

Global Optimization with GAMS/LGO

Introduction, Usage, and Applications

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Atlanta, GA October 2003**

Introduction

- Nonlinear processes and phenomena: NLP models in the sciences, engineering, and economics
- Nonlinear models frequently possess multiple optima: hence, their solution requires a suitable global scope search approach
- The objective of global optimization is to find the “absolutely best” solution of decision models, in the possible presence of local (sub-)optima

Introduction (continued)

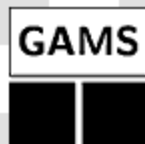
- Global optimization is a relatively new field: “noticeable interest” and research efforts only since ~ 1970
- Solid general theoretical foundations since ~ 1980: globally convergent, exact deterministic and stochastic algorithms; as well as a range of heuristic approaches
- Availability of several gradually developed/improved, “professional level” global optimization codes; in addition, a significant variety of “research” codes since ~ 1990
- Collaboration between the GAMS Corp. & the developers of BARON, LGO, and OQNLP → GAMS/GO solvers (~ 2000)

Introduction (continued)

- Topics covered
 - Brief GAMS review
 - General GO model statement and related notes
 - LGO solver: current implementations
 - LGO solver: key features
 - GAMS/LGO implementation and usage
 - Illustrative numerical example(s)
 - Existing and prospective applications
 - Illustrative references

GAMS (General Algebraic Modeling System)

- GAMS: development started as a research project at the World Bank (1976); GAMS Devpt. Corp. (1988)
- GAMS book by Brooke, Kendrick & Meeraus (1988); subsequent revisions; currently, extensive further documentation is available: papers, technical reports, lectures, model libraries, web sites...
- GAMS: integrates core model development/parser system and a range of connected solver options
- 10,000+ users in over 100 countries (2003)



[[Home](#) | [Support](#) | [Sales](#) | [Solvers](#) | [Documentation](#) | [Model Library](#) | [Search](#) | [Contact](#)]

Welcome to the GAMS Home Page!

The General Algebraic Modeling System (GAMS) is a high-level modeling system for mathematical programming problems. It consists of a language compiler and a stable of integrated high-performance solvers. GAMS is tailored for complex, large scale modeling applications, and allows you to build large maintainable models that can be adapted quickly to new situations.

- [An Introduction to GAMS](#)
- [Documentation](#) (including [FAQ](#))
- [Contributed Documentation](#)
- [Presentations, Books, Posters](#)
- [Contributed Software](#)
- [Courses and Workshops](#)
- [The GAMS Mailing List and Newsletter](#)
- [Visit our European Web Site](#)
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- [Solution Specialists](#)
- [Other Sites on the Web](#)

GAMS Solver Options

(October 2003)

<u>BARON</u>	Branch-And-Reduce Optimization Navigator for proven global solutions from The Optimization Firm
<u>BDMLP</u>	LP solver that comes with any GAMS system
<u>CONOPT</u>	Large scale NLP solver from ARKI Consulting and Development
<u>CPLEX</u>	High-performance LP/MIP solver from Ilog
<u>DECIS</u>	Large scale stochastic programming solver from Stanford University
<u>DICOPT</u>	Framework for solving MINLP models. From Carnegie Mellon University
<u>LGO</u>	Lipschitz global optimizer from Pinter Consulting Services
<u>MILES</u>	MCP solver from University of Colorado at Boulder that comes with any GAMS system
<u>MINOS</u>	NLP solver from Stanford University
<u>MOSEK</u>	Large scale LP/MIP plus conic and convex non-linear programming system from EKA Consulting
<u>MPSGE</u>	Modeling Environment for CGE models from University of Colorado at Boulder
<u>MPSWRITE</u>	MPS file generator that comes with any GAMS System
<u>OQNLP</u>	Multi-start method for global optimization from Optimal Methods Inc.
<u>OSL</u>	High performance LP/MIP solver from IBM
<u>OSLSE</u>	OSL Stochastic Extension for solving stochastic models
<u>PATH</u>	Large scale MCP solver from University of Wisconsin at Madison
<u>SBB</u>	Branch-and-Bound algorithm from ARKI for solving MINLP models
<u>SNOPT</u>	Large scale SQP based NLP solver from Stanford University
<u>XA</u>	Large scale LP/MIP system from Sunset Software
<u>XPRESS</u>	High performance LP/MIP solver from Dash

GAMS NL Solvers

- Nonlinear modeling capabilities have been available in GAMS from the beginning
- Some 3000 licensed GAMS systems are equipped with NLP solver capability + educational users
→ significant potential interest in GO solvers
- Availability of **local NL** solvers (2003): NLP (MINOS, CONOPT, SNOPT, LSGRG, PATHNLP, MOSEK, TRAMP) and **MINLP** solvers (DICOPT, SBB)
- Implementation of a range of algorithms and methods (B&B, GRG, OA, SLP, SQP,...) improves the overall reliability of nonlinear modeling and solution efforts, **to the benefit of modelers/users**

GAMS Global Solvers

Available solvers:

- **BARON** Branch-and-reduce solver system
by The Optimization Firm
- **LGO** Global/nonlinear optimization solver suite
by PCS
- **OQNLP** OptQuest/NLP solver system
by OptTek Systems and Optimal Methods

Key differences between the global solver options:

- underlying algorithms
- external solver needs
- optimality guarantees provided
- model forms they can handle
- model sizes they can handle

GAMS Global Solvers (continued)

- All solvers benefit from GAMS system services:
 - Search algorithms often have difficulties with equalities:
(automatic) defining equation elimination by GAMS
 - Dual solution unavailable, approximate solution only:
optional “polishing” call (CONOPT or other local solvers)
from solution found by global solver
 - Currently, no MINLP capability (in LGO):
B&B code SBB can make use of GAMS NLP sub-solvers
- Seamless exchange of solvers is supported: for example, setting **option nlp=lg** activates LGO
- Other benefits: benchmarking (model libraries such as GlobalLib, PAVER), model converter utility, additional model examples, documentation & technical notes

Continuous GO Model

$$\min f(x)$$

$$f: R^n \rightarrow R^1$$

$$g(x) \leq 0$$

$$g: R^n \rightarrow R^m$$

$$x_l \leq x \leq x_u$$

$x, x_l, x_u, (x_l < x_u)$ are real n -vectors

“Minimal” analytical assumptions:

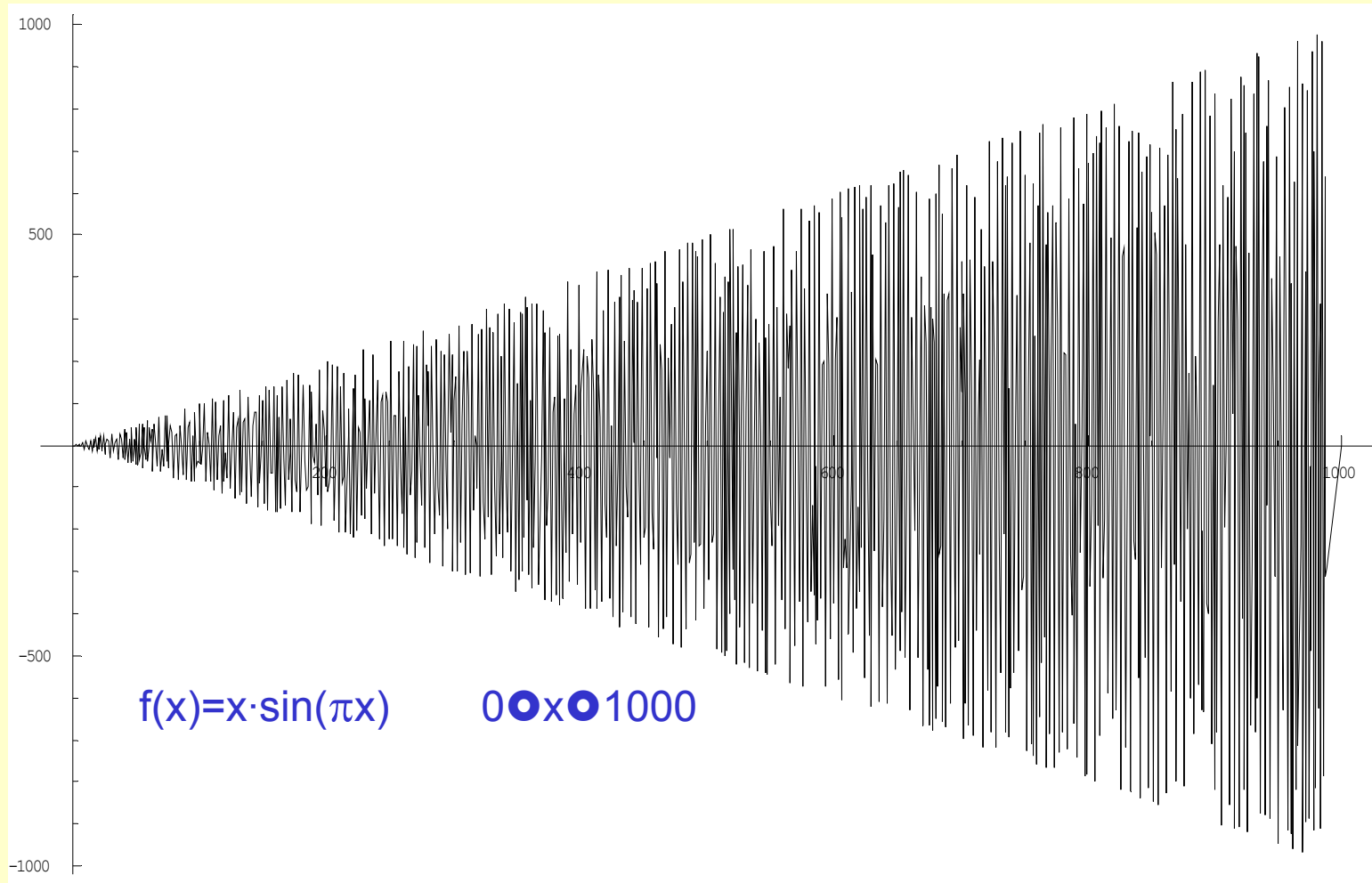
- x_l, x_u finite
- feasible set $D = \{x_l \leq x \leq x_u: g(x) \leq 0\}$ non-empty
- f, g continuous

These guarantee the existence of global solution set X^*

Typically, we assume that X^* is finite (at most, countable)

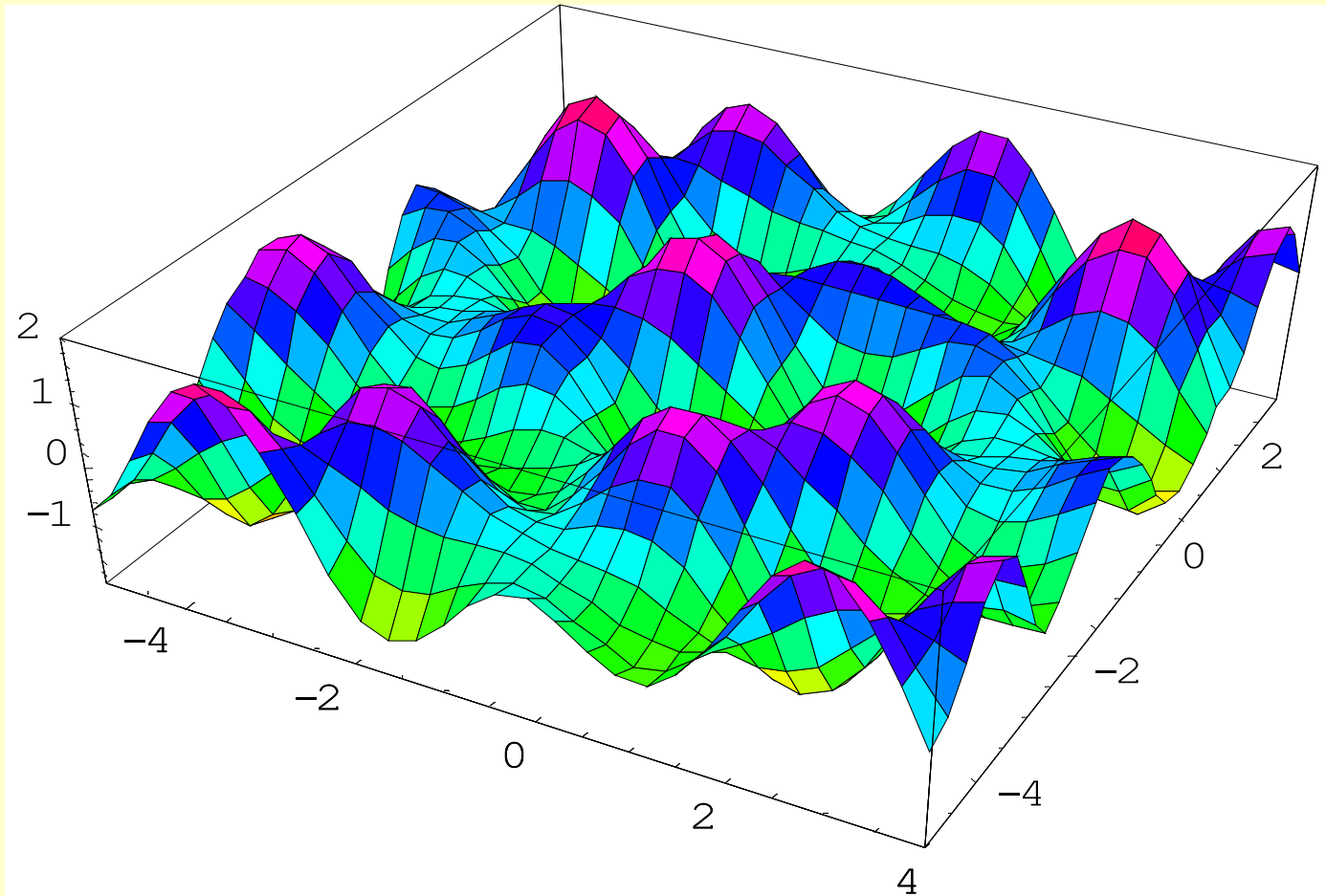
CGO covers a very general class of models, including all combinatorial and mixed integer (MIP) problems

Illustrative GO Model, $n=1$



GO models can be arbitrarily difficult to solve,
even in (very) low dimensions...

Illustrative GO Model, $n=2$



Numerical difficulty may increase exponentially,
as model size (n, m) grows

Solving GO Models

- Practical objective: solution estimates, on the basis of finite sequences of search points and corresponding function values $\{x_1, \dots, x_k\}$, $\{f_1, \dots, f_k\}$, $\{g_1, \dots, g_k\}$ (Note: higher order information, as a rule, has no general value in the context of global search)
- If the functions f and g are Lipschitz-continuous, then - based on the sample sequence - deterministic lower/upper bounds can be derived; hence, a rigorous convergence theory can be developed $(|h(x_1) - h(x_2)| \leq L \|x_1 - x_2\| \text{ for } x_1, x_2 \in D; L = L(D, h) > 0)$
- If the model functions are “only” continuous, then stochastic convergence is the best one can aim for - and it can be properly established
- Implementation issues and challenges: “only a few details”...

LGO Solver Suite: General Scope

- The LGO (Lipschitz Global Optimizer) solver suite has been designed and developed with GO models in mind that do not - or may not - have an easily identifiable and “exploitable” special structure
- This scope specifically includes “black box” models, and models with computable (perhaps non-analytical) functions: examples will be shown and listed later on
- Note that more structured GO models - e.g. with convex, concave or indefinite QP objectives over convex domains - also belong to the scope of LGO (as a rule, with a relatively small performance “hit” - due to its “broad-minded” global search methods, followed by classical local searches)

LGO Solver Suite: Components

- LGO integrates several global search algorithms
 - adaptive partition and search (branch-and-bound)
 - adaptive random search
 - adaptive multi-start
 - heuristic (continuous) scatter search
 - and “traditional” local optimization methods
 - exact penalty method, box-constrained local search
 - generalized reduced gradient method
- This approach flexibly supports the search for global or local solutions in optimization models defined by continuous (or Lipschitz) functions, w/o further necessary analytical conditions

LGO Solver Suite:

Characteristics and Implementations

- No external solver needs; combinations with local solvers are possible, and easy to add
- Optimality guarantees provided: exact and stochastic (deterministic, if model Lipschitz constants are known or can be properly over-estimated during optimization)
- Model forms handled: arbitrary continuous model functions
- Model sizes: in principle, arbitrary (depending on processor and RAM), but: “curse of dimensionality”... typical current delivery versions: thousand(s) of variables and constraints
- Customized versions of LGO implemented for several major modeling platforms; used in advanced scientific, engineering, and economic applications for over a decade

LGO Solver Implementations

(continued)

- LGO “silent” version with a text I/O interface (compiler-based; with Fortran, C, Delphi, API,... connectivity)
- LGO IDE, with a Windows interface (as above)
- LGO solver engine for Excel (PSP solver family)
- LGO external solver link to *Mathematica* (MathOptimizer Professional)
- Further similar development in progress



**Computational Global Optimization
in Nonlinear Systems — An Interactive Tutorial**
By János D. Pintér, Ph.D., D.Sc.



October 2000

SOFTWARE REVIEW



LGO

*Versatile tool for global
optimization*

By Harold P. Benson and Erjiang Sun

A large variety of quantitative decision problems in applied mathematics, engineering, the sciences, business, and economics can be described by constrained optimization models. In many applications, including problems in production.

MathOptimizer and MathOptimizer Professional

Software review in Scientific Computing World (July - August 2003)

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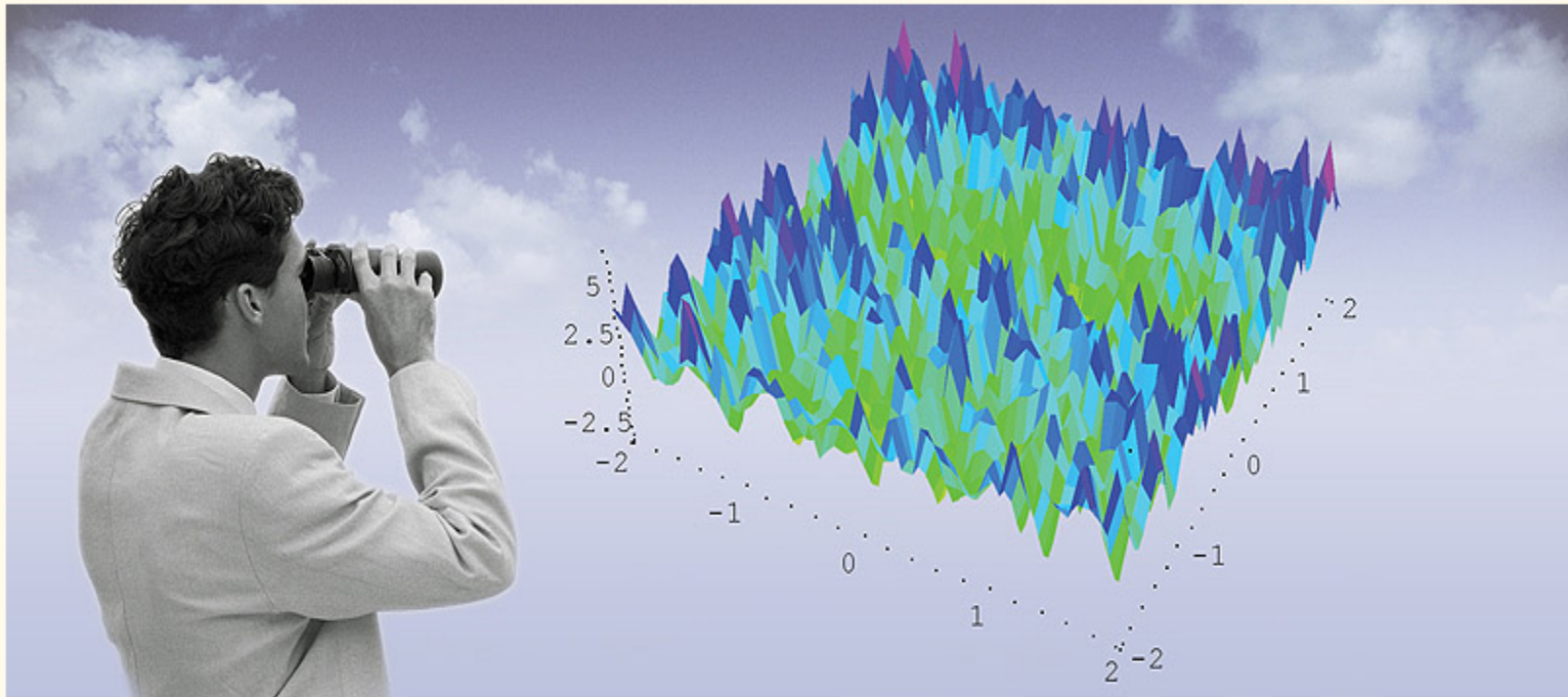
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www.gams.com

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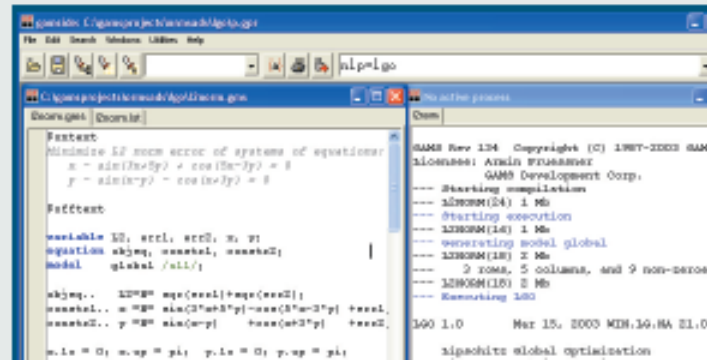


High performance and reliability of solvers come as a result of technological and theoretical developments in solution technology and modeling systems.

LGO is an integrated solver suite for handling nonlinear optimization models. The LGO solution approach is based on the seamless combination of globally convergent and traditional local search strategies.

LGO has been successfully applied in many application areas:

- Advanced Engineering Design



GAMS/LGO Solver Operations

GAMS Model

Model description/preprocessing

GAMS/LGO

Solver options

Solution report



LGO Solver Suite

Global search methods

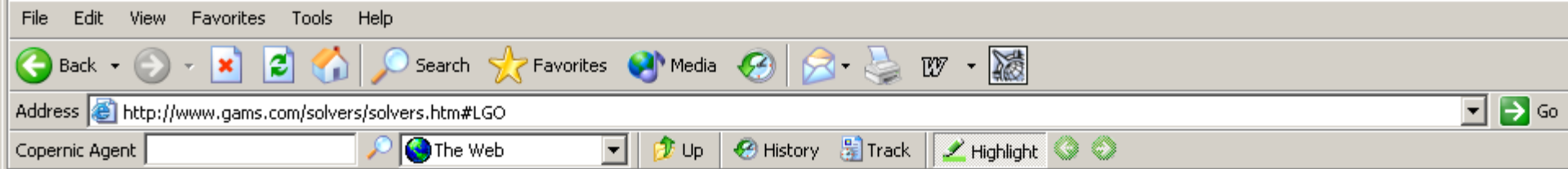
for multiextremal models

Local search methods

for convex nonlinear models



Optional calls to other GAMS solvers
and to external program systems



GAMS/LGO

The LGO solver integrates algorithms of global and local scope. It serves to analyze and solve complex nonlinear models, under minimal analytical assumptions. In order to use LGO, only computable function values are required, without a need for gradient or higher order information.

LGO can be used in several search modes, providing a robust, effective, and flexible solver suite approach to a broad range of nonlinear models. The solver suite approach increases the reliability of the overall solution process.

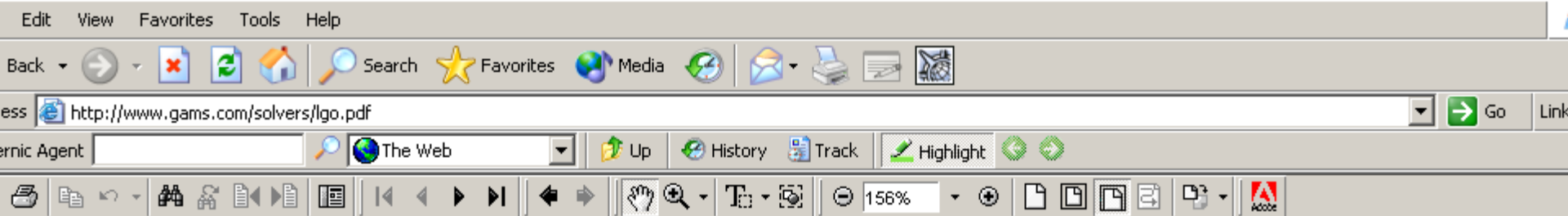
LGO Solver Components

- branch-and-bound based global search
- global adaptive random search
- multistart global search
- exact penalty function based local search
- constrained local optimization

LGO has been successfully applied even to highly complex, 'black box' system models. Tractable model sizes depend only on available hardware.

Illustrative Application Areas

- advanced engineering design
- econometrics and finance
- medical research and biotechnology
- chemical and process industries
- scientific modeling



GAMS/LGO

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2.1	General Options	3
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1 Introduction

1.1 The LGO Solver Suite

GAMS/LGO

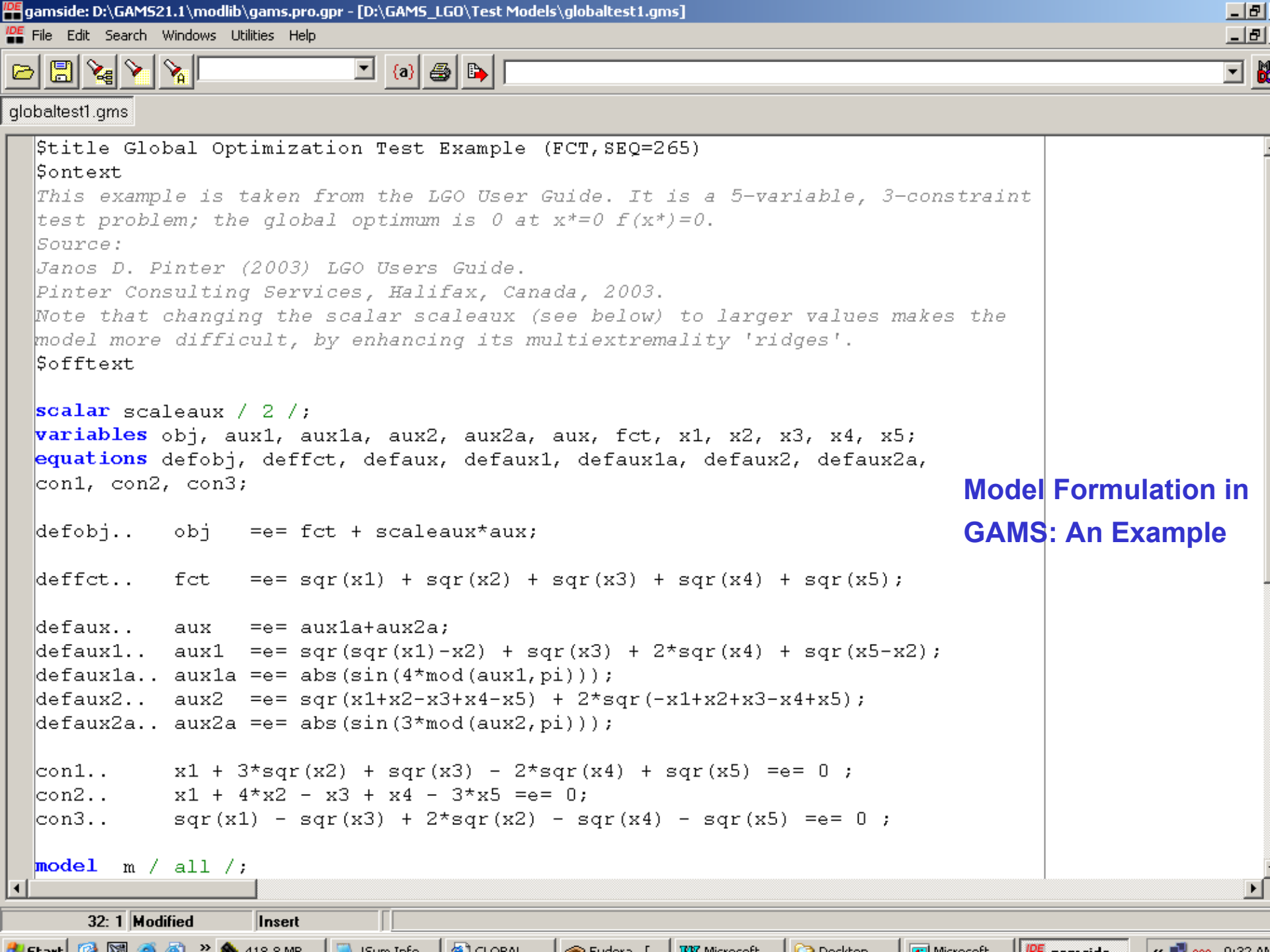
General Options

Option	Description	Default
Opmode	Specifies the search mode used. 0 Local search from the given nominal solution without a preceding global search (LS) 1 Global branch-and-bound search and local search (BB+LS) 2 Global adaptive random search and local search (GARS+LS) 3 Global multistart random search and local search (MS+LS)	3
Tlimit	Time limit in seconds. This is equivalent to the GAMS option reslim. If specified, this overrides the GAMS reslim option.	1000
Log_time	Iteration log time interval in seconds. Log output occurs every log_time seconds.	0.5
Log_iter	Iteration log time interval. Log output occurs every log_iter iterations.	1
Log_err	Iteration log error output. Error reported (if applicable) every log_err iterations.	10
Debug	Debug option. Prints out complete LGO status report to listing file. 0: No 1: yes	0
CallConopt	Number of seconds given for cleanup phase using CONOPT. CONOPT terminates after at most CallConopt seconds. The cleanup phase determines duals for final solution point.	10
Help	Prints out all available GAMS/LGO solver options in the log and listing files.	

GAMS/LGO

Limits and Tolerances

Option	Description	Default
G_maxfct	Maximum number of merit function evaluations before termination of global search phase (BB, GARS, or MS). The difficulty of global optimization models varies greatly: for difficult models, g_maxfct can be increased to 1.0E+6, or even larger values, as needed.	1000 * (n+m)
Max_nosuc	Maximum number of merit function evaluations in global search phase (BB, GARS, or MS) where no improvement is made. Algorithm terminates upon reaching this limit.	200 * (n+m)
Penmult	Constraint penalty multiplier. Global merit function is defined as objective + the violated constraints weighted by penmult.	1
Acc_tr	Global search termination criterion parameter (acceptability threshold). The global search phase (BB,GARS, or MS) ends, if an overall merit function value found in the global search phase is less than acc_tr.	-1.0E10
Fct_trg	Partial stopping criterion in second local search phase.	-1.0E10
Fi_tol	First local search merit function improvement.	1.0E-6
Con_tol	Maximal constraint violation in local search.	1.0E-6
Kt_tol	Kuhn-Tucker local optimality condition tolerance.	1.0E-6
Irngs	Random number seed.	0
Var_lo	Smallest lower bound	-1.0E10
Var_up	Largest uppder bound	1.0E10
Bad_obj	Default value for objective function given evaluation errors.	1.0E10



```
$title Global Optimization Test Example (FCT,SEQ=265)
```

```
$ontext
```

```
This example is taken from the LGO User Guide. It is a 5-variable, 3-constraint  
test problem; the global optimum is 0 at x*=0 f(x*)=0.
```

```
Source:
```

```
Janos D. Pinter (2003) LGO Users Guide.
```

```
Pinter Consulting Services, Halifax, Canada, 2003.
```

```
Note that changing the scalar scaleaux (see below) to larger values makes the  
model more difficult, by enhancing its multiextremality 'ridges'.
```

```
$offtext
```

```
scalar scaleaux / 2 /;
```

```
variables obj, aux1, aux1a, aux2, aux2a, aux, fct, x1, x2, x3, x4, x5;
```

```
equations defobj, deffct, defaux, defaux1, defaux1a, defaux2, defaux2a,  
con1, con2, con3;
```

```
defobj..    obj    =e= fct + scaleaux*aux;
```

```
deffct..    fct    =e= sqr(x1) + sqr(x2) + sqr(x3) + sqr(x4) + sqr(x5);
```

```
defaux..    aux    =e= aux1a+aux2a;
```

```
defaux1..   aux1    =e= sqr(sqr(x1)-x2) + sqr(x3) + 2*sqr(x4) + sqr(x5-x2);
```

```
defaux1a..  aux1a   =e= abs(sin(4*mod(aux1,pi)));
```

```
defaux2..   aux2    =e= sqr(x1+x2-x3+x4-x5) + 2*sqr(-x1+x2+x3-x4+x5);
```

```
defaux2a..  aux2a   =e= abs(sin(3*mod(aux2,pi)));
```

```
con1..      x1 + 3*sqr(x2) + sqr(x3) - 2*sqr(x4) + sqr(x5) =e= 0 ;
```

```
con2..      x1 + 4*x2 - x3 + x4 - 3*x5 =e= 0;
```

```
con3..      sqr(x1) - sqr(x3) + 2*sqr(x2) - sqr(x4) - sqr(x5) =e= 0 ;
```

```
model m / all /;
```

Model Formulation in GAMS: An Example

globaltest1

```

--- Starting compilation
--- globaltest1.gms (51) 1 Mb
--- Starting execution
--- globaltest1.gms (45) 1 Mb
--- Generating model m
--- globaltest1.gms (51) 2 Mb
--- 10 rows, 12 columns, and 43 non-zeroes.
--- globaltest1.gms (51) 2 Mb
--- Executing LGO

LGO 1.0      Sep  3, 2003 WIN.LG.NA 21.2 001.000.000.VIS

LGO Lipschitz Global Optimization
(C) Pinter Consulting Services, Inc.
129 Glenforest Drive, Halifax, NS, Canada B3M 1J2
E-mail : jdpinter@hfx.eastlink.ca
Website: www.dal.ca/~jdpinter

7 defined, 0 fixed, 0 free
3 LGO equations and 5 LGO variables

  Iter      Objective      SumInf      MaxInf      Seconds      Errors
28380    7.634679E-18    0.00E+00    0.0E+00        0.311

--- LGO Exit: Normal completion - Global solution
    0.311 LGO Secs (0.17 Eval Secs, 0.006 ms/eval)

```

← GAMS Preprocessing Step

← LGO Solver Result Summary

Close Open Log ☐ Summary only ☒ Update

1: 1 Insert



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GLOBAL Reproducibility of Results

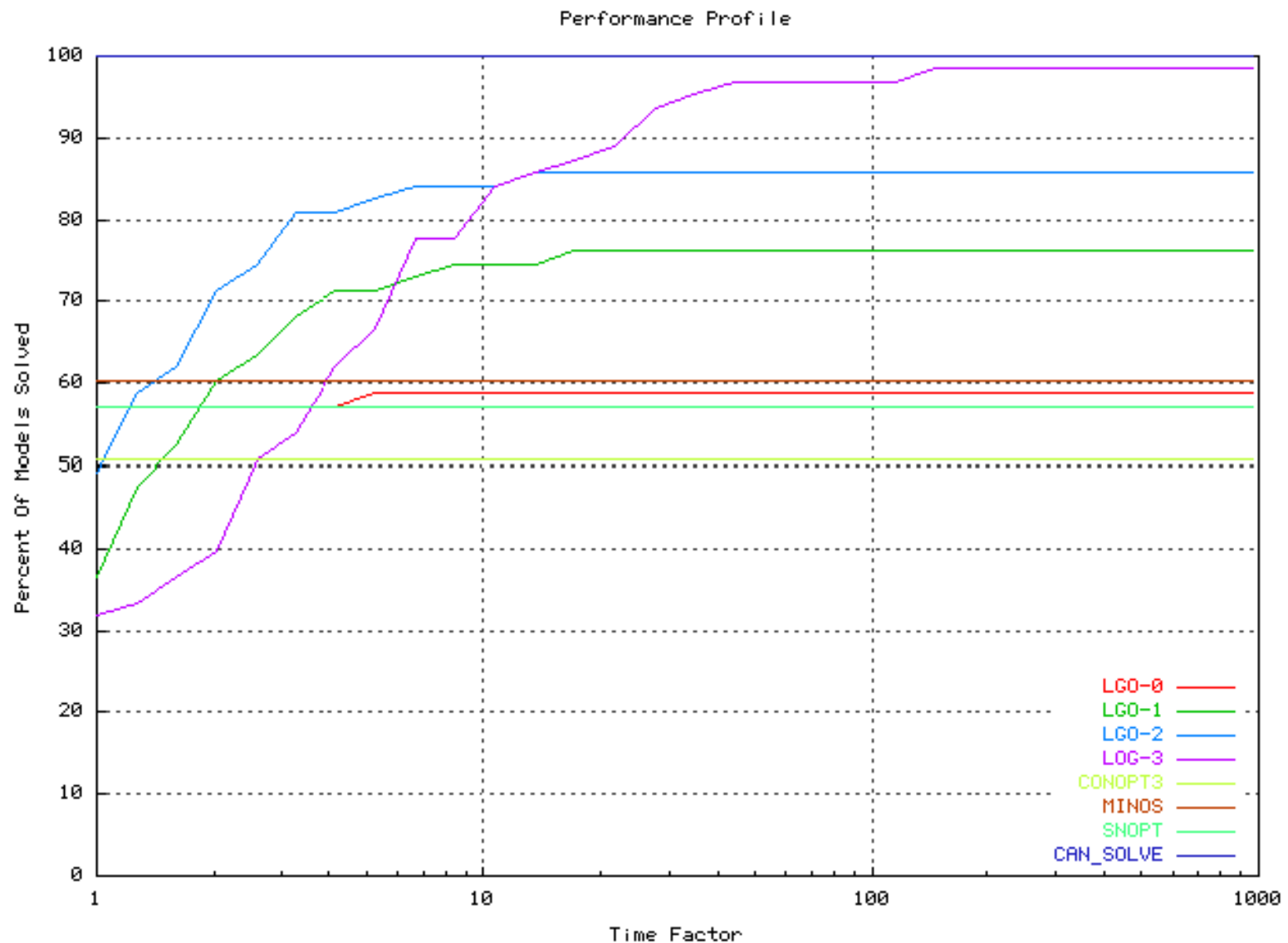
This section gives instructions on reproducing the numerical results in the paper

- Pinter, J (2003), GAMS/LGO Nonlinear Solver Suite: Key Features, Usage, and Numerical Performance, Submitted.

Preliminaries

- Download the GLOBALLib library of models: [globallib.zip](#)
- Download the GAMS scripts to run the experiments
 - [go_test_fig1.gms](#)
 - [go_test_fig2.gms](#)
- Download the termination routines described in the section on [Program Termination](#) These tools are necessary to independently ensure that solvers are allocated a set resource solver time.
 - The GAMS routine [schulz.gms](#)
 - [PSList](#) (Download and place into your GAMS system directory)
 - [PSKill](#) (Download and place into your GAMS system directory)

Performance Profile Summary for a Set of GLOBALLib Models: GAMS/LGO Solver Modes vs. Three State-of-Art Local Solvers



Advanced GO Applications

An Illustrative List

The list below is primarily based on actual (PCS client) case studies and projects, and a few articles / reports

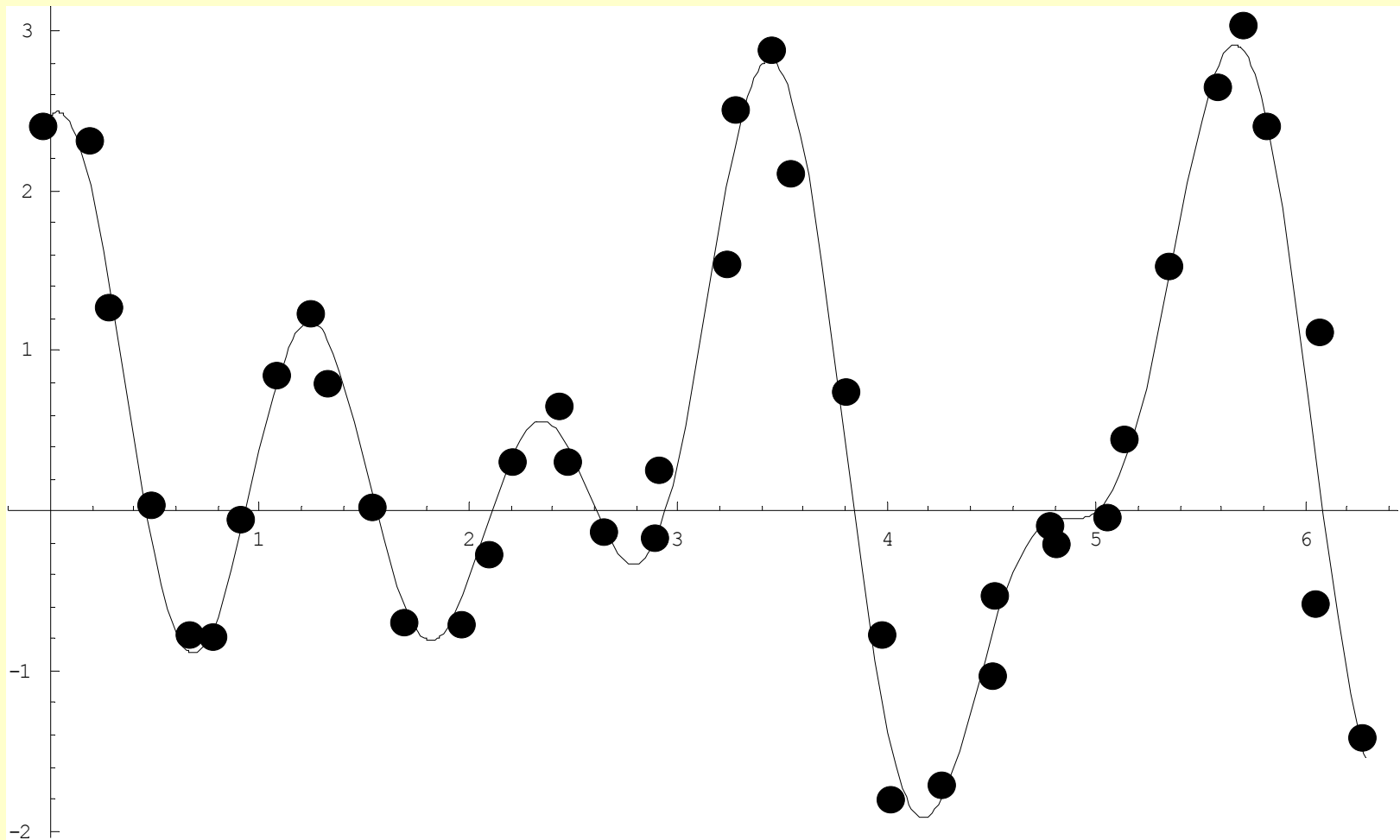
- Chemotherapy and radiotherapy design
- Computational physics and chemistry
- Chemical data and process analysis
- Data classification (clustering) and visualization
- Differential equations (numerical solution)
- Engineering design
- Experimental design
- Environmental engineering
- Financial model development

Application Examples (Continued)

- Laser design
- Model fitting to experimental data (in various calibration or forecasting contexts)
- Packing and loading configuration design
- Resource / population (fish stock,...) management
- Robotics (grasp) design
- Staff scheduling
- Systems of (nonlinear) equations and inequalities
- Vehicle routing and scheduling
- Waste-water engineering system design
and some other areas

Several advanced applications are highlighted on the next slides

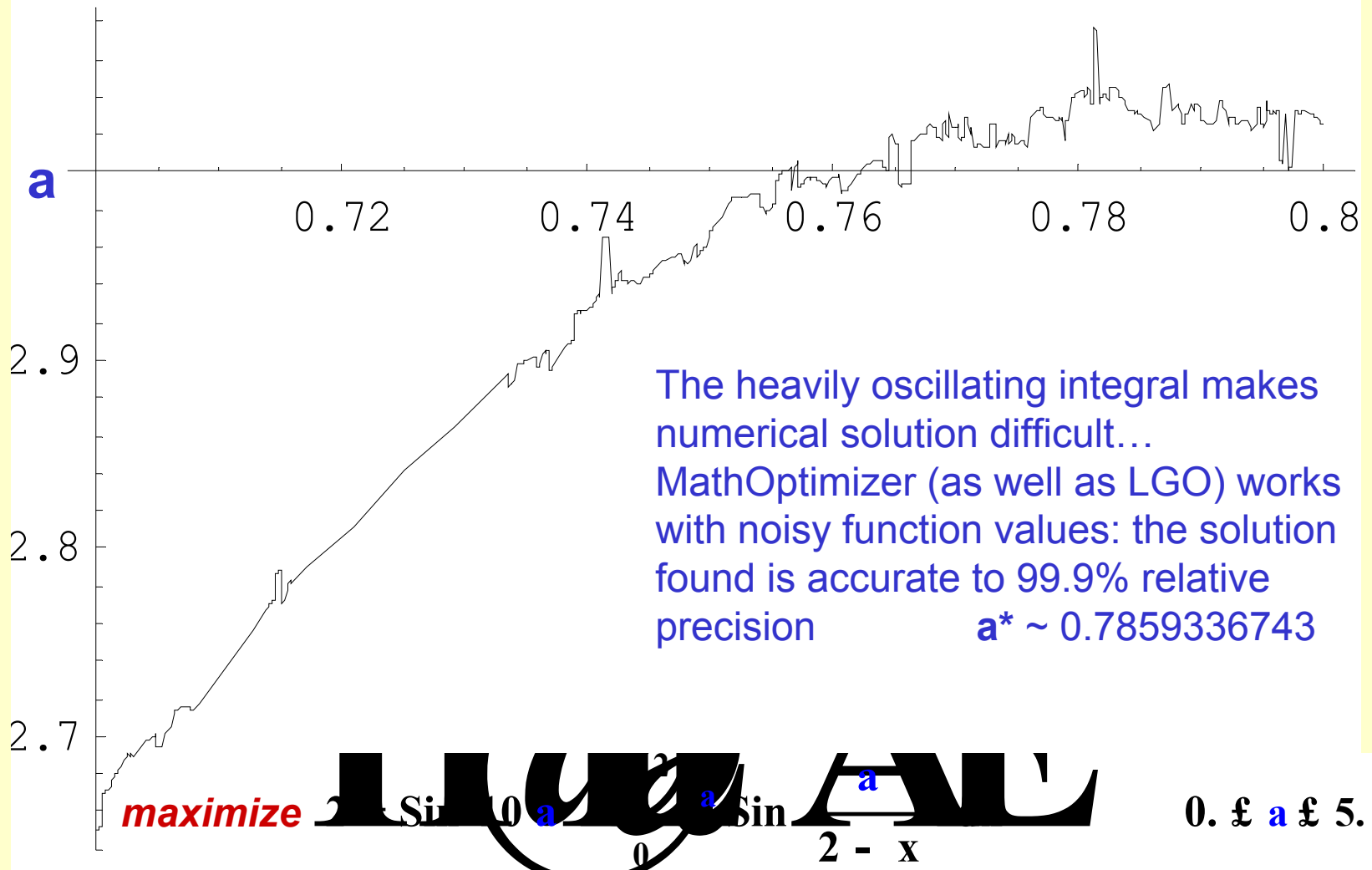
Calibration of Nonlinear Model in Presence of Noise



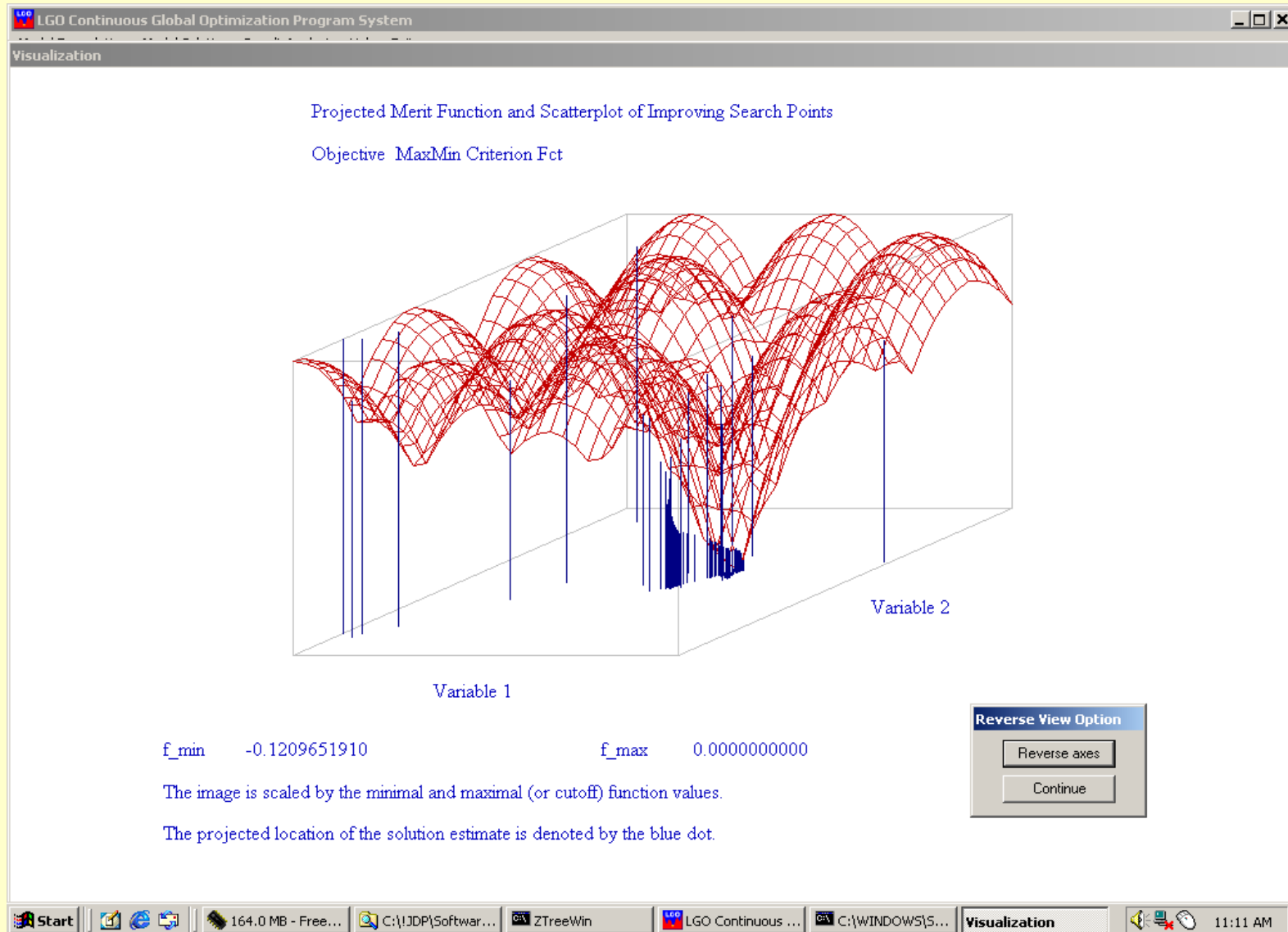
$$a + \sin[b \cdot (\pi \cdot t) + c] + \cos[d \cdot (3\pi \cdot t) + e] + \sin[f \cdot (5\pi \cdot t) + g] + \xi$$

Globally optimized model fit (found by MathOptimizer)

HDHD Challenge, Problem 9: Parametric Integration



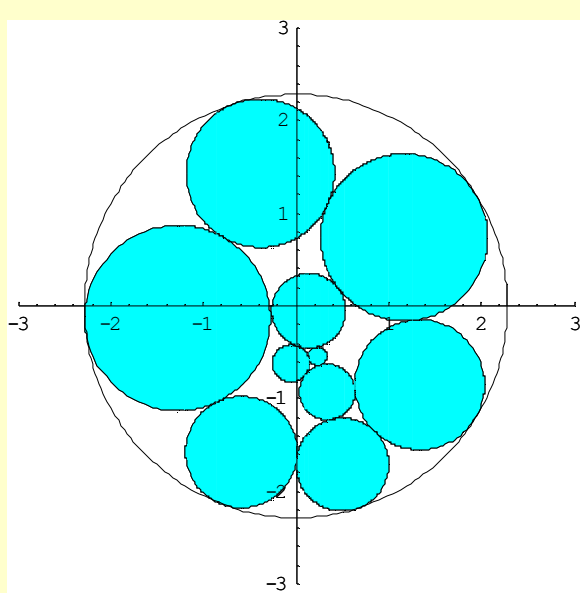
MaxiMin Point Arrangement Problem (U. of Tilburg, NL)



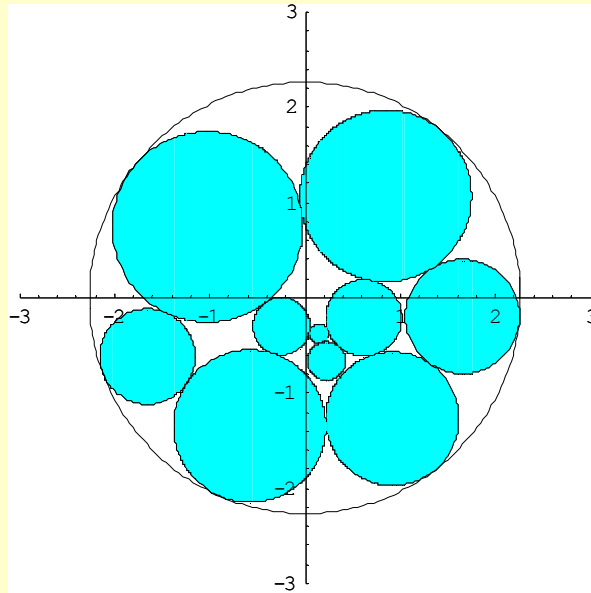
LGO IDE: model visualization ($m=13$, $d=2$)

Non-Uniform Circle Packings

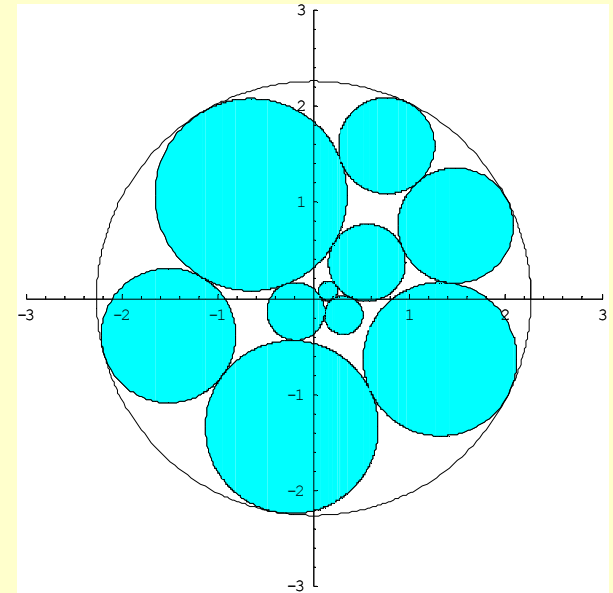
On the pictures shown below, 10 circles - with radii 0.1, 0.2,..., 1.0 - are arranged; equal consideration is given to the “tightness” of the configuration and the radius of the embedding circle (emb_c_rad)



emb_c_rad=2.28345



emb_c_rad=2.26559



emb_c_rad=2.26107

Three different configurations found (MOP runtimes ~ 2 to 5 min)

Laser Equipment Design (NRC, Canada)

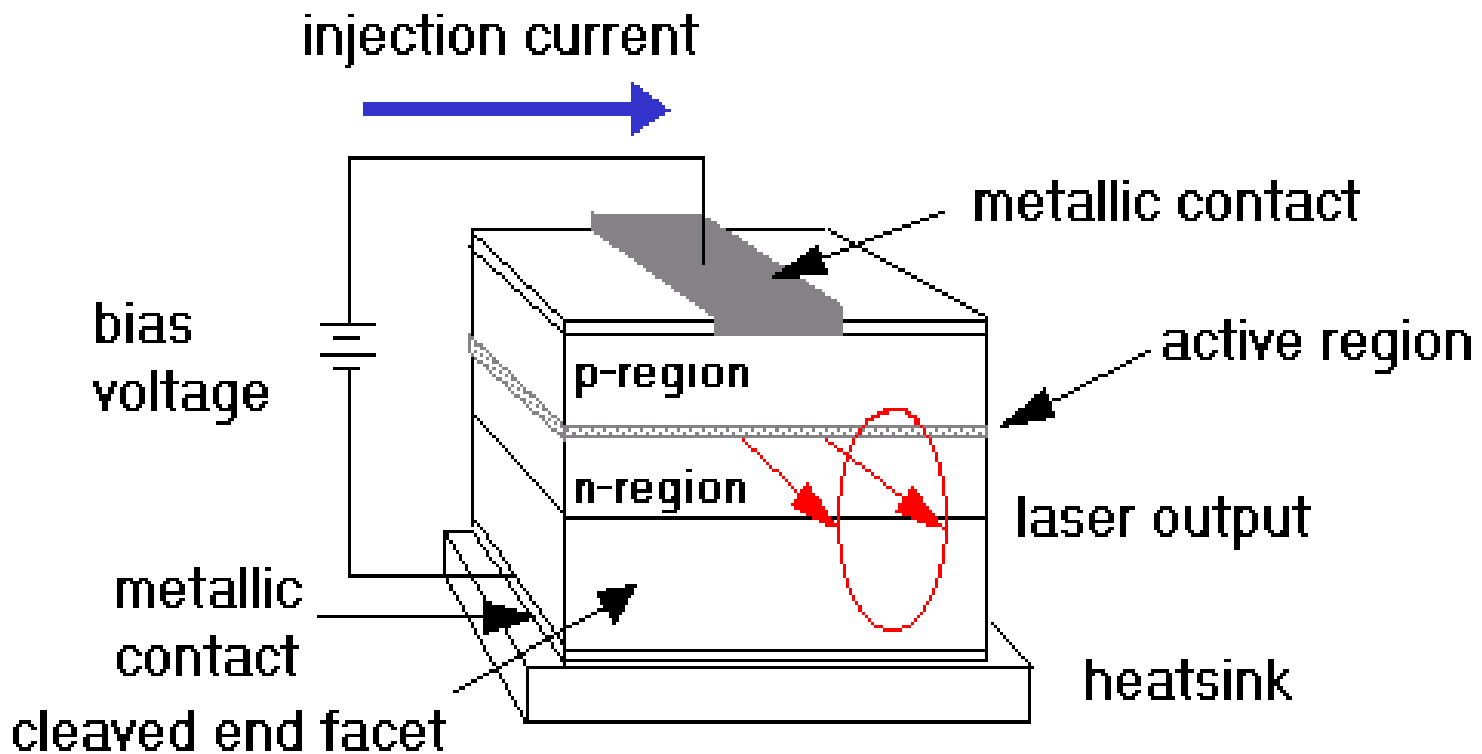
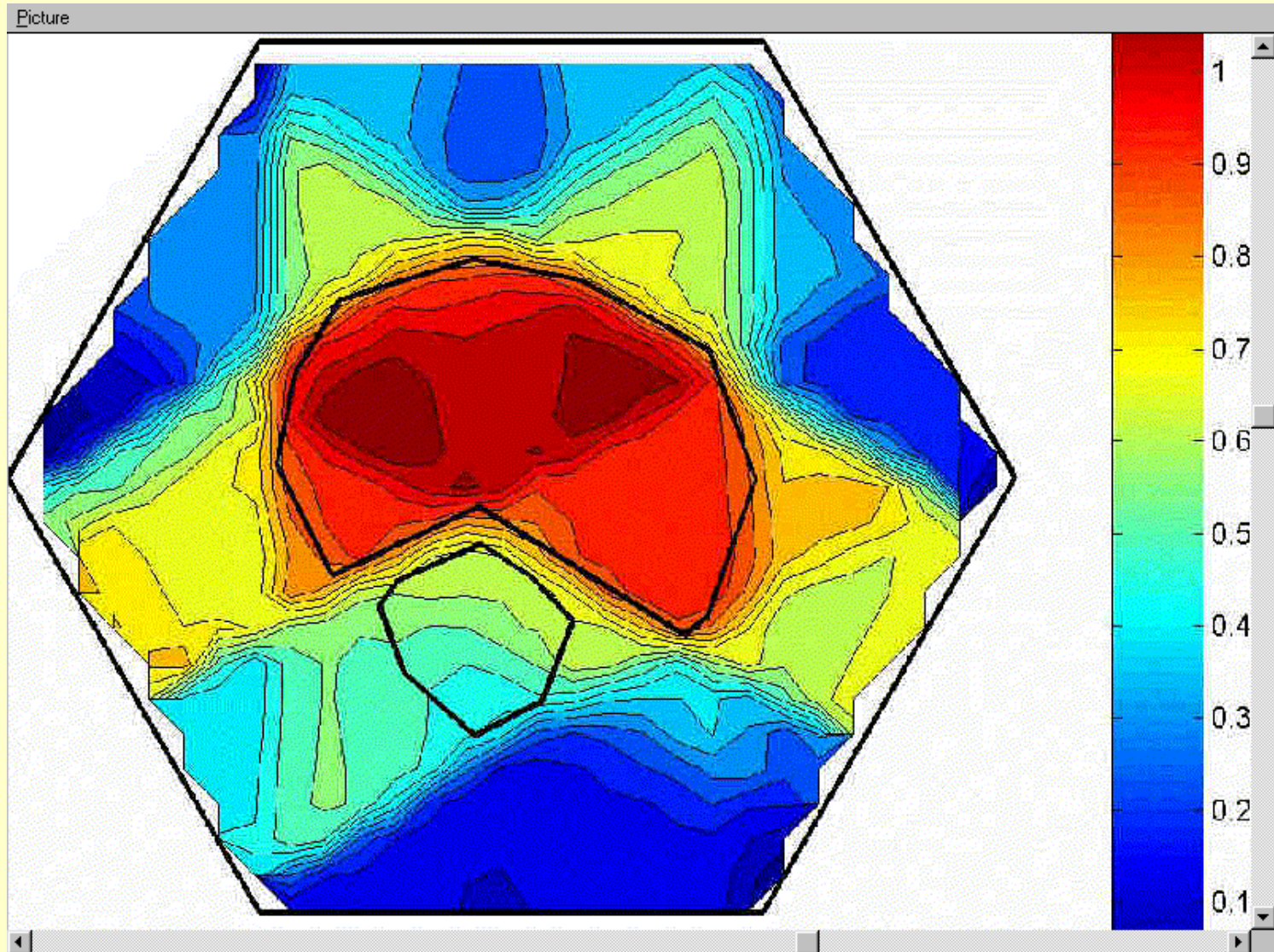


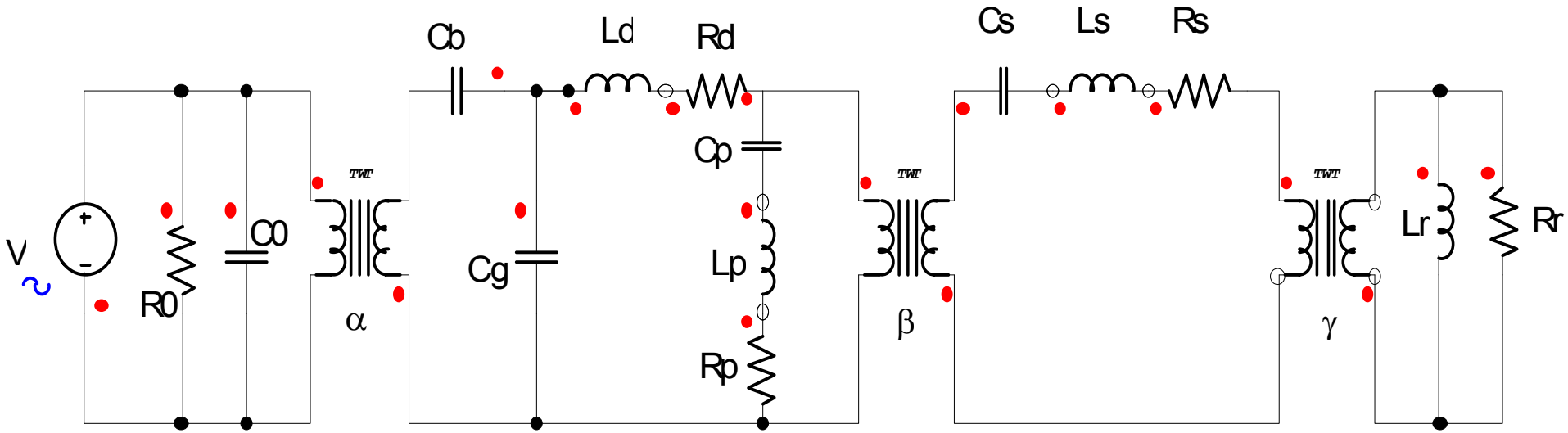
Fig. 1
Semiconductor Laser Diode
(simplified diagram)

Intensity Modulated Radiotherapy (U. of Kuopio, FI)



Globally optimized dose distribution

Sonar Transducer Design: Numerical Model (DRDC)



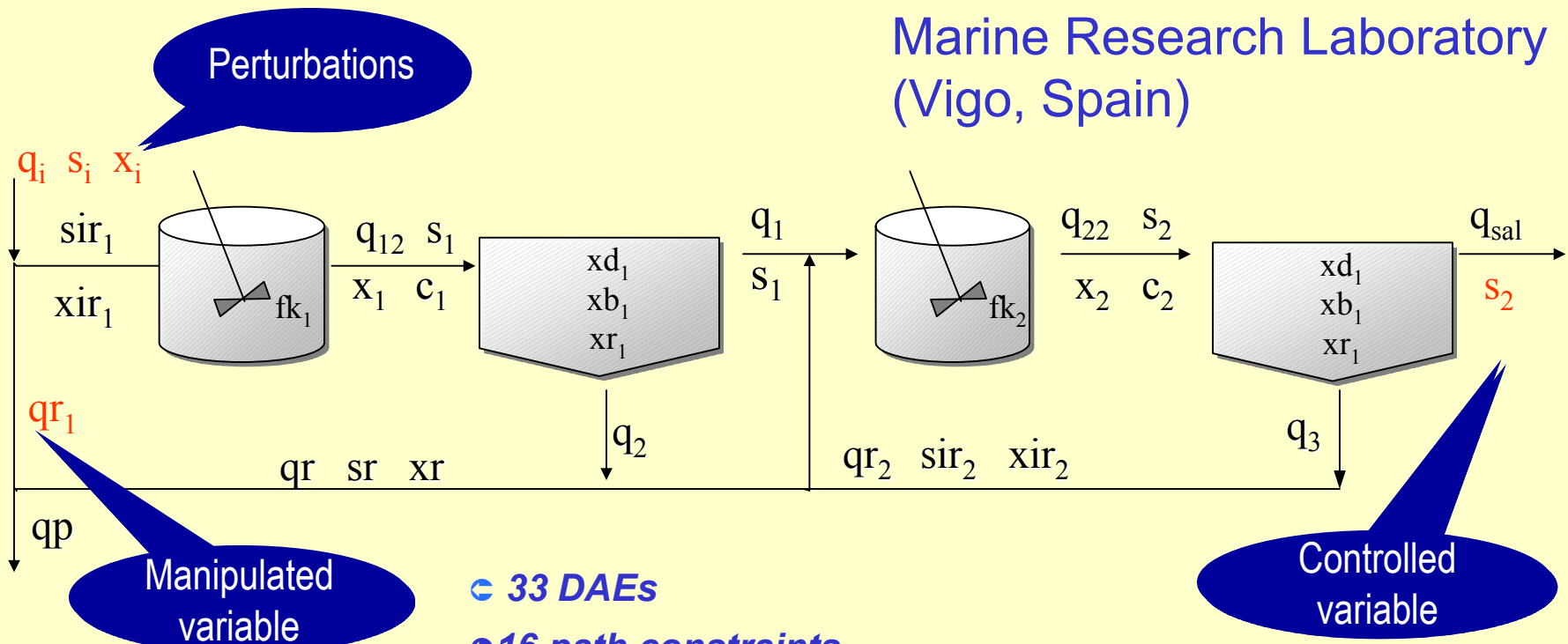
This electric circuit simulates a piezoelectric sonar projector

The optimization problem consists of finding a set of circuit design parameters such that the sonar projector gives a broad efficiency vs. frequency


This model has been solved using MathOptimizer: the results have been applied to the actual design of sonar equipment

Integrated Design and Operation of Waste-Water Plant

Marine Research Laboratory
(Vigo, Spain)



- 33 DAEs
- 16 path constraints
- 30 bounds on states
- 44 states, 8 design variables

 **Objective:** to find the design of the units, the operating conditions and the parameters of the controller which minimize a weighted sum (C) of economic terms (ϕ_{econ}) and a controllability measure (ISE)

This model has been analyzed and solved using an LGO DLL version

Conclusions

- Brief Review:
GAMS, GO, LGO
LGO solver modes
Illustrative applications
- Benefits of Global Optimization:
Global scope search: hence, often improved solutions
Added modeling freedom: more realistic models, new possibilities (and markets)
- Models welcome; additional documentation available
Workshops, consulting services offered

Illustrative References

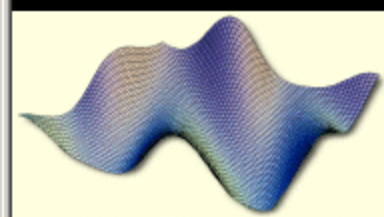
- Over one hundred GO books
- Kluwer AP Non-Convex Opt. And its Appls. series: 60+ volumes (2003)
- *Handbook of GO*, Vols. 1-2
- *J. of Global Optimization*
- *J. of Heuristics*
- Many thousands of research articles and reports
- ...

Illustrative References (cont'd)

Some recent work by JDP & colleagues

- Chapter 15 in *Handbook of Global Optimization*, Vol. 2
- Radiotherapy: Tervo et al; *Annals of Opns. Res.* (2003)
- Model calibration: JDP; *Opt. Methods & Software* (2003)
- Laser design: Isenor, JDP & Cada, *Opt. & Engrg.* (2003)
- MathOptimizer / MathOptimizer Pro applications:
Kampas and JDP; *The Mathematica Journal* (to appear)
- *Global Optimization: Selected Case Studies*
JDP, ed.; Kluwer AP (forthcoming)
- Nonlinear optimization in modeling environments (in ed. Vol.)
- Numerical GO tests: Khompatraporn, JDP & Zabinsky;
J. of Glob. Opt. (to appear)
- GAMS/LGO: JDP (submitted)
- MathOptimizer and ModelMaker: JDP and Purcell (w.i.p.)

...



Nonlinear Systems Modeling and Optimization Software Development and Applications

Links

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- Research Interests and Expertise
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- LGO For Excel
- MathOptimizer For Mathematica
- Downloads
- Selected Professional Links
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- Dalhousie University Homepage
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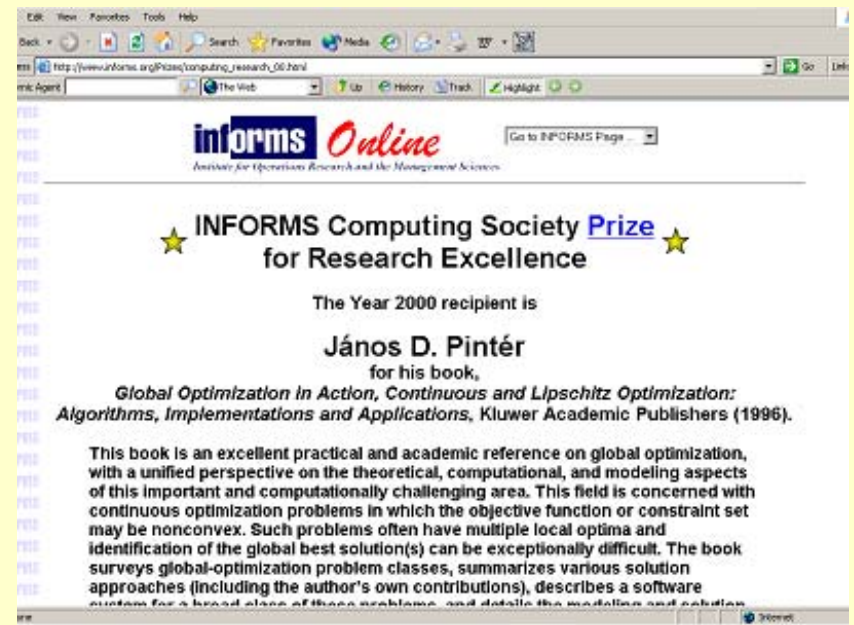
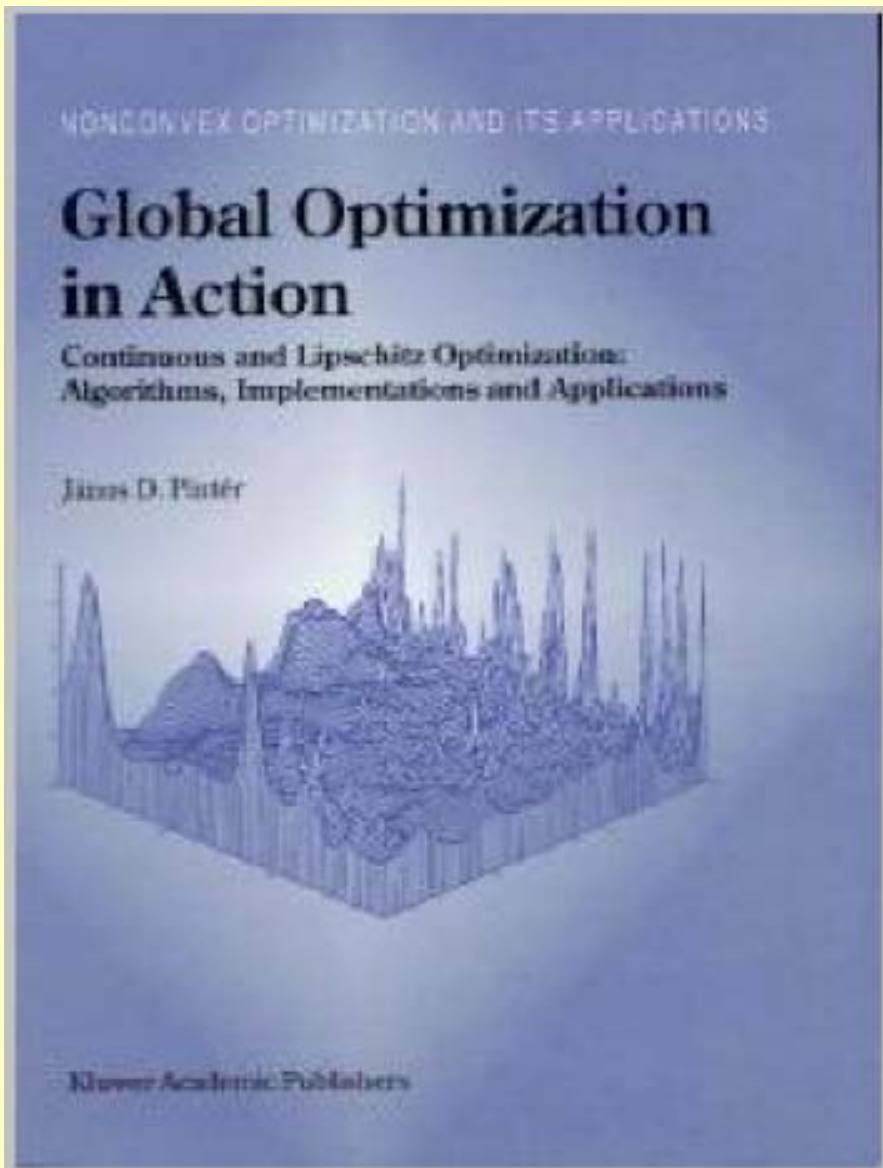
János D. Pintér

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Adjunct Professor, Dalhousie University

129 Glenforest Drive
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E-mail: jdpinter@is.dal.ca
URL address: <http://www.dal.ca/~jdpinter/>



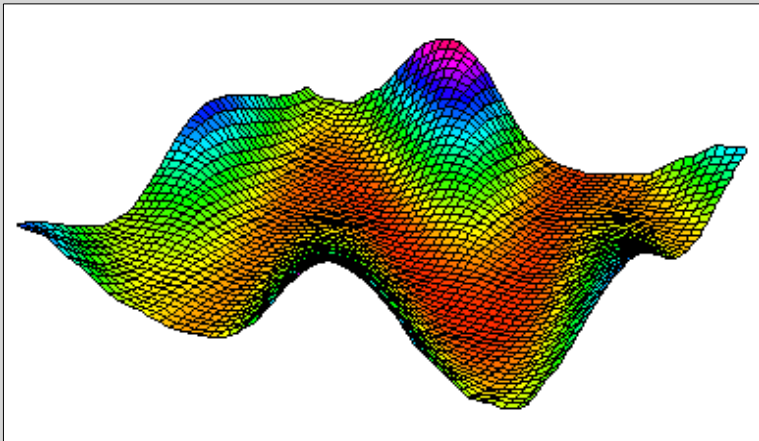
Research monograph: theoretical background of LGO solvers; applications



“Easygoing” e-book; GO review, includes demos of the LGO IDE

Global Optimization

*Scientific and Engineering Applications
with Mathematica® Examples*

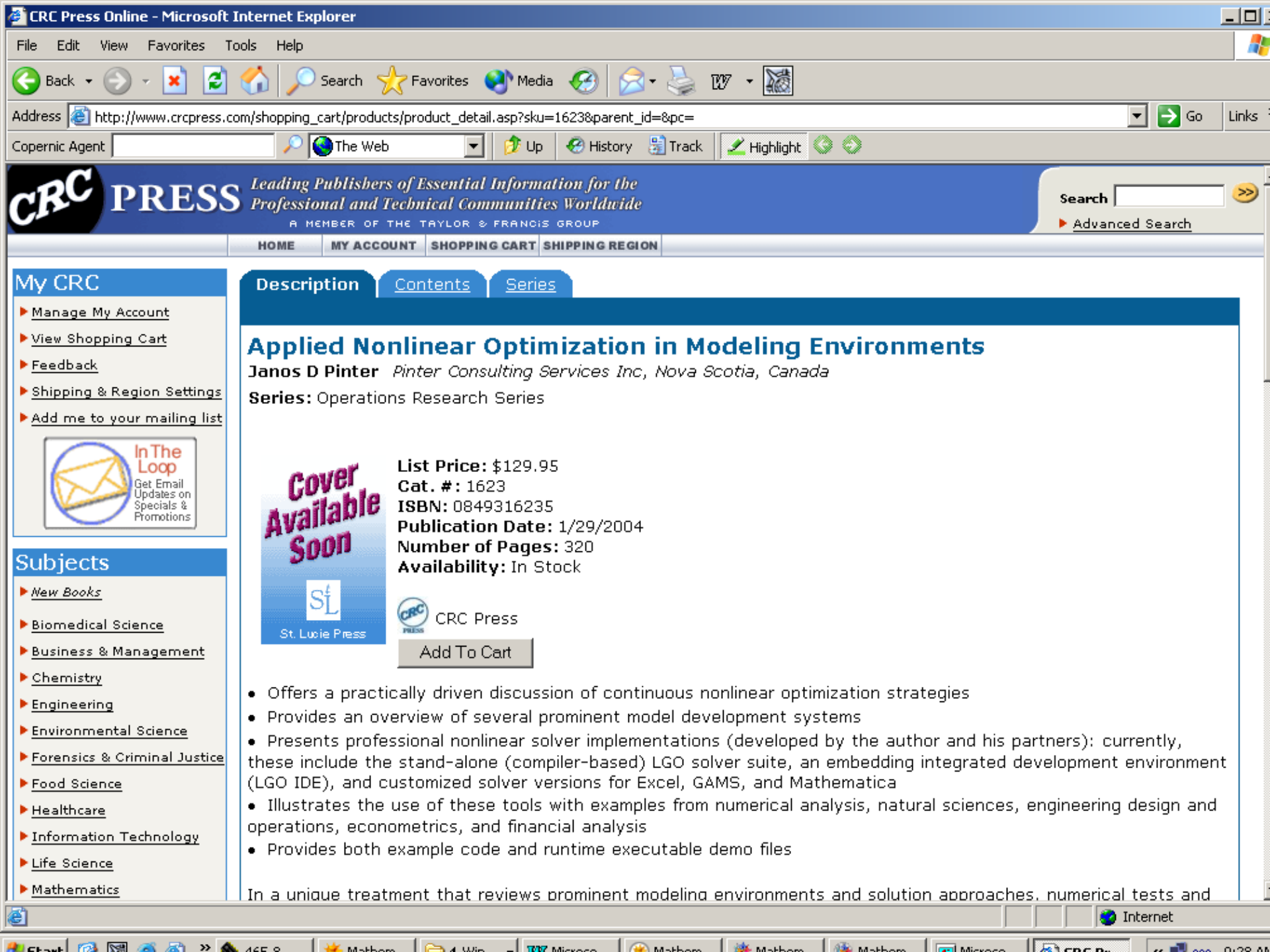


János D. Pintér

ELSEVIER

SCIENCE

Joint work with
Frank J. Kampas
(forthcoming, 2004)



Acknowledgements

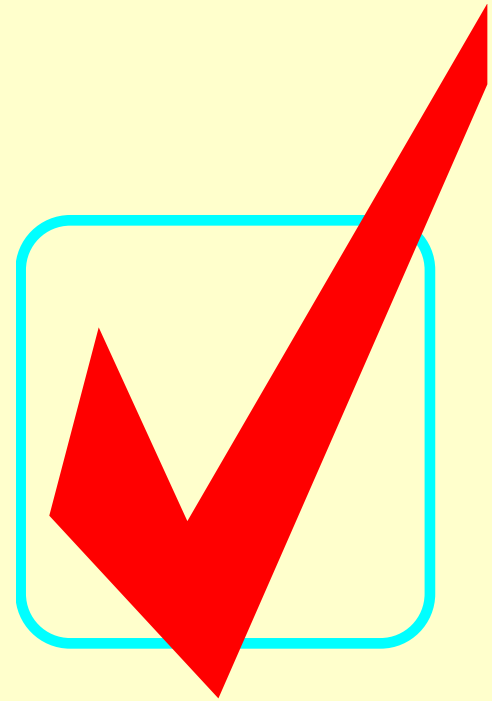
- GAMS team: Alex Meeraus, Steven Dirkse, Michael Bussieck, Armin Pruessner, et al.

for contributions to the development of GAMS/LGO

for information on GAMS

for assistance with testing GAMS/LGO

Thanks for your attention!



Global Optimization with GAMS/LGO

Introduction, Usage, and Applications

János D. Pintér

PCS Inc., Halifax, NS, Canada

**GAMS Global Optimization Workshop
Washington, DC Sept 18, 2003**

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GAMS Dev

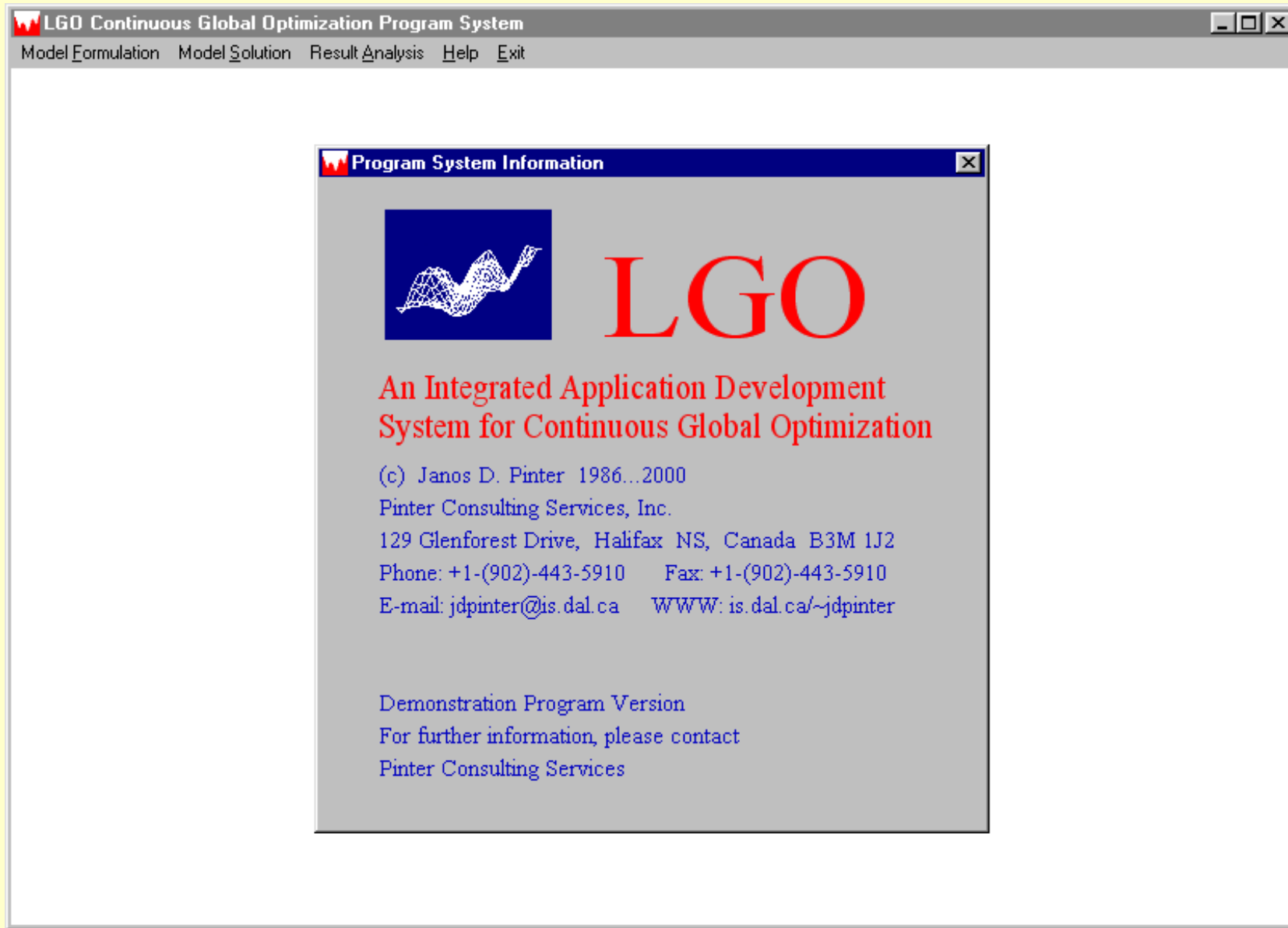
Welcome to **GAMS Software GmbH**, the European branch of [GAMS Development Corporation](#). This web site informs about the **General Algebraic Modeling System (GAMS)** and how we help our clients face modeling challenges and increase their productivity.

GAMS is the leading tool for the development, solution, and management of large scale optimization problems. Our clients use GAMS to implement and maintain complex models in various areas.

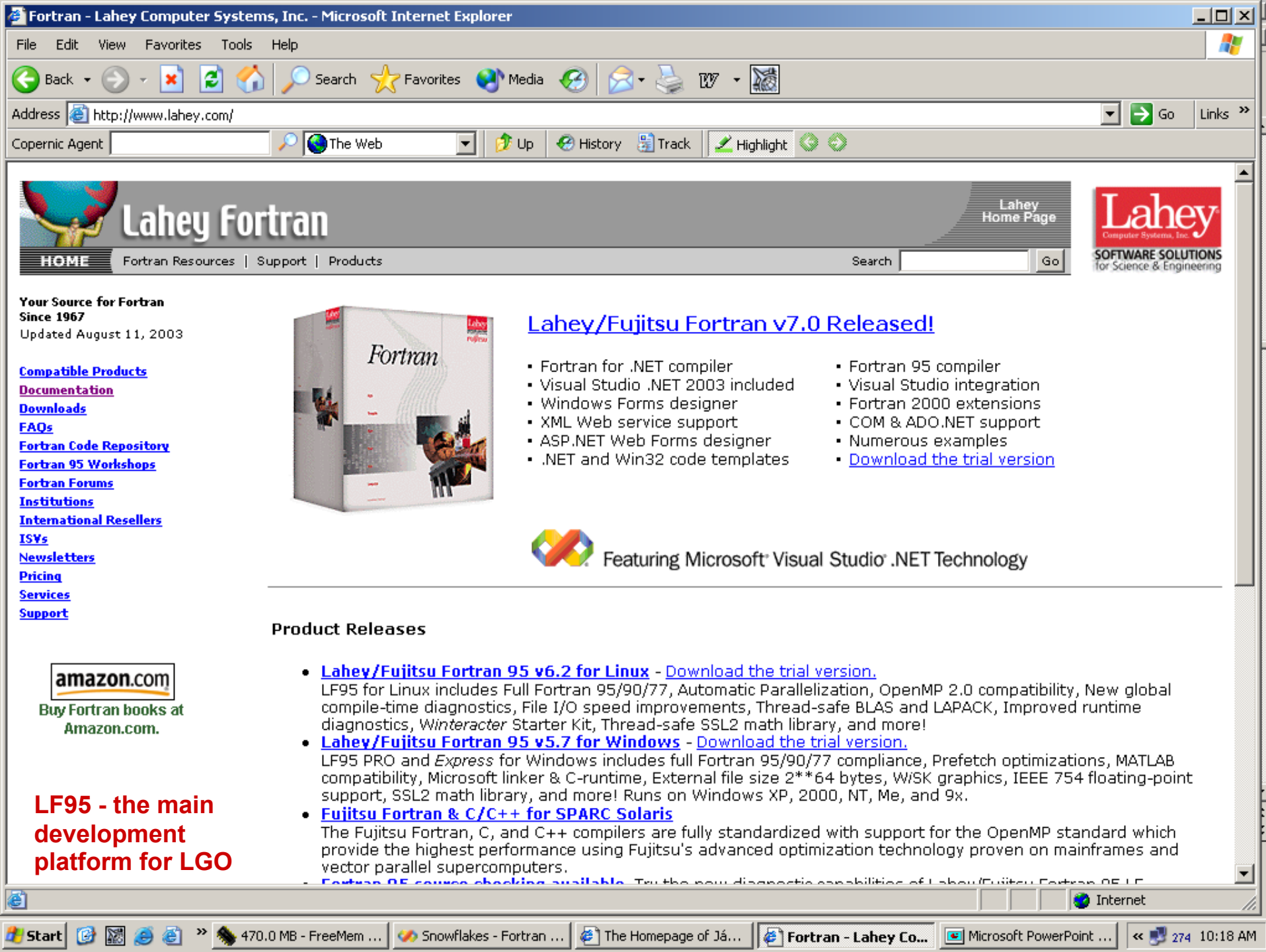
GAMS has been continually evolved to reflect insights gained from over a decade of close collaboration with both the world's leading software creators for optimization techniques and successful clients from academia and industry.

News and Events:

- August 19, 2003: [RANK](#), a new GAMS utility for sorting numeric data, is now on the [Contributed Software](#) page
- July 30, 2003: [OR News](#) advertisement: [LGO Global Optimization Solver](#)
- June 30, 2003: OR/MS Today advertisement: [Conic programming in GAMS](#)
- June 18, 2003: The announcement for the book [Modeling Languages in Mathematical Optimization](#) is now on the [Presentations](#) page.
- June 02, 2003: The presentations from the GOR Workshop on Modeling Languages are added to the [Presentations](#) page.
- May 29, 2003: The [twelfth issue](#) of [Bruce McCarl's Newsletter](#) is now available.
- May 28, 2003: The distribution 21.0 is out, here are the [Release Notes](#).
- January 17, 2003: [Bruce McCarl's GAMS User Guide 2003](#) is online.



LGO modeling and solver system (+ IDE): MS Windows version



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Fujitsu Fortran & C/C++ for SPARC Solaris
Fortran 95 source checking available

amazon.com Buy Fortran books at Amazon.com.

LF95 - the main development platform for LGO



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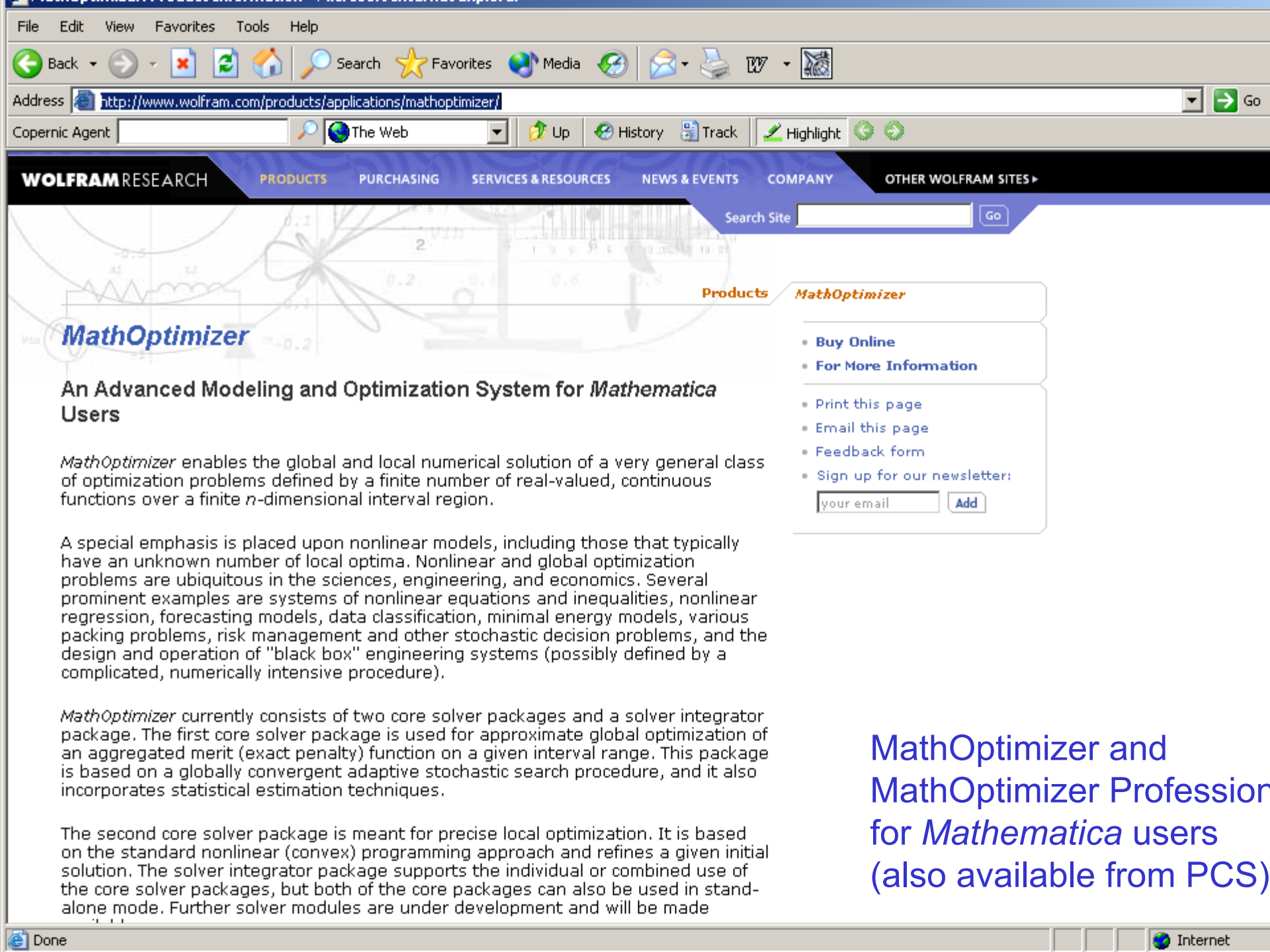
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MathOptimizer

An Advanced Modeling and Optimization System for *Mathematica* Users

MathOptimizer enables the global and local numerical solution of a very general class of optimization problems defined by a finite number of real-valued, continuous functions over a finite n -dimensional interval region.

A special emphasis is placed upon nonlinear models, including those that typically have an unknown number of local optima. Nonlinear and global optimization problems are ubiquitous in the sciences, engineering, and economics. Several prominent examples are systems of nonlinear equations and inequalities, nonlinear regression, forecasting models, data classification, minimal energy models, various packing problems, risk management and other stochastic decision problems, and the design and operation of "black box" engineering systems (possibly defined by a complicated, numerically intensive procedure).

MathOptimizer currently consists of two core solver packages and a solver integrator package. The first core solver package is used for approximate global optimization of an aggregated merit (exact penalty) function on a given interval range. This package is based on a globally convergent adaptive stochastic search procedure, and it also incorporates statistical estimation techniques.

The second core solver package is meant for precise local optimization. It is based on the standard nonlinear (convex) programming approach and refines a given initial solution. The solver integrator package supports the individual or combined use of the core solver packages, but both of the core packages can also be used in stand-alone mode. Further solver modules are under development and will be made

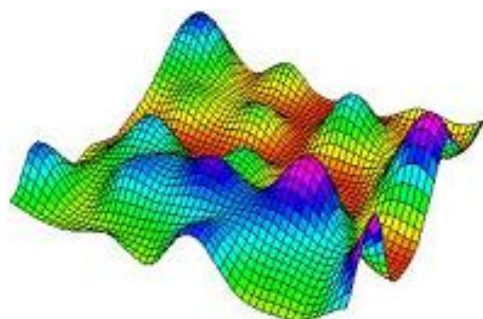
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MathOptimizer Professional

An Advanced Nonlinear Optimization Environment for *Mathematica*
with an External LGO Solver Link



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MathOptimizer Professional enables the global and local solution of a general class of continuous optimization problems. The model form considered is:

$$\min f(x) \text{ subject to } x \in D \subset \mathbb{R}^n \quad D := \{x: x_l \leq x \leq x_u \quad g(x) \leq 0\}$$



abel.gms | abel.lst | abel.log

\$Title Linear Quadratic Control Problem (ABEL,SEQ=64)

\$Ontext

*Linear Quadratic Riccati Equations are solved as a General
Nonlinear Programming Problem instead of the usual Matrix
Recursion.*

*Kendrick, D, Caution and Probing in a Macroeconomic Model. Journal of
Economic Dynamics and Control 4, 2 (1982).*

\$Offtext

Sets n states / consumpt, invest /
 m controls / gov-expend, money /
 k horizon / 1964-i, 1964-ii, 1964-iii, 1964-iv
 1965-i, 1965-ii, 1965-iii, 1965-iv /
ku(k) control horizon
ki(k) initial period
kt(k) terminal period ;

Alias (n,np), (m,mp) ;

ku(k) = **yes**\$(ord(k) lt card(k));

ki(k) = **yes**\$(ord(k) eq 1);

kt(k) = **not** ku(k);

Display k, ki, kt, ku;

Table a(n,np) state vector matrix

	consumpt	invest
consumpt	.914	-.016
invest	.097	.424

About

GAMS

Integrated Development Environment

GAMS IDE	2.0.23.10
Module	GAMS Rev 135
Lic date	Jun 2, 2003
Build	VIS 21.1 135

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3: 50

Insert

Automatic solver performance analysis and visualization

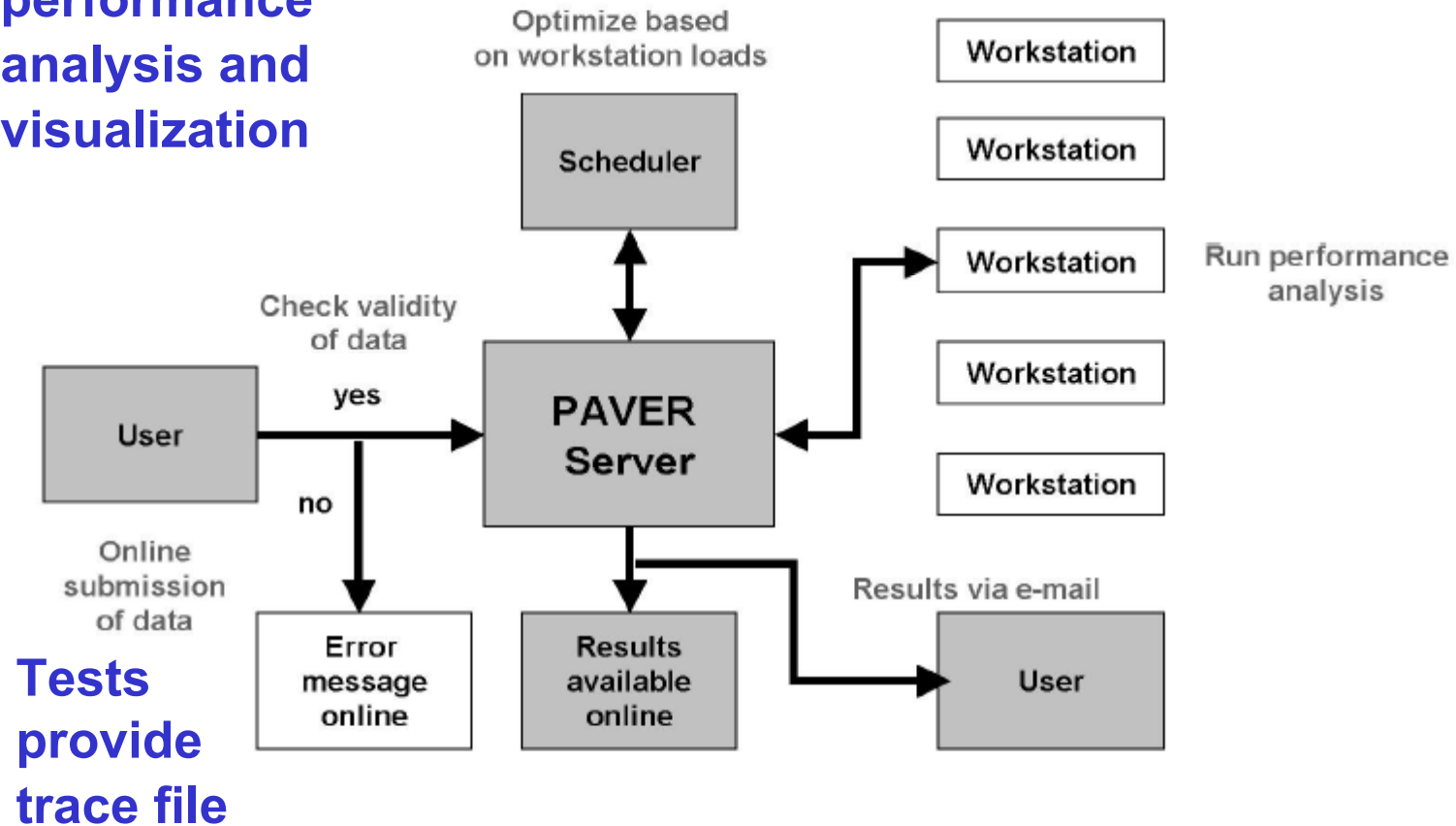


Figure 1: PAVER: Process Overview

GAMS Performance World: PAVER Benchmarking Tool (Mittelmann and Pruessner, 2003)

Metrics based on solver

- **robustness**
- **efficiency**
- **quality of solution (objective value, feasibility level)**

Elements of subjectivity

- **test models used**
- **solver parameterizations and options**

“Remedies”

- **standard test collections, including randomized model-classes**
- **compare solver performance trends**

References: DTOS, GAMS, NEOS; JDP paper

An Illustrative Test Model (N.L. Trefethen)

A Hundred-dollar, Hundred-digit Challenge

Each October, a few new graduate students arrive in Oxford to begin research for a doctorate in numerical analysis. In their first term, working in pairs, they take an informal course called the “Problem Solving Squad.” Each week for six weeks, I give them a problem, stated in a sentence or two, whose answer is a single real number. Their mission is to compute that number to as many digits of precision as they can.

Ten of these problems appear below. I would like to offer them as a challenge to the SIAM community. Can you solve them?

I will give \$100 to the individual or team that delivers to me the most accurate set of numerical answers to these problems before May 20, 2002. With your solutions, send in a few sentences or programs or plots so I can tell how you got them. Scoring will be simple: You get a point for each correct digit, up to ten for each problem, so the maximum score is 100 points.

Fine print? You are free to get ideas and advice from friends and literature far and wide, but any team that enters the contest should have no more than half a dozen core members. Contestants must assure me that they have received no help from students at Oxford or anyone else who has already seen these problems.

Hint: They’re hard! If anyone gets 50 digits in total, I will be impressed. The ten magic numbers will be published in the July/August issue of *SIAM News*, together with the names of winners and strong runners-up.—Nick Trefethen, Oxford University.

The Hundred-dollar, Hundred-digit Challenge Problems

1. What is $\lim_{\epsilon \rightarrow 0} \int_{\epsilon}^1 x^{-1} \cos(x^{-1} \log x) dx$?
2. A photon moving at speed 1 in the x - y plane starts at $t = 0$ at $(x,y) = (0.5, 0.1)$ heading due east. Around every integer lattice point (i,j) in the plane, a circular mirror of radius $1/3$ has been erected. How far from the origin is the photon at $t = 10$?
3. The infinite matrix A with entries $a_{11} = 1$, $a_{12} = 1/2$, $a_{21} = 1/3$, $a_{13} = 1/4$, $a_{22} = 1/5$, $a_{31} = 1/6$, etc., is a bounded operator on ℓ^2 . What is $\|A\|$?
4. What is the global minimum of the function
$$\exp(\sin(50x)) + \sin(60e^y) + \sin(70 \sin(x)) + \sin(\sin(80y)) - \sin(10(x+y)) + \frac{1}{4}(x^2 + y^2) ?$$



Trefethen's Problem 4 (continued)

\$title Trefethen's HDHD 2002 Challenge, Problem 4

\$ontext

Trefethen's GO model: a 2-var, 0-cons problem, with many local optima

Source: SIAM News, Jan - Feb 2002 issue

Solution: $x^* \sim (-0.0244030796, 0.2106124272)$; $f(x^*) \sim -3.306868647$

\$offtext

variables f, x1, x2;

equations objf;

objf.. f=e= 0.25*(x1*x1+x2*x2) + exp(sin(50.*x1)) + sin(60.*exp(x2))
+ sin(70.*sin(x1)) + sin(sin(80.*x2)) - sin(10.*(x1+x2));

model trefethen4 / all / ;

x1.lo = -3;

x1.l = -2;

x1.up = 3;

x2.lo = -3;

x2.l = 2;

x2.up = 3;

solve trefethen4 **using** nlp **minimizing** f; // nlp=lgo option is set

Trefethen's Problem 4 (continued)

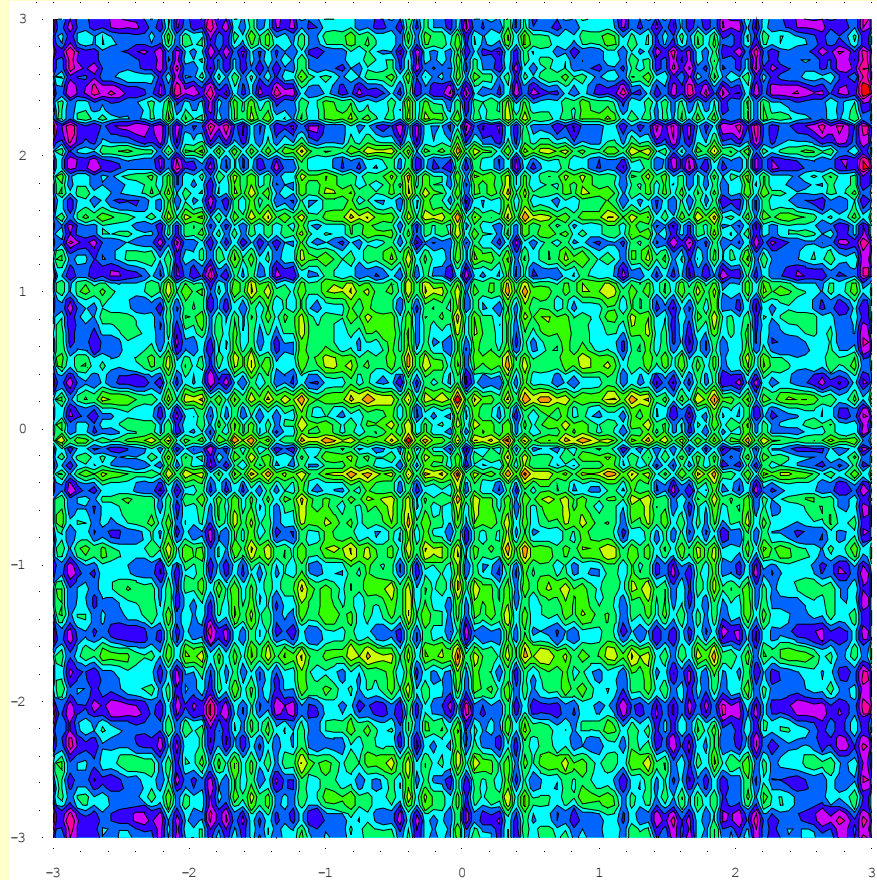
- Solution found by LGO:

$$x^* \sim (-0.0244030796, 0.2106124272);$$

$$f(x^*) \sim -3.306868647$$

- This solution is identical to at least 10 decimal digits to the “true” solution reported by SIAM News (July-August 2002)
- LGO runtime \ll 1 second (P4 desktop PC)

Contour Plot of Objective Function



**Thousands of local optima;
a single global solution...**

$$\frac{1}{4} \left(1 + y^2 - \tilde{a}^{n+5} - \sin^2 y - \sin^2 \tilde{a} y - \sin^2 \delta y \right)$$