Economic Equilibrium Analysis with GAMS/MPSGE

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GAMS/MPSGE: A Mathematical Programming System for General Equilibrium Analysis

- specifically designed for applied general equilibrium analysis, including both models represented as systems of equations and those which involved *complementarity* between inequalities and bounded variables.

- system is particularly useful for large, complex models based on benchmark equilibrium datasets

- GAMS/MPSGE provides a highly structured framework for inexperienced analysts, yet GAMS/MPSGE models can be customized through the use of *auxiliary variables*.

Economic Models in the Policy Arena

Economic models produce results which can play a central role in political dialogue.

Although economic models are based on formal mathematics, it is important to recognize that the origins of economic analysis are in social philosophy rather than physical science.

Everyone participates in economic transactions, so non-specialist audiences can be influenced by populist appeals to "common sense".

Economic Equilibrium Ideas

Agents in an economic model include consumers, producers and governments, collectively representing all participants in market transactions in a given economy.

A central concept in economic equilibrium models is that agents *optimize subject to constraints*.

Systematic errors are logically inconsistent with individually rational choice.

A typical starting point for *dynamic* economic models is that consumers are fully informed and hold *consistent expectations of the future*. This approach differentiates economic models from models of physical systems.

GAMS/MPSGE Equilibrium Framework

Variables

$p \in R^N$ Prices for all goods and factors (possibly indexed by commodity, sector, region, household, time period etc.)

$y \in R^M$ Production activity levels (also indexed)

$M \in R^H$ Income levels for each consumer in the model

Given Functions and Data

$\tilde{p}_j$ is a vector of producer prices, net (gross) of applicable taxes for outputs (inputs, resp.)

$r_j(\tilde{p}_j)$ is the unit revenue function for sector j

$c_j(\tilde{p}_j)$ is the unit cost function for sector j

$d_{ih}(p, M_h)$ is the demand function for household h, derived from budget-constrained utility maximization. By definition, these functions satisfy Walras' law:

$$\sum_i p_i d_{ih}(p, M_h) = M_h.$$

ω_{ih}, θ_{jh} are matrices of commodity endowments and tax revenue allocations.

Dual Equilibrium: Zero Profit

$$c_j(\tilde{p}_j) \geq r_j(\tilde{p}_j) \quad \perp \quad y_j \geq 0$$

Primal Equilibrium: Market Clearance

$$\sum_j \left(\frac{\partial r_j}{\partial \tilde{p}_{ij}} - \frac{\partial c_j}{\partial \tilde{p}_{ij}} \right) y_j + \sum_h \omega_{ih} \geq \sum_h d_{ih}(p, M_h) \quad \perp \quad p_i \geq 0$$

Income Balance

$$M_h = \sum_i p_i \omega_{ih} + \sum_j \theta_{jh} \; y_j \left[\sum_i (\tilde{p}_{ij} - p_{ij}) \left(\frac{\partial r_j}{\partial \tilde{p}_{ij}} - \frac{\partial c_j}{\partial \tilde{p}_{ij}} \right) \right]$$

Calibrated Functions

Data for specification of functions is typically based on the first two terms in a Taylor approximation:

$\bar{a}_{ij}^{U} = \frac{\partial c_j}{\partial \tilde{p}_i}\Big|_{\bar{p}}$ benchmark producer inputs (the "use matrix")

$\bar{a}_{ij}^{M} = \frac{\partial r_j}{\partial \tilde{p}_i}\Big|_{\bar{p}}$ benchmark producer output (the "make matrix"),

$\bar{d}_{ih} = d_{ih}(\bar{p}, \bar{M}_h)$ benchmark consumer demands at reference prices

ω_{ih}, θ_{jh} benchmark initial endowments and tax shares.

Mission for Public Income – Colombian Fedesarrollo

- 1997 input-output table supplemented with additional data from 1999, 2000, 2001.

- 56 production sectors

- 6 categories of labor:

ufs	Urban formal salaried work
ufn	Urban formal non-salaried work
utc	Urban traditional contract work
umc	Urban modern contract work (consulting)
rsw	Rural salaried work (organized farming work)
rnw	Rural non-salaried work (farming)

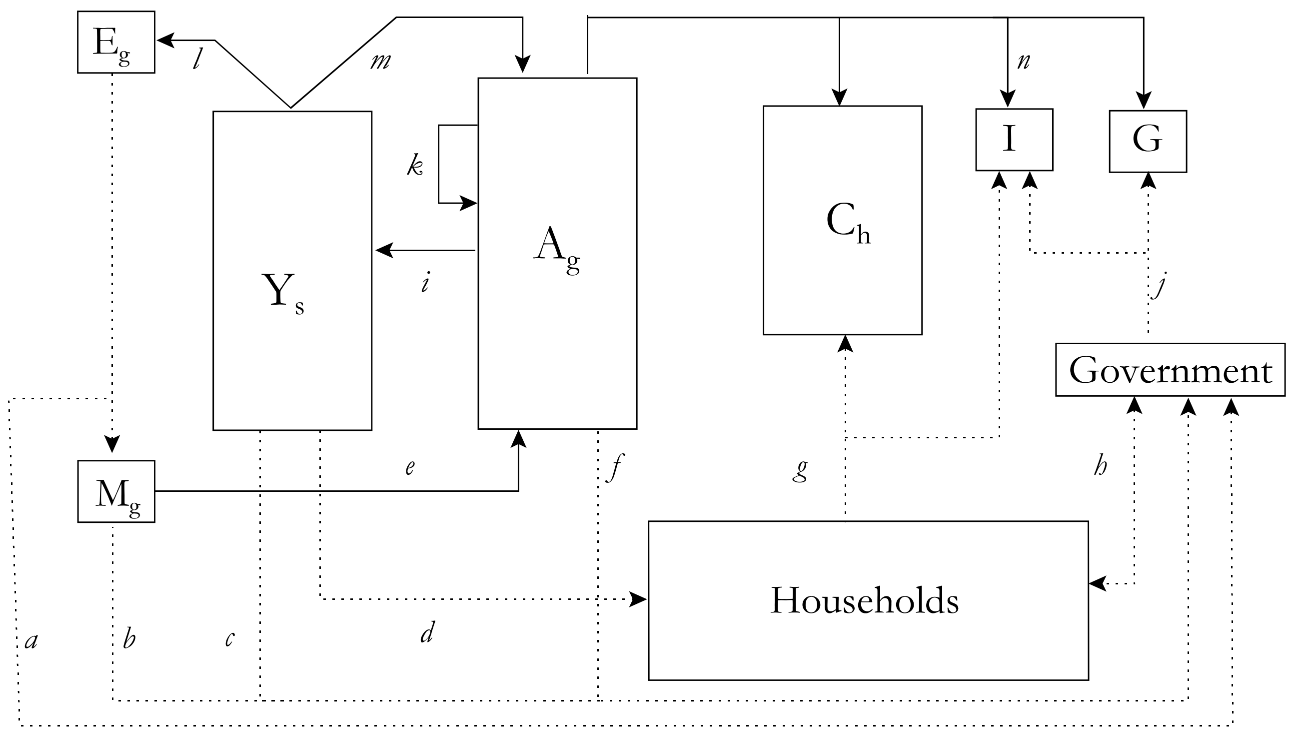
- 10 representative households

	n	n%	c	c/n	$/day
h1	2980	33	2980	907	2
h2	1980	22	2701	2245	4
h3	1410	15	4444	4029	7
h4	800	9	5661	8522	16
h5	520	6	6785	15773	29
h6	340	4	8145	29615	54
h7	210	2	9931	54643	100
h8	180	2	11615	82257	150
h9	100	1	14936	226664	414
h10	90	1	22000	246362	901

Tax Revenue

	Revenue	%	Base	Tax Rates (%) Collected	Posted
VAT	5.6	33	216	2.6	16.5
Corporate Income	4.3	26	24	17.8	34
Excise	2.1	12	216	1.0	
Tariffs	1.4	8	24	5.8	5-15
Payroll	1.0	6	51	1.9	
Indirect Output	0.9	5	200	0.5	
State/Local	0.8	5	200	0.4	
Individual Income	0.7	4	51	1.4	17-34
Subsidies	-0.034	-0.2	17	-0.2	
Total	16.8	100			
Social Security	6.6		104	6.3	
Central Govt. Income:	20.1				
Local Govt. Income:	12.5				
Soc. Security Expend:	9.5				

Equilibrium Framework



Key: Physical flow of goods:: ————————

Financial flows of factor earnings,
tax payments and transfers: ....

Model Formulation

Sets:

s, g	Sectoral and commodity identifiers
h	Households
ℓ	Labor types

Activity Levels:

Y_s	Production activity level
A_g	Aggregate supply to domestic and export markets
C_h	Aggregate consumption demand by household h
K	Capital stock
I	Investment
G	Public demand

Prices:

e	Real exchange rate
r^K	Rental price of capital
w_ℓ	Wage rate for labor type ℓ
p_g^Y	Supply price of good g (gross of indirect taxes)
p_g^A	Market price of good g (gross of excise and VAT)
p_h^C	Consumer price index
p^G	Public provision price index
p^I	Investment cost index

Other variables:

κ	Capital stock
G_B	Government external balance

Leontief Demand and Supply Coefficients

a^M_{gs}	Output of good g per unit activity of sector s, the *make matrix*.
a^U_{gs}	Input of good g per unit activity of sector s, the *use matrix*.
a^G_g	Demand for good g per unit of government activity
a^I_g	Demand for good g per unit of aggregate investment
$a^\mu_{gg'}$	Trade and transport margin net demand per unit aggregate supply of good g'

Cost and Revenue Functions

$c_s^Y(\tilde{w}_s, r^K)$	Unit cost of value-added in sector s (Y_s)
$R_g^A(\tilde{p}_r^A, \tilde{p}_g^X)$	Unit revenue per unit of aggregate supply (A_g)
$c_g^A(p_g^Y, \tilde{p}_g^M)$	Unit cost of aggregate supply (A_g)
$c_h^C(\tilde{w}_h, p^A)$	Unit cost of final demand for leisure and goods (C_h)

Arbitrage (Zero-Profit) Conditions

- Domestic production (Y_s):

$$\sum_g \tilde{p}^Y_{gs} a^M_{gs} = \sum_g \tilde{p}^A_{gs} a^U_{gs} + c^Y_s(\tilde{w}_{\ell s}, r^K)$$

- Aggregate Supply (A_g):

$$R^A_g(\tilde{p}^A_g, \tilde{p}^X_g) = c^A_g(\tilde{p}^Y_g, \tilde{p}^M_g) + \sum_{g'} p^A_{g'} a^\mu_{gg'}$$

- Consumption Cost (C_h):

$$p^C_h = c^C_h(p^A, \tilde{w}_{\ell h})$$

- Cost of Investment (I):

$$p^I = \sum_g a^I_g p^A_g$$

- Cost of Public Provision (G):

$$p^G = \sum_g a_g^G p_g^A$$

Market Clearance Conditions

- Domestic Output

$$\sum_s Y_s a^M_{gs} = A_g \frac{\partial c^A_g}{\partial p^Y_g}$$

- Domestic Demand

$$A_g = \sum_{g'} a^{\mu}_{gg'} A_{g'} + \sum_s a^U_{gs} Y_s + \sum_h \frac{\partial c^C_h}{\partial p^A_g} + G a^G_g + I a^G_g$$

- Labor Markets

$$\sum_h \bar{L}_{\ell h} = \sum_h C_h \frac{\partial c^C_h}{\partial \tilde{w}_{\ell h}} + \sum_s Y_s \frac{\partial c^Y_s}{\partial \tilde{w}_{\ell s}}$$

- Capital Market

$$\kappa \sum_h \bar{K}_h = \sum_s Y_s \frac{\partial c^Y_s}{\partial r^K}$$

- Household Demand

$$C_h = \frac{M_h}{c_h^C}$$

- Investment-Savings

$$I = \sum_h S_h + S_G$$

- Current account

$$B_G + \sum_h \bar{B}_h + \sum_g A_g \frac{\partial R_g^A}{\partial \tilde{p}_g^X} = \sum_g A_g \frac{\partial c_g^A}{\partial \tilde{p}_g^M}$$

Income Balance

- Household Income

$$M_h = \sum_\ell \bar{L}_{\ell h}\tilde{w}_{\ell h} + \kappa\tilde{r}^K\bar{K}_h + e\bar{B}_h - \bar{T}_h - p^I\bar{S}_h$$

- Government

$$\begin{aligned} M_G \quad = \quad & \sum_g A_g \frac{\partial c_g^A}{\partial \tilde{p}_g^M}(\tilde{p}_g^M - p_g^M) + \\ & + \sum_g A_g \frac{\partial R_g^A}{\partial \tilde{p}_g^X}(\tilde{p}_g^X - p_g^X) \\ & + \sum_s Y_s \frac{\partial c_s^Y}{\partial \tilde{w}_{\ell s}}(\tilde{w}_{\ell s} - w_{\ell s}) \\ & + \sum_{\ell h}\left(L_{\ell h} - \frac{\partial c_h^C}{\partial \tilde{w}_{\ell h}}\right)(w_{\ell s} - \tilde{w}_{\ell h}) \\ & + \ldots \end{aligned}$$

Auxiliary Constraints

- Steady-state model (κ):

$$q = \frac{\tilde{r}^K}{p^I} = 1$$

- Budget Balance (B_G)

$$G = 1$$

Data Management in GAMS

```
*       Units of the 1997 SAM are Millions of 1997 Pesos
*       (current price). After scaling, we get Billions of Pesos

set     r       SAM Rows        /11*210/;

alias (r,c);

sam(r,c) = sam(r,c)/1000;

parameter       samchk  Check of SAM consistency;
samchk(r) = round(sum(c, sam(r,c)-sam(c,r)), 5);
display samchk;

set     Goods(r) /11*68/,                Sectors(r) /70*127/,
        Labor(r) /128*133/,              Capital(r) /134/,
        Households(r) /186*195/,         Government(r) /135*158,159*185,197*199/,
        Firms(r) /196,200*201,202*206/, Row(r) /69,207/,
        Investment(r) /208*210/;
```

Aggregated Social Accounts

	G	S	L	C	H	G	F	R	I
Goods		87.1			78.8	24.2	0.2	16.8	25.5
Sectors	199.0								
Labor		74.3							
Capital		35.5							
Households			74.3	7.6	0.1	6.2	10.7	4.8	
Government	9.8	2.1		3.5	9.6	27.2	8.7		
Firms				24.5	9.5	2.1	15.3	0.8	
row	23.8				0.2	1.3	2.8	0.2	
Investment					5.4	-0.2	14.5	5.8	25.5

```
$model:sam97

$sectors:
        y(s)            !       Production
        x(g)            !       Domestic supply
        a(s)            !       Aggregate supply
        gov             !       Public expenditure
        inv             !       Investment
        con(h)          !       Household consumption ...

$commodities:
        py(g)           !       Output price
        pa(g)           !       Aggregate price
        pc(h)           !       Household consumption
        w(l)            !       Wage rates
        rk              !       Return to capital (gross of tax)
        pfx             !       Foreign exchange ...

$consumers:
        ra(h)           !       Households
        govt            !       Government
```

Production Functions

```
$prod:y(s)      s:0     va:1  t:0
        o:py(g)         q:make(s,g)     a:govt  t:(tcm(s)+tif(s)-sub(s)+ty(s))
        i:pa(g)         q:use(g,s)      a:govt  t:-vat(s)
        i:w(l)          q:ld0(l,s)      p:pl0(l,s)  a:govt  t:pyrl(l,s) va:
        i:rk            q:kd0(s)                                         va:

$prod:x(g)  t:4
        o:pd(g)         q:d0(g) p:1
        o:pfx           q:x0(g) p:px0(g)        a:govt t:crt(g)
        i:py(g)         q:y0(g) p:py0(g)        a:govt t:tp(g)

$prod:a(g)  s:0  dm:4
        o:pa(g)         q:a0(g) p:pa0(g) a:govt t:txs(g) t:vat(g)
        i:pmg(gg)       q:margin(gg,g)
        i:pd(g)         q:d0(g)          dm:
        i:pfx           q:m0(g) p:pm0(g) dm:    a:govt t:tm(g)
```

Dual Equilibrium: Zero Profit

$$c_j(\tilde{p}_j) \geq r_j(\tilde{p}_j) \quad \perp \quad y_j \geq 0$$

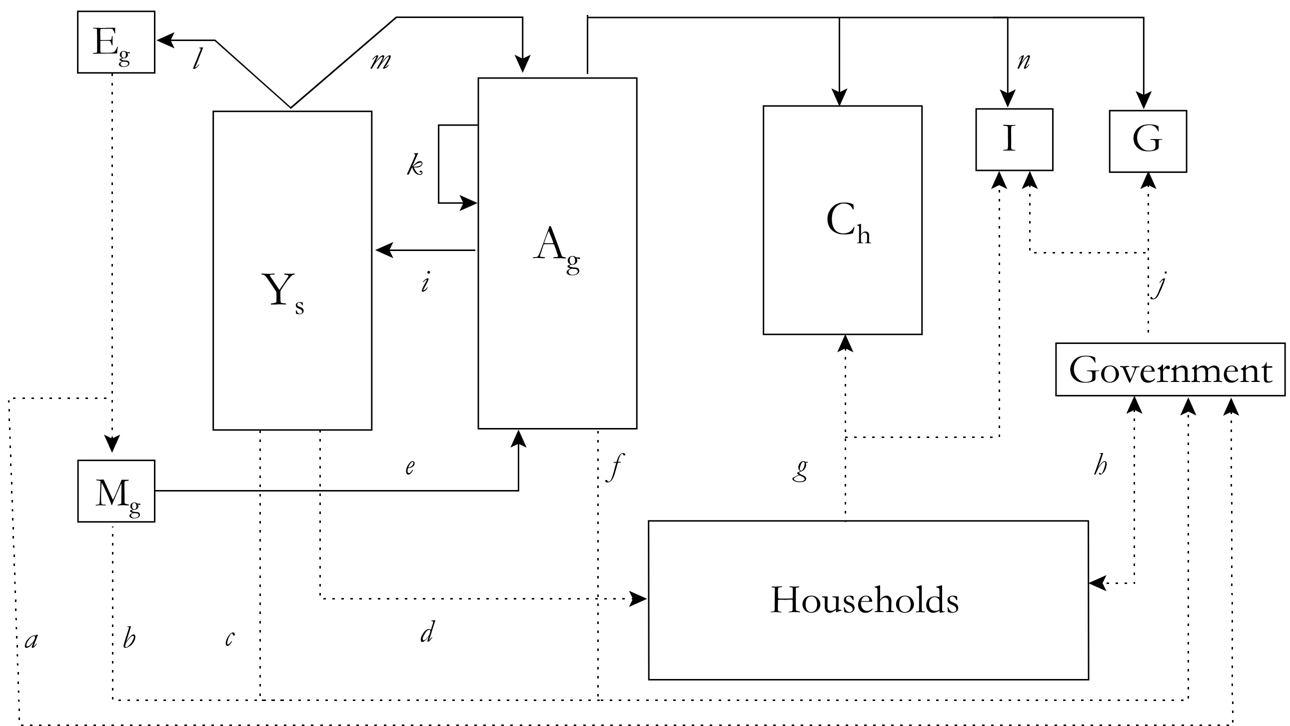
Primal Equilibrium: Market Clearance

$$\sum_j \left(\frac{\partial r_j}{\partial \tilde{p}_{ij}} - \frac{\partial c_j}{\partial \tilde{p}_{ij}} \right) y_j + \sum_h \omega_{ih} \geq \sum_h d_{ih}(p, M_h) \quad \perp \quad p_i \geq 0$$

Income Balance

$$M_h = \sum_i p_i \omega_{ih} + \sum_j \theta_{jh} \ y_j \left[\sum_i (\tilde{p}_{ij} - p_{ij}) \left(\frac{\partial r_j}{\partial \tilde{p}_{ij}} - \frac{\partial c_j}{\partial \tilde{p}_{ij}} \right) \right]$$

Equilibrium Framework



Key:

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Corresponding GAMS Algebra

```
*   Demand for domestic input into Armington production:

DEF_A_Y(d,i,re)$(vd(d,i,re))..

A_A_Y(d,i,re) =E=
        vd(d,i,re) *
        {[ ((1- thetaa_m(d,i,re)) * py(i,re)**(1-esubdm(i,re))
          +      thetaa_m(d,i,re) * PM(i,re)**(1-esubdm(i,re))
          )**(1/(1-esubdm(i,re)))
         ]/ (py(i,re))
        }**esubdm(i,re);
```

Constraints Associated with Auxiliary Variables

```
$constraint:tau
        gov =e= 1;

$constraint:kf
        pinv =e= pinv0 * rkf;
```

Proportional Tax Increases: Steady-State Model

Value-Added Taxes

	ACF(σ)			Revenue		
Rate	0	1	∞	Pesos(T)	%GDP	%Yield
$\times$1.2	1.29	1.36	1.47	1.1	0.6	95
$\times$1.6	1.33	1.40	1.51	3.2	1.9	95
$\times$2.0	1.37	1.44	1.55	5.3	3.2	94

Import Tariffs

	ACF(σ)			Revenue		
Rate	0	1	∞	Pesos(T)	%GDP	%Yield
$\times$1.2	2.03	1.98	1.92	0.2	0.1	68
$\times$1.6	2.09	2.03	1.97	0.5	0.3	64
$\times$2.0	2.14	2.08	2.02	0.8	0.5	61

Tax Revenue Yield in the Refined Petroleum Market