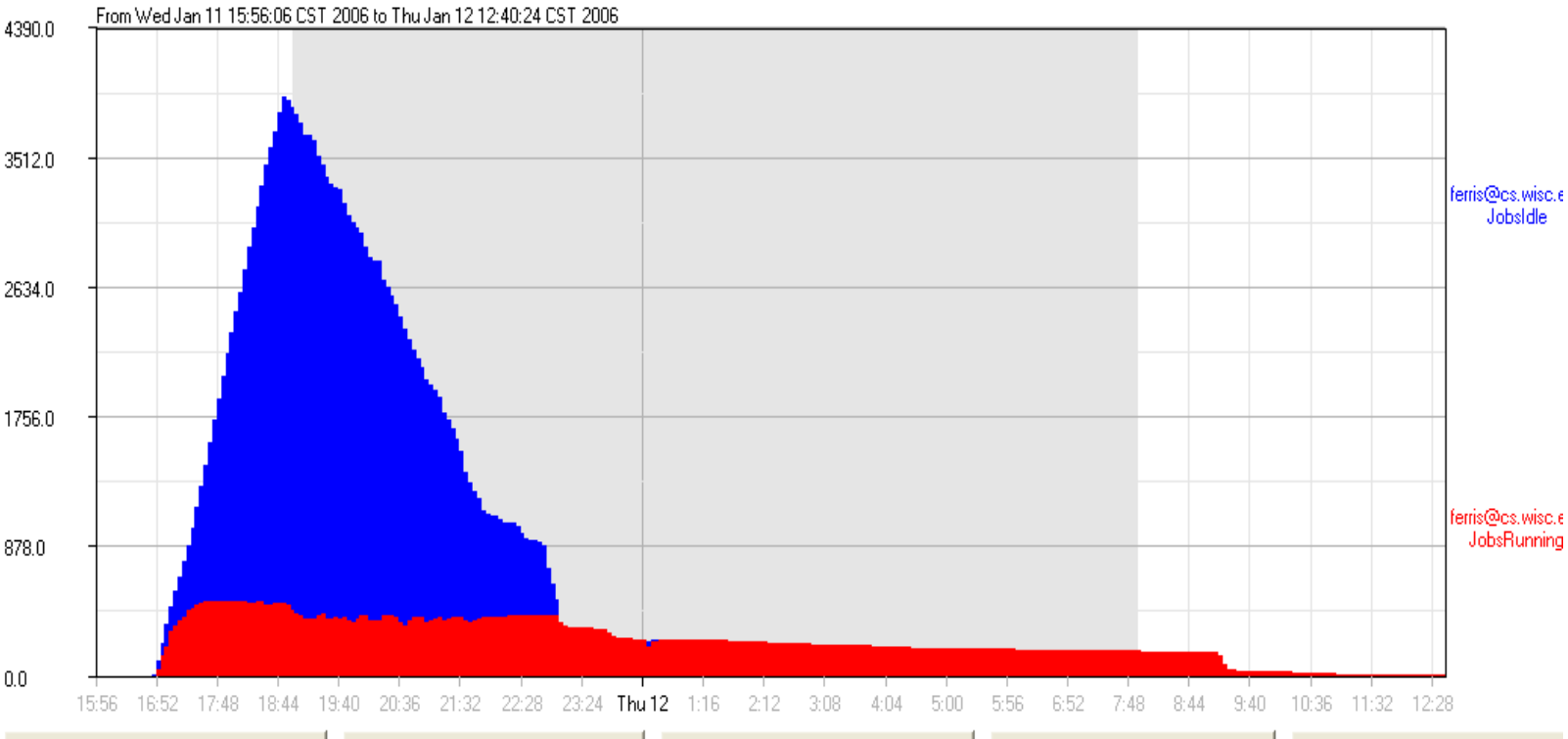
GRID Specific Enhancements

1. Submission of jobs
2. "Grid Middleware"
   - Distribution of jobs
   - Job execution
3. Collection of solutions
4. Processing of results



Results for 4096 MIPS on Condor Grid

- Submission started Jan 11,16:00
- All jobs submitted by Jan 11, 23:00
- All jobs returned by Jan 12, 12:40
  - 20 hours wall time, 5000 CPU hours
  - Peak number of CPU's: 500



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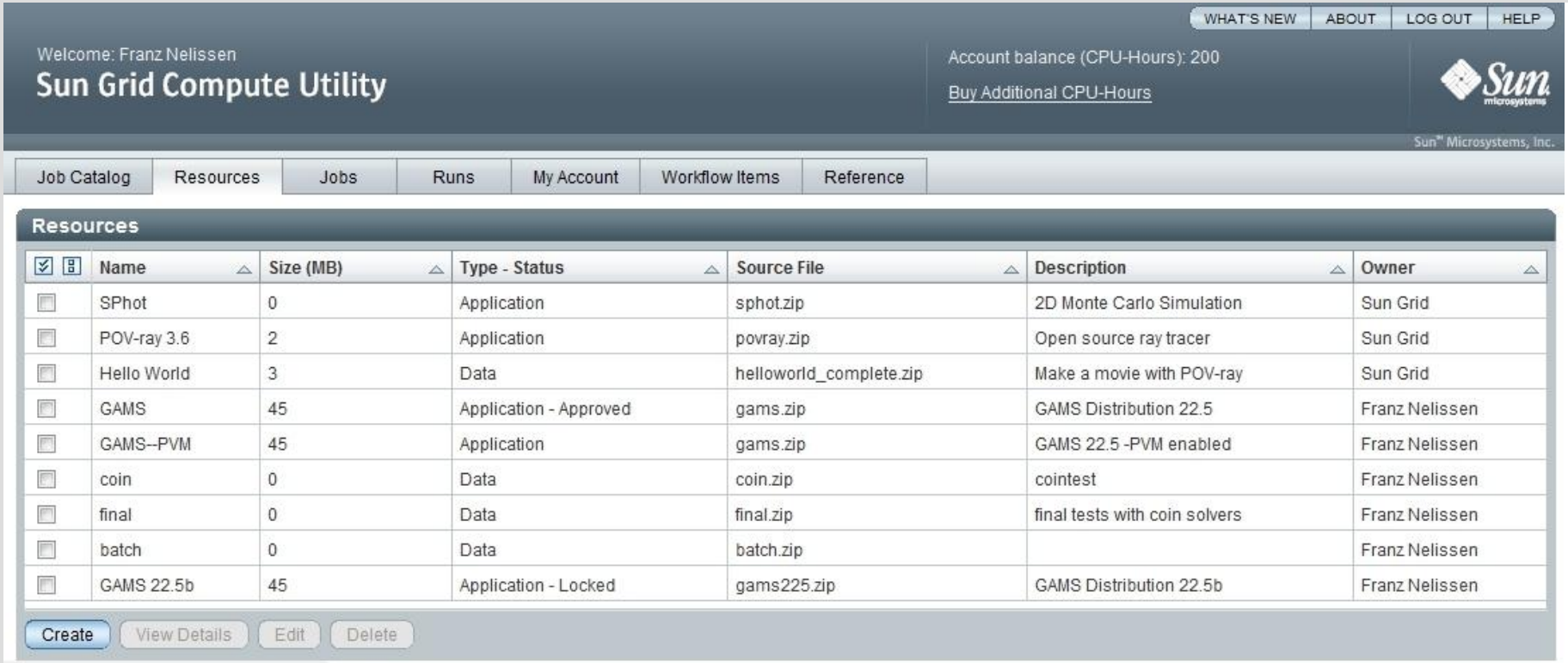
Network.com

Network.com operated by Sun:

- On-demand grid computing service
- Pay as you go utility: All-inclusive price of 1 USD per CPU-hour (Hardware)
- A few hundred CPU's (AMD Opteron, 2 CPU SMP, 2 *4 GB RAM) running Solaris 10
- *Similar Services:*
  - *Amazon: "Elastic Compute Cloud (EC2)"*
  - *ZeroC: IceGrid*

Using Network.com

Using Network.com



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Welcome: Franz Nelissen

Sun Grid Compute Utility

Account balance (CPU-Hours): 200

Buy Additional CPU-Hours

Sun microsystems

Sun™ Microsystems, Inc.

Job Catalog | Resources | Jobs | Runs | My Account | Workflow Items | Reference

Resources

Name	Size (MB)	Type - Status	Source File	Description	Owner
SPhot	0	Application	sphot.zip	2D Monte Carlo Simulation	Sun Grid
POV-ray 3.6	2	Application	povray.zip	Open source ray tracer	Sun Grid
Hello World	3	Data	helloworld_complete.zip	Make a movie with POV-ray	Sun Grid
GAMS	45	Application - Approved	gams.zip	GAMS Distribution 22.5	Franz Nelissen
GAMS--PVM	45	Application	gams.zip	GAMS 22.5 -PVM enabled	Franz Nelissen
coin	0	Data	coin.zip	cointest	Franz Nelissen
final	0	Data	final.zip	final tests with coin solvers	Franz Nelissen
batch	0	Data	batch.zip		Franz Nelissen
GAMS 22.5b	45	Application - Locked	gams225.zip	GAMS Distribution 22.5b	Franz Nelissen

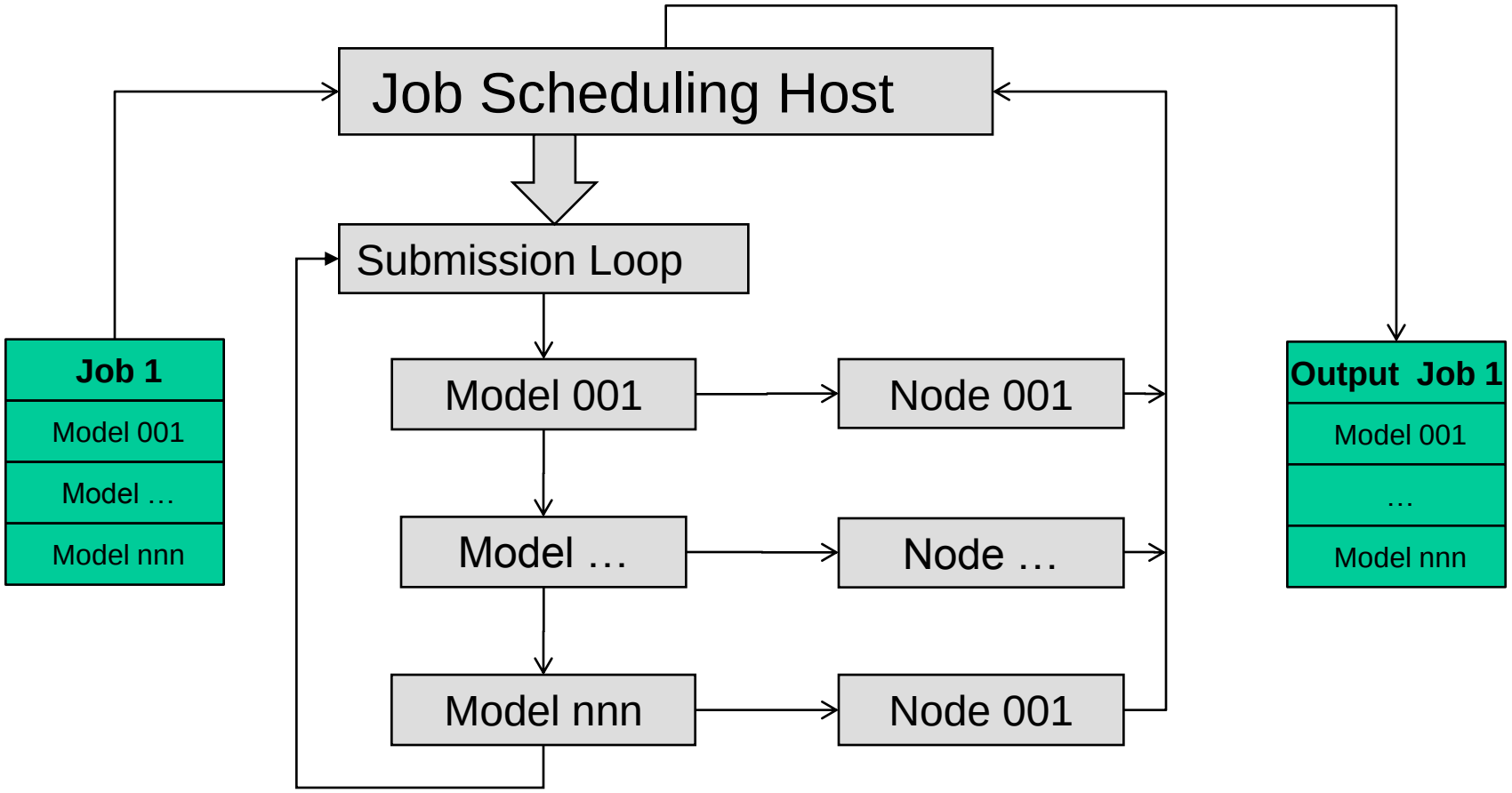
Create | View Details | Edit | Delete

Total Resources Size (MB): 135.000

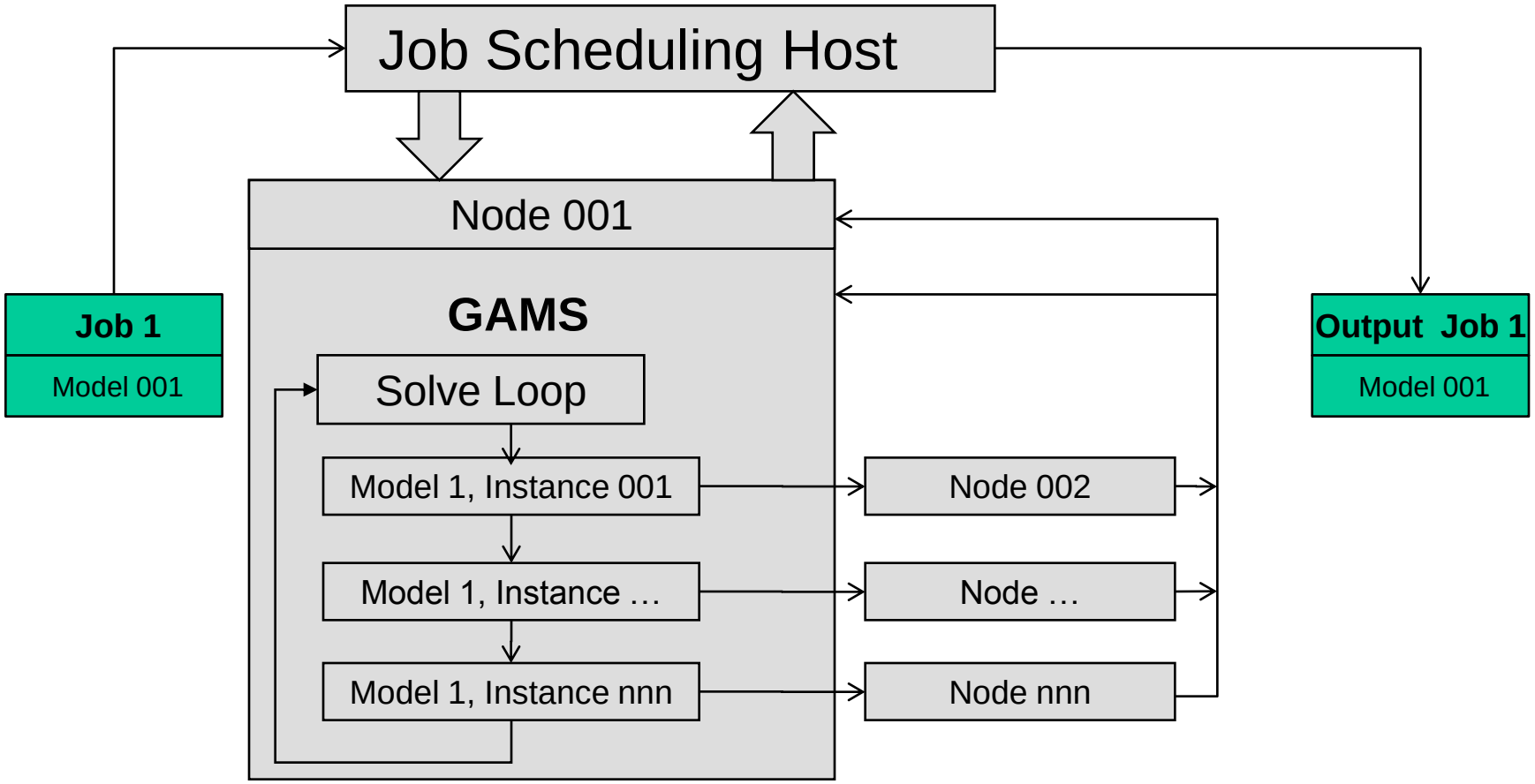
Account Storage limit (MB): 10240

→More Information at: http://www.gams.com/sungrid/

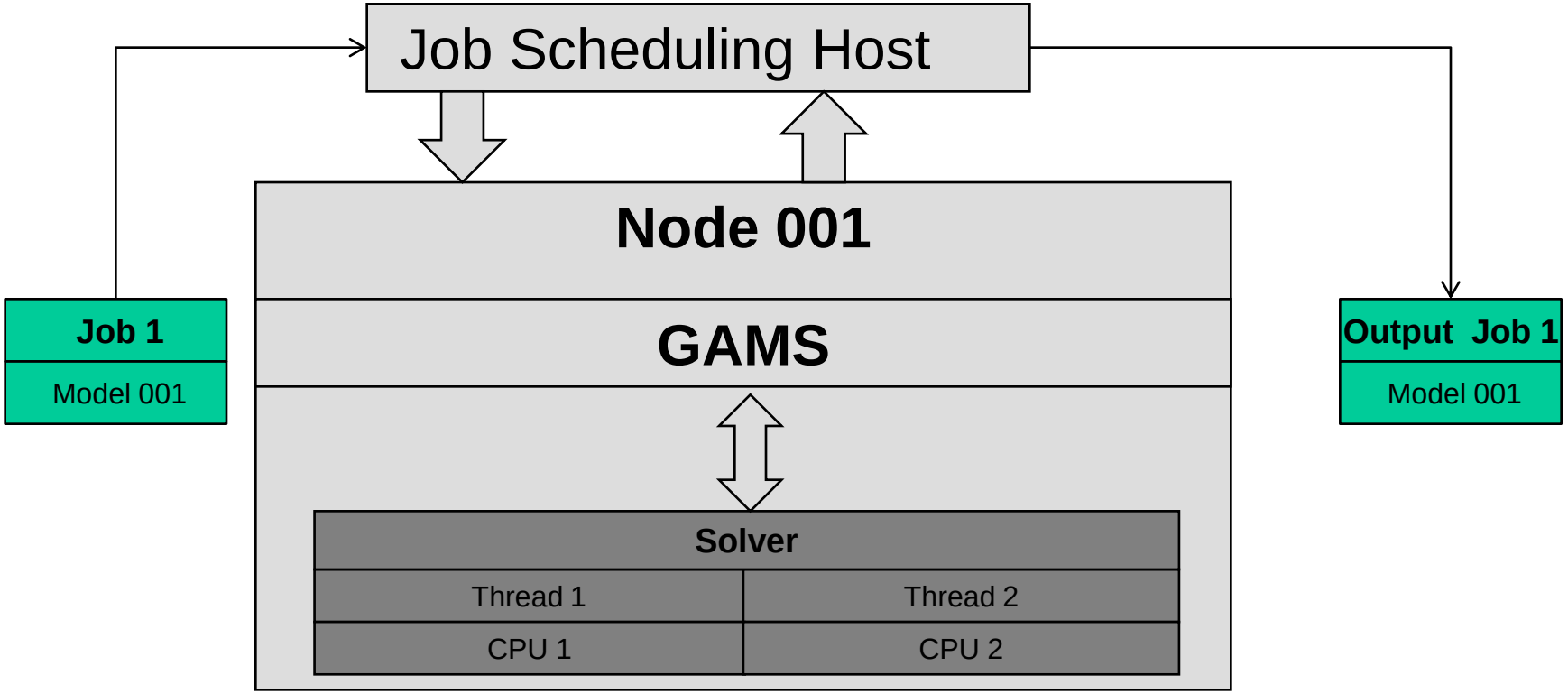
Parallel Execution of Job Arrays



Using the GAMS GRID Facilities



Parallel Solver Threads (SMP)



- **Restricted to two parallel Threads**
- **Not available with all Solvers**

Further Developments

- Better Interfaces to different Grid Architectures
- Parallel Submission of Jobs from within GAMS
- Better Job Control
- …

Conclusions and Summary

- Finance is a success story for OR applications
- Rich set of different risk models available
- Incorporating business rules essential
- Stochastic programming still challenging

- Grid Computing offers lots of promising developments
- Algebraic Modeling Languages fully support parallel enviroments

- Sun's Network.com interesting commercial approach
- Currently 250 free CPU hours and free access to GAMS (COIN) at network.com: http://www.gams.com/sungrid

The End

Thank you!
… Questions?

More Theory and Templates

Theory	• **Practical Financial Optimization** (forthcoming) by S. Zenios • **A Library of Financial Optimization Models** (forthcoming) by A. Consiglio, S. Nielsen, H. Vladimirou and S. Zenios • **Financial Optimization** by S. Zenios (ed.)
Templates available online	• **GAMS Model Library:** http://www.gams.com/modlib/libhtml/subindx.htm • **Course Notes „Financial Optimization“:** http://www.gams.com/docs/contributed/financial/

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