Complementarity at GAMS Development

Steven Dirkse

GAMS Development Corporation

Washington DC

How Important is CP?

- In 1989, no CP possible in GAMS
- In 1999, CP comprises over 10% of solver sales
- Active modeling & application area
- Many different parties involved

GAMS

Talk Outline

- The MCP model type
  - Modeling syntax
  - Solution algorithms
- MPSGE
- New Directions
  - reformulation tools
  - MPEC models

Why Use GAMS for CP?

- Benefits for the modeler
  - Powerful set-based indexing, large scale
  - Symbolic differentiation
  - Self-documenting model: portable, flexible, extendable, readable
  - Solver independence
  - Access to model library

Why Use GAMS for CP?

- Benefits for the algorithm developer
  - Readily available test problems: GAMSLIB, MCPLIB
  - Immediate access to market, user community
  - Environment allows algorithm comparison, enforces rigor

MCP: Definition

Given $F : \Re^n \mapsto \Re^n$, $-\infty \leq \ell \leq u \leq +\infty$,

find $\quad z \in \Re^n \qquad$ s.t.

$$\begin{array}{lrll} \text{either} & z_i = \ell_i & \text{and} & F_i(z) \geq 0 \\ \text{or} & z_i = u_i & \text{and} & F_i(z) \leq 0 \\ \text{or} & \ell_i < z_i < u_i & \text{and} & F_i(z) = 0. \end{array}$$

Extending GAMS for CP

- CP requires:
  - (nonlinear) functions - ✓
  - (bounded) variables - ✓
  - complementary pairing - ??
- MCP model type introduces pairing F.z
- Allows reuse of functions
- Builds on user expertise in NLP

A Walrasian Equilibrium Example

Find a price $p \in \Re^m$ and an activity level $y \in \Re^n$ such that

$$S(p,y) := b - d(p) + Ay \geq 0, \quad p \geq 0, \quad \perp$$

$$L(p) := -A^\top p \geq 0, \quad y \geq 0, \quad \perp$$

An equivalent MCP:

$$F(p,y) := \begin{bmatrix} S(p,y) \\ L(p) \end{bmatrix}, \qquad B := \Re^m_+ \times \Re^n_+,$$

The GAMS Model

```
parameters b(I), s(I), A(I,J);

positive variables p(I), y(J);
equations S(I), L(J);

S(I) .. b(I) + sum{J, A(I,J)*y(J)}
        - s(I)*sum{K, b(K)*p(K)} / p(I)
        =g= 0;
L(J) .. - sum{I, p(I)*A(I,J)}
        =g= 0;

model walras / S.p, L.y /;

solve walras using mcp;
```

CPLIB (Rutherford)

- An interface layer between GAMS & solver
  - Provides function F and its Jacobian, box B, initial iterate, solution reporting
  - Allows integration of MPSGE
- Fortran code, supports C solvers as well
- Cornerstone of CP solvers at GAMS

MPSGE: Intro

- A language for GE models
  - Economists say a lot with a few words
  - MPSGE "speaks economics"
- This language foreign to GAMS
  - Preprocessor reads MPSGE code
  - GAMS code and MPSGE code integrated

MPSGE: Advantages

- Shorthand that reduces errors, tedium
- Efficient constraint representation
- Structures and manages complexity
- Consequence: increased solver demand
  – Forces robustness, speed
  – Drives sales of MCP solvers

Intl. Impact Assessment Model

- 80-country GE model
- Evaluate effects of environmental policy
- User input of key parameters
- Displayed data and graphics allows convenient, rapid comparison b/t policies

MCP Solvers: MILES, PATH

- (Josephy)Newton-based algorithms
  - Local quadratic convergence
  - Require exact Jacobians
  - Line/pathsearch techiques increase robustness
- Use sparse linear algebra (LUSOL)
  - Scale to large problems, efficient pivoting
  - Employ dynamic memory allocation

Why use PATH?

- Crashing
  - Quickly finds near-optimal basis
  - Major speed boost on large models
- Proximal point term to handle rank-deficiency
- Robustness: restarts, pathsearch, merit functions
- Diagnostics

GAMS

A Simple NLP

Let $c_i, i = 1..1000$ be in $[0, 2]$, $C = \sum_i c_i$.

$$\begin{array}{ll} \text{minimize} & \sum_i (c_i - x_i)^2 \\ \text{subject to} & .4C = \sum_i x_i \\ & x \leq .85 \end{array}$$

NLP solution: 673 superbasic vars, 1 basic, 326 nonbasic.

First order (KKT) conditions for the above: a simple complementarity problem.

$$\begin{array}{rcl} 2(x - c) + e\lambda \leq 0 & \perp & x \leq .85 \\ .4C - \sum_i x_i = 0 & \perp & \lambda \ \text{ free} \end{array}$$

MCP solution: 674 basic vars, 326 nonbasic

NLP vs. KKT

- Problems with NLP solvers
  - expect “few” superbasics
  - slow convergence: add superbasics singly
  - memory usage quite high
- Advantages of KKT system
  - Uses second order information
  - no superbasics, exact basis identification

NLP2MCP (Ferris, Horn)

- Output KKT conditions for an NLP
  - Requires taking derivatives symbolically
  - Automates error-prone process
- Makes interesting projects possible
  - Integrate optimization and equilibrium concepts
  - Large NLP models may solve better
  - Build large models out of components

GAMS

MPEC model type

- Superset of NLP and MCP
  - Application may come from either direction
  - Syntax reflects this
- Some models and solver links exist
  - Bundle code
  - SolvOpt
- Better solvers required

MCP2NLP (Drud)

- May want to reformulate CP as NLP
  - Improve robustness of arsenal
  - Diagnostics for unsolvable models
  - Starting point for MPEC models
- Initial results:
  - Can improve robustness, speed
  - Comprehensive test requires more automation

Summary

- MCP/MPSGE models no longer “exotic”
  - Large base of demanding users
  - System delivers dependable results
- Active work on new applications in CP
  - MPEC model type
  - reformulation tools: NLP2MCP, MCP2NLP

GAMS