Solving MPEC's via Automatic Reformulation as NLP's

S. P. Dirkse, M.C. Ferris, and A. Meeraus

GAMS Development Corporation & UW-Madison

ISMP-2003 Copenhagen
August 17-22, 2003

Background & Motivation

- GAMS/Convert – scalar model tools
  - GAMS -> other formats/languages
  - Why not MPEC -> NLP?
- Function libraries
- Solid suite of NLP solvers
- Existing reformulation work
  - Tin-Loi, Que
  - Scholtes, Billups, C-M, F-B, Anitescu
  - Hamish Mac-MPEC

Introduction

- Background and motivation
- MPEC reformulation
  - Strategies & variations
  - Function libraries
- NLPEC & testing environment
- Computational experiments

MPEC Definition

$$\begin{array}{ll} \min & f(x,y) \\ \text{s.t.} & g(x,y) \ge 0, \quad x \in C \\ & y \ \text{ solves } \ \text{MCP } (h(x,\cdot\,), B) \end{array}$$

- The NLP constraints are left alone
- Force a solution y of MCP(h,B) via reformulation

MCP Definition

Let $B = \{y = \{y_L, y_U, y_B\} \mid$

$$a_L \le y_L, y_U \le b_U, a_B \le y_B \le b_B\}.$$

y solves MCP $(h(x,\cdot\,), B)$

$$\Leftrightarrow$$

$$y \in B$$

$$w_L = h_L(x,y),\ \text{-}v_U = h_U(x,y),\ w_B - v_B = h_B(x,y),$$

$$\left\langle w_L, y_L - a_L \right\rangle = 0,\ \left\langle v_U, b_U - y_U \right\rangle = 0,\ w_L \ge 0,\ v_U \ge 0$$

$$\left\langle w_B, y_B - a_B \right\rangle = 0,\ \left\langle v_B, b_B - y_B \right\rangle = 0,\ w_B \ge 0,\ v_B \ge 0$$

Alternative MCP

$$y \quad \text{solves} \quad \text{MCP}\ (h(x,\cdot\ ), B)$$

$$\leftrightarrow$$

$$y \in B$$

$$\langle h_L(x,y), y_L - a_L \rangle \le m, \quad h_L(x,y) \ge 0$$

$$\langle -h_U(x,y), b_U - y_U \rangle \le m, \ -h_U(x,y) \ge 0$$

$$w_B - v_B = h_B(x,y), \ w_B \ge 0, \ v_B \ge 0$$

$$\langle w_B, y_B - a_B \rangle \le m, \ \langle v_B, b_B - y_B \rangle \le m$$

Inner Product Reformulation

- Assume *y* in *B*
- Summation vs. componentwise
  - – Sum {i=1..M, <,>} = 0
  - – <,> = 0,   i=1..M
- Equality vs. inequality
- Parameter on RHS
- Double-bounded pairs are a special case

Slack Variables

- We can use slack variables for *h(x,y)*
  - – Bounded or not
  - – One or two slacks per h_i
- Computationally, slacks are significant

Scholtes Reformulation

$$h_{\mathrm{B}} \quad \perp \quad a_B \leq y_B \leq b_B$$

$$\leftrightarrow$$

$$y \in B$$

$$\langle h_i(x,y), y_i - a_i \rangle \leq m, \quad \forall i \in B$$

$$\langle -h_i(x,y), b_i - y_i \rangle \leq m, \quad \forall i \in B$$

Smoothed NCP-functions

- An NCP-function has the following property:

$$j\,(r,s)=0 \;\Leftrightarrow\; r\geq 0, s\geq 0, rs=0$$

- Various smoothed NCP-functions proposed

$$j_{FB}(r,s)=\sqrt{r^2+s^2+2m}-r-s$$

$$j_{CM}(r,s)=r-m\ln(1+\exp(\frac{r-s}{m}))$$

Billups Reformulation

- NCP-functions assume the single-bounded case
- Can be adapted to handle the double-bounded case

$$h_{\mathrm{B}} \quad \perp \quad a_B \le y_B \le b_B$$

$$\leftrightarrow$$

$$j\,(y_i - a_i\,, j\,(-h_{\mathrm{i}}(x, y), b_i - y_i\,)) = 0, \quad \forall i \in B$$

Special Functions

Consider $j_{CM}(r,s) = r - m\ln(1+\exp(\frac{r-s}{m}))$

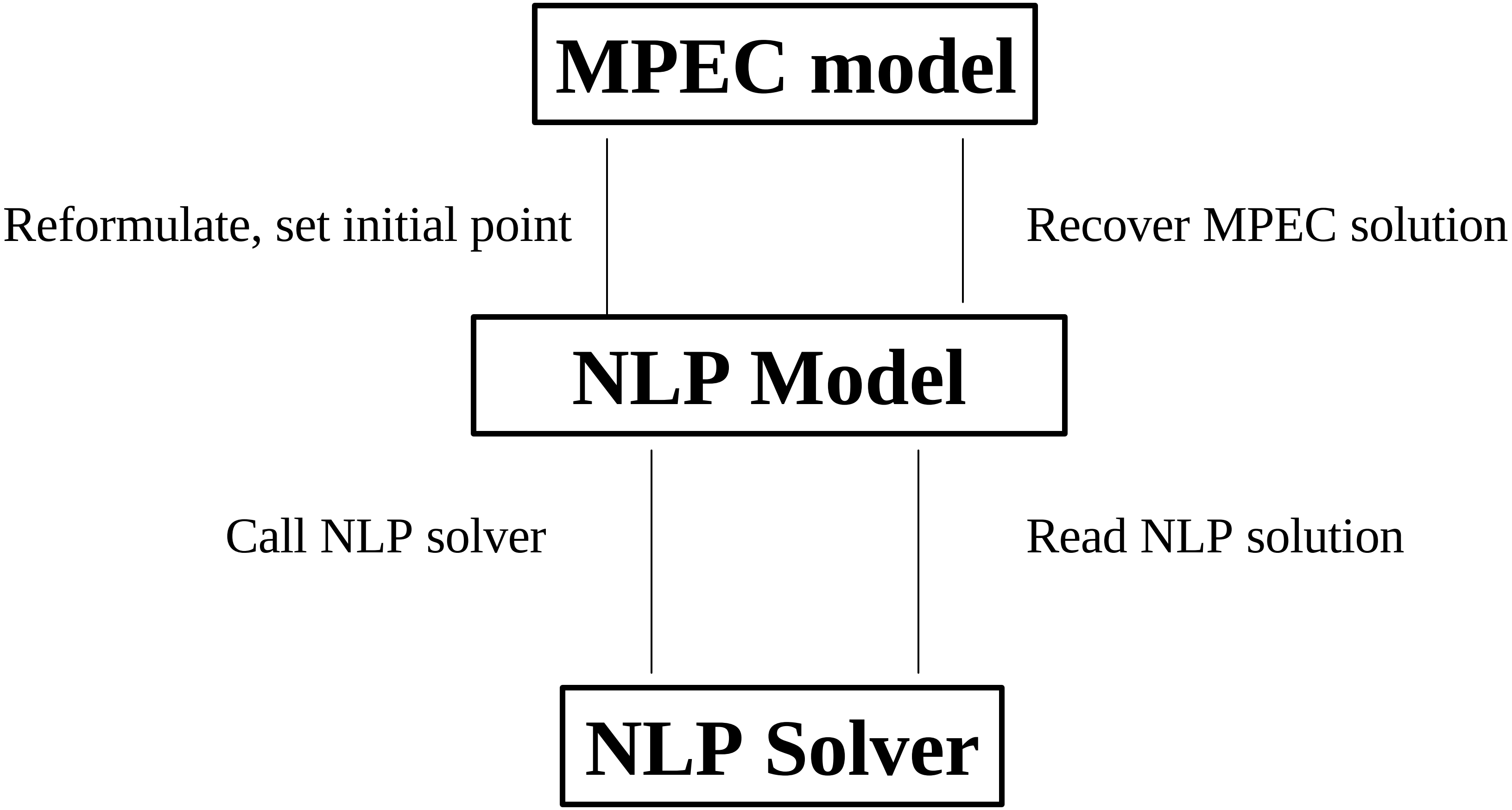
As $m \to 0$, $\frac{r-s}{m} \to \infty$ and $\exp(\frac{r-s}{m})$ overflows

- Functions are well-defined in spite of the overflow
- Smoothed NCP-functions implemented in GAMS
- Code shared between GAMS and solvers

GAMS/NLPEC

- Based on scalar-mode GAMS format
- Rewrite model based on reformulation rules
- Call NLP solver
- Recover MPEC solution from NLP solution

GAMS/NLPEC



MPEC model

Reformulate, set initial point

Recover MPEC solution

NLP Model

Call NLP solver

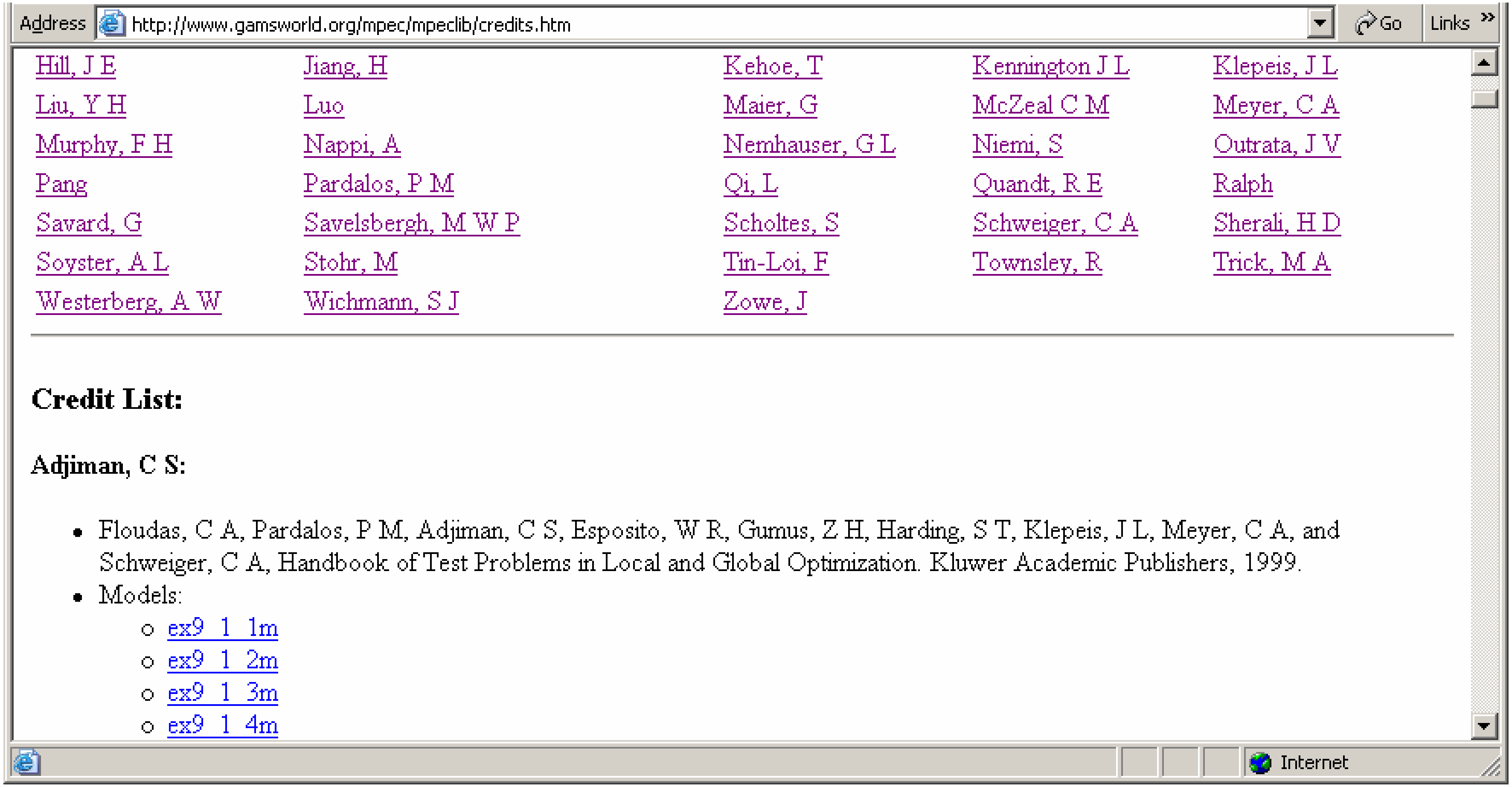
Read NLP solution

NLP Solver

NLPEC Options

- Set the reformulation type!
- Set parameters for penalization, smoothing
- Drive penalties down in loop
- Set initial values for slack variables
- Control parameters for automated tests

MPEC Lib



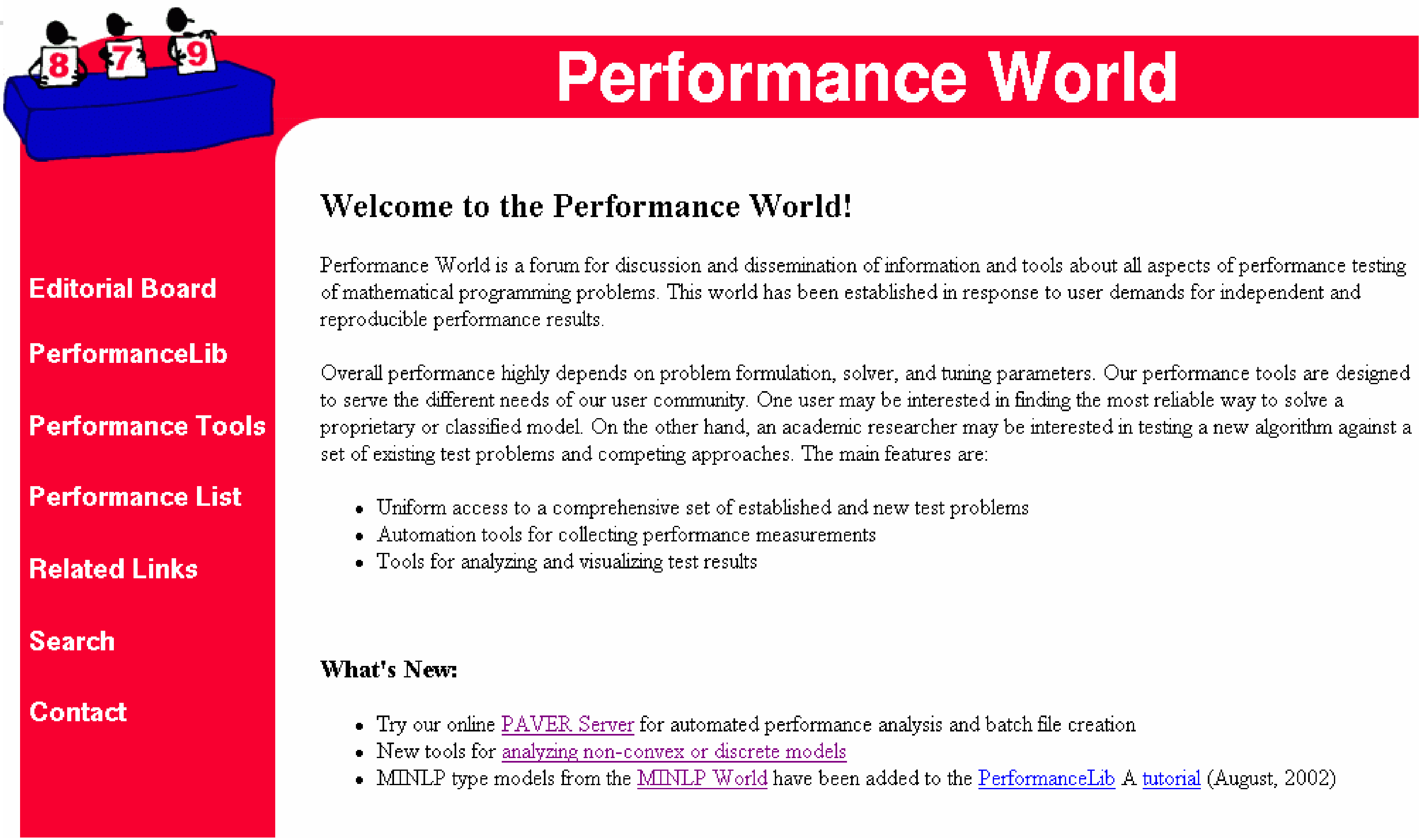
Hill, J E	Jiang, H	Kehoe, T	Kennington J L	Klepeis, J L
Liu, Y H	Luo	Maier, G	McZeal C M	Meyer, C A
Murphy, F H	Nappi, A	Nemhauser, G L	Niemi, S	Outrata, J V
Pang	Pardalos, P M	Qi, L	Quandt, R E	Ralph
Savard, G	Savelsbergh, M W P	Scholtes, S	Schweiger, C A	Sherali, H D
Soyster, A L	Stohr, M	Tin-Loi, F	Townsley, R	Trick, M A
Westerberg, A W	Wichmann, S J	Zowe, J		

Credit List:

Adjiman, C S:

- Floudas, C A, Pardalos, P M, Adjiman, C S, Esposito, W R, Gumus, Z H, Harding, S T, Klepeis, J L, Meyer, C A, and Schweiger, C A, Handbook of Test Problems in Local and Global Optimization. Kluwer Academic Publishers, 1999.
- Models:
  - ex9_1_1m
  - ex9_1_2m
  - ex9_1_3m
  - ex9_1_4m

Performance World



Welcome to the Performance World!

Performance World is a forum for discussion and dissemination of information and tools about all aspects of performance testing of mathematical programming problems. This world has been established in response to user demands for independent and reproducible performance results.

Overall performance highly depends on problem formulation, solver, and tuning parameters. Our performance tools are designed to serve the different needs of our user community. One user may be interested in finding the most reliable way to solve a proprietary or classified model. On the other hand, an academic researcher may be interested in testing a new algorithm against a set of existing test problems and competing approaches. The main features are:

- Uniform access to a comprehensive set of established and new test problems
- Automation tools for collecting performance measurements
- Tools for analyzing and visualizing test results

What's New:

- Try our online PAVER Server for automated performance analysis and batch file creation
- New tools for analyzing non-convex or discrete models
- MINLP type models from the MINLP World have been added to the PerformanceLib A tutorial (August, 2002)

PTools: Performance Profiles

Performance Profiles (Dolan and Moré, 2002):

- Cumulative distribution function for a performance metric
- Performance metric: ratio of current solver time over best time of all solvers for “success”
- Intuitively: probability of success if allowed τ multiples of the best time (τ=ratio)

PTools: Performance Profiles

Interpretation (for *t=ratio*, *P=profile*):

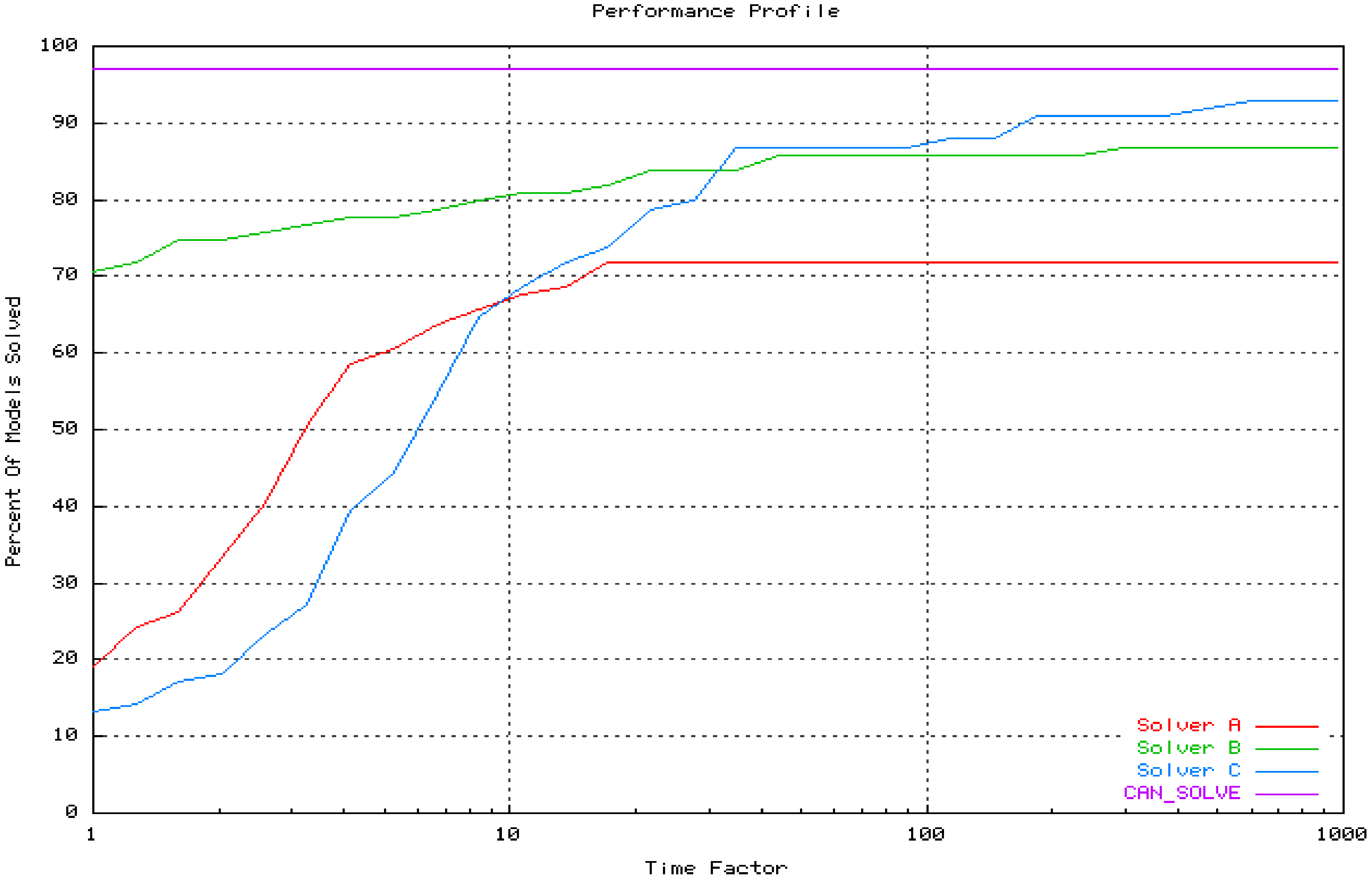
- Efficiency: $P(t)$ for $t = 1$

- Probability of success:

$$\lim P(t) \text{ as } t \to \infty$$

- Compact graphs summarize all information

Example Profile



GAMS/Examiner

- Independent check of solution accuracy
- Check feasibility and optimality
  - Can check a subset of these
  - User can set tolerances used, etc.
- Runs in between GAMS and a solver
  - Acts as a silent observer

Sample Experiment II

- 92 models from MPECLIB
- Run “all” NLPEC reformulations - 52
  - NLP solvers: CONOPT, MINOS, SNOPT.
  - Use Examiner to verify
- Compare performance using:
  - GAMS
  - PTools.

Feasibility %

	Conopt	MINOS	SNOPT	anysolv
er1.0	76	41	83	87
er1.1	71	23	66	83
er2.0	67	40	75	87
er2.1	72	45	76	89
er3.0	75	41	83	87
er3.1	72	23	64	83
er4.0	70	67	75	89
er4.1	57	63	78	87
er5.0	77	58	72	92
er5.1	59	40	60	80
er6.0	75	58	72	92
er7.0	76	63	76	90
er8.0	66	40	61	86

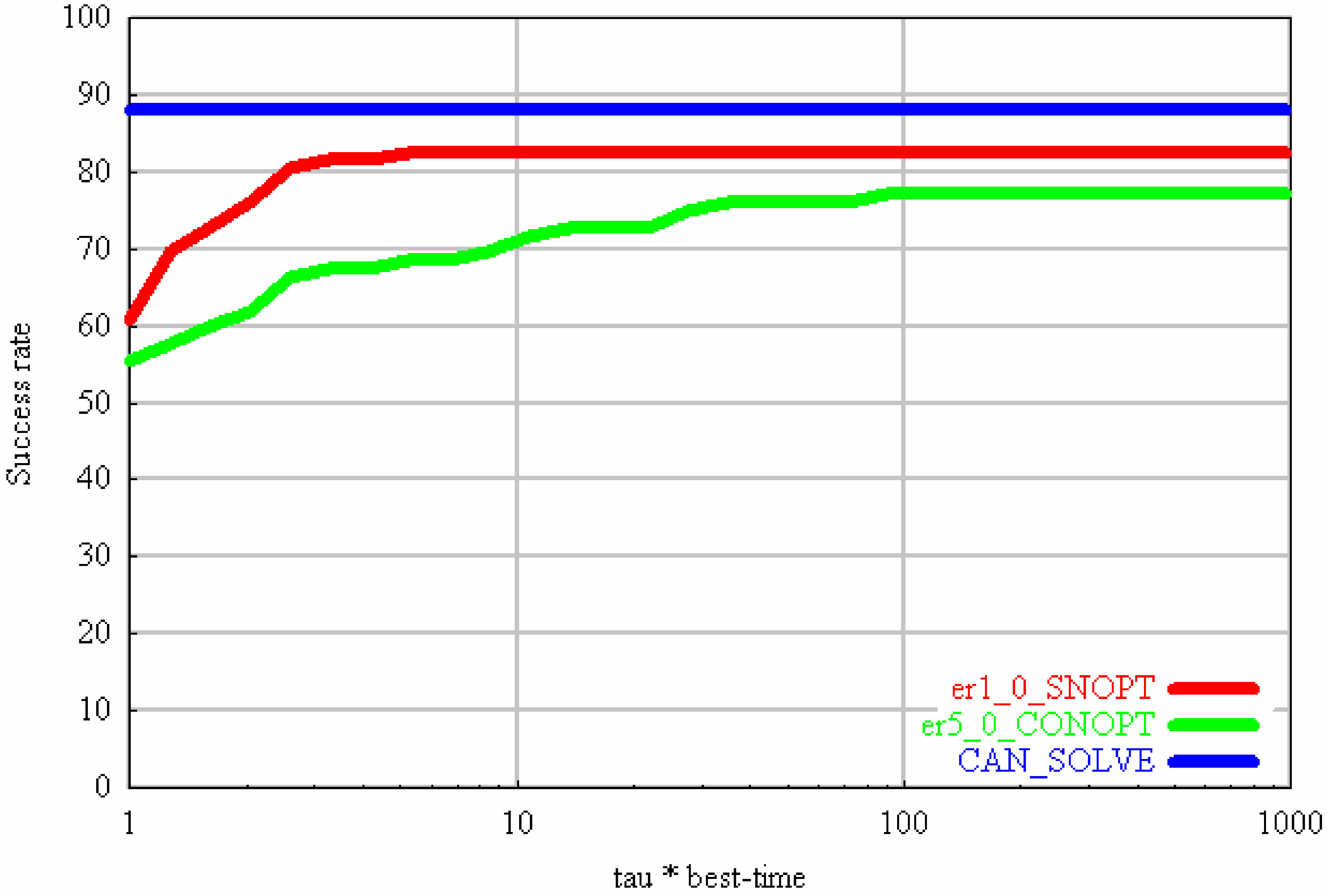
Notable Reforms

- Reforms
  - 1: $<,>_i = \mu$, bnd slacks for L,U,B
  - 2: $<,>_i <= \mu$, bnd slacks for L,U,B
  - 3: $<,>_i = \mu$, bnd slacks for L,U + Scholtes,slack
  - 5: $<,>_{L+U+B} = \mu$, no slacks for L,U, bnd slack for B
  - 21: $<,>_{L+U+B} <= \mu$, bnd slacks for L,U,B
  - 22: $F\text{-}B_i = 0$, bdd slacks for L,U,B
- Option files
  - 1: initmu = .01, numsolves = 5, finalmu = 0
  - 1: initmu = 1, numsolves = 5, finalmu = 0

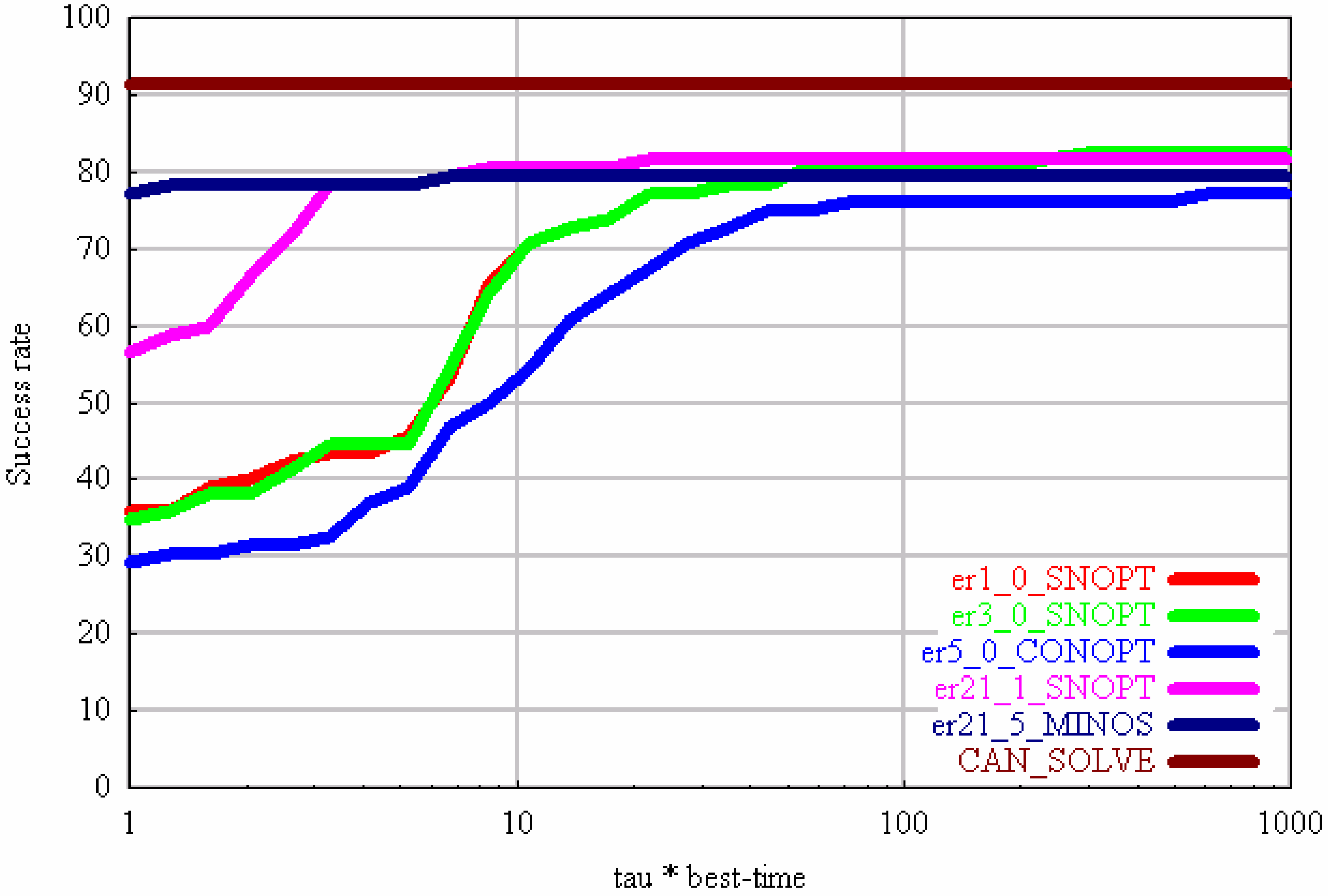
Summary statistics II

- 94% of models solved by some means
- “Best” technique for feasibility:
  - reform 5.0 - 92%
  - 6 reforms >= 90% , 28 reforms >= 80%
- “Best” technique for optimality:
  - reform 21.5 or 22.5 - 74%
  - 5 reforms >= 70%, 15 reforms >= 60%

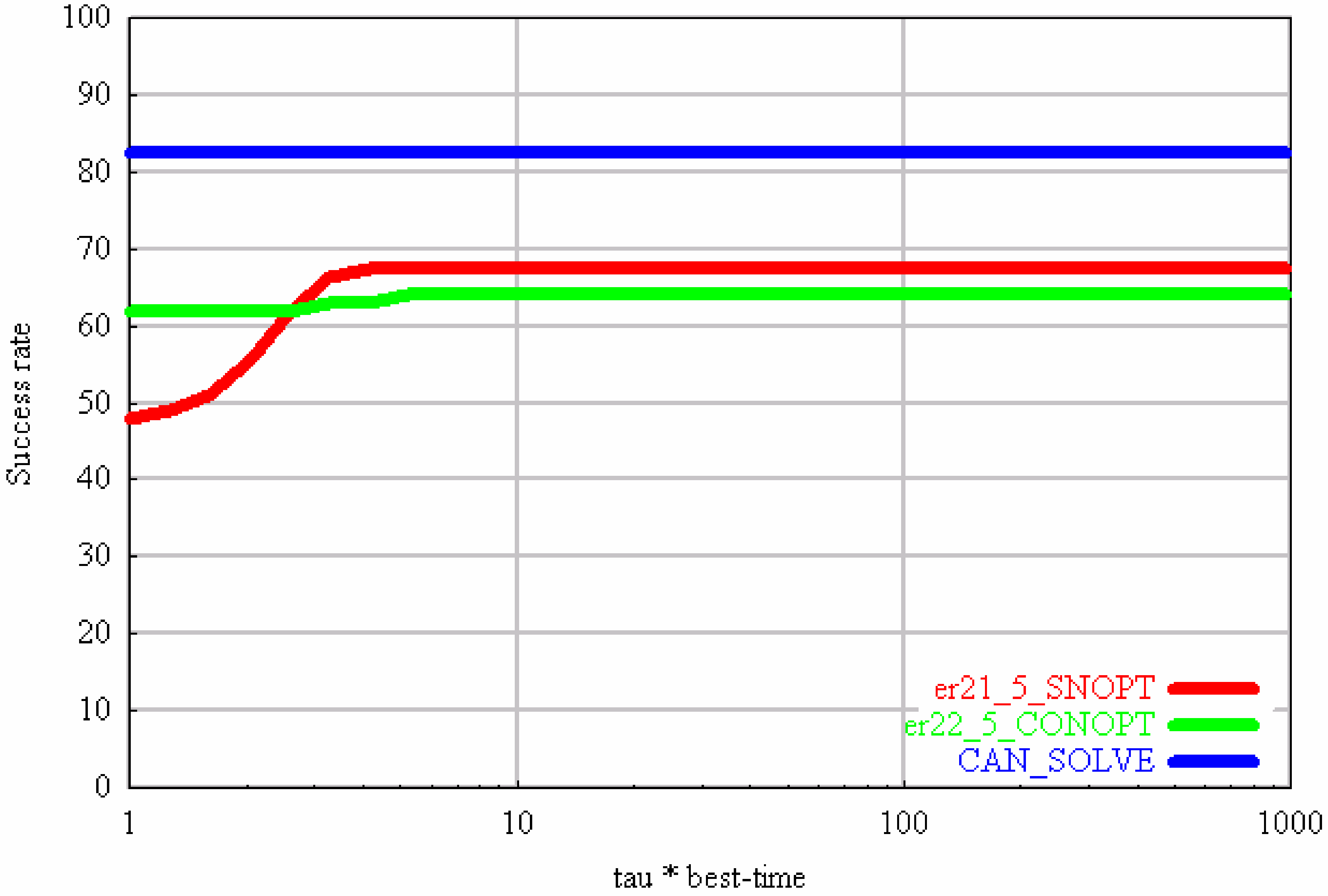
Success = Feasibility



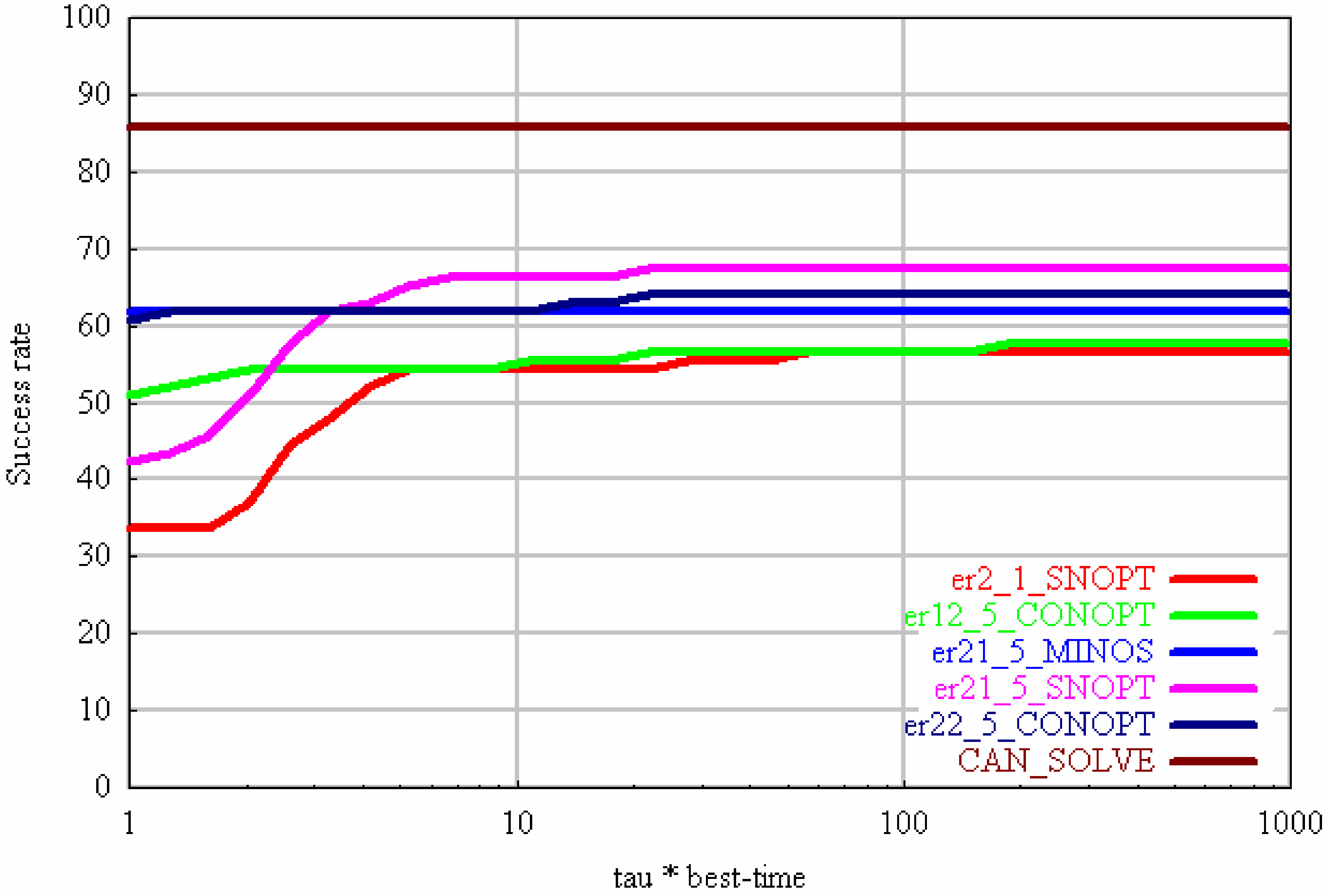
Success = Feasibility



Success = BestFound



Success = BestFound



Conclusions

- Reformulation a viable MPEC solution strategy.
- Special functions required for some reforms.
- Very different strategies competitive:
  - Component-wise inner products
  - Fischer-Burmeister
  - Scholtes, Billups
- Presentation with code to reproduce results: `http://www.gams.com/presentations`