

# **Global Optimization with GAMS/LGO**

**Introduction, Usage, and Applications**

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**GAMS Global Optimization Workshop  
Washington, DC      Sept 18, 2003**

# Introduction

- Acknowledgement to GAMS team  
for contributions to the development of GAMS/LGO  
for assistance in organizing this GO workshop
- Part of these lecture notes based on Michael Bussieck's  
related talk at the MPS Conference (Copenhagen, Aug  
2003), co-authored with the three other GAMS GO solver  
developers (Leon Lasdon, Nick Sahinidis, and JDP)
- Nonlinear processes and phenomena surround us  
→ NL models are ubiquitous in the sciences, engineering,  
and econometrics  
→ NLP/GO modeling and solver needs

# Introduction (continued)

- Global optimization is a relatively new field: “noticeable interest” and research efforts only since ~ 1970
- Solid general theoretical foundations since ~ 1980: globally convergent, exact deterministic and stochastic algorithms; as well as a range of heuristic approaches
- Availability of several gradually developed/improved, “professional level” global optimization codes; in addition, a significant variety of “research” codes since ~ 1990
- Collaboration between the GAMS Corp. & the developers of BARON, LGO, and OQNLP → GAMS/GO solvers (~ 2000)

# Introduction (continued)

- Topics covered
  - Brief GAMS review
  - General CGO model statement and related notes
  - LGO solver: current implementations
  - LGO solver: key features
  - GAMS/LGO implementation and usage
  - Illustrative numerical example(s)
  - Existing and prospective applications
  - Illustrative references

# GAMS (General Algebraic Modeling System)

- GAMS: development started as a research project at the World Bank (1976); GAMS Devpt. Corp. (1988)
- GAMS book by Brooke, Kendrick & Meeraus (1988); subsequent revisions; currently, extensive further documentation is available (papers, lectures, models,...)
- GAMS: integrates core model development/parser system and a range of connected solver options
- 10,000+ users in over 100 countries (2003)



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# Welcome to the GAMS Home Page!

The General Algebraic Modeling System (GAMS) is a high-level modeling system for mathematical programming problems. It consists of a language compiler and a stable of integrated high-performance solvers. GAMS is tailored for complex, large scale modeling applications, and allows you to build large maintainable models that can be adapted quickly to new situations.

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GAMS Dev

Welcome to **GAMS Software GmbH**, the European branch of [GAMS Development Corporation](#). This web site informs about the **General Algebraic Modeling System (GAMS)** and how we help our clients face modeling challenges and increase their productivity.

GAMS is the leading tool for the development, solution, and management of large scale optimization problems. Our clients use GAMS to implement and maintain complex models in various areas.

GAMS has been continually evolved to reflect insights gained from over a decade of close collaboration with both the world's leading software creators for optimization techniques and successful clients from academia and industry.

## News and Events:

- August 19, 2003: [RANK](#), a new GAMS utility for sorting numeric data, is now on the [Contributed Software](#) page
- July 30, 2003: [OR News](#) advertisement: [LGO Global Optimization Solver](#)
- June 30, 2003: OR/MS Today advertisement: [Conic programming in GAMS](#)
- June 18, 2003: The announcement for the book [Modeling Languages in Mathematical Optimization](#) is now on the [Presentations](#) page.
- June 02, 2003: The presentations from the GOR Workshop on Modeling Languages are added to the [Presentations](#) page.
- May 29, 2003: The [twelfth issue](#) of [Bruce McCarl's Newsletter](#) is now available.
- May 28, 2003: The distribution 21.0 is out, here are the [Release Notes](#).
- January 17, 2003: [Bruce McCarl's GAMS User Guide 2003](#) is online.



# GAMS Solver Options

<u><b>BARON</b></u>	Branch-And-Reduce Optimization Navigator for proven global solutions from The Optimization Firm
<u><b>BDMLP</b></u>	LP solver that comes with any GAMS system
<u><b>CONOPT</b></u>	Large scale NLP solver from ARKI Consulting and Development
<u><b>CPLEX</b></u>	High-performance LP/MIP solver from Ilog
<u><b>DECIS</b></u>	Large scale stochastic programming solver from Stanford University
<u><b>DICOPT</b></u>	Framework for solving MINLP models. From Carnegie Mellon University
<u><b>LGO</b></u>	Lipschitz global optimizer from Pinter Consulting Services
<u><b>MILES</b></u>	MCP solver from University of Colorado at Boulder that comes with any GAMS system
<u><b>MINOS</b></u>	NLP solver from Stanford University
<u><b>MOSEK</b></u>	Large scale LP/MIP plus conic and convex non-linear programming system from EKA Consulting
<u><b>MPSGE</b></u>	Modeling Environment for CGE models from University of Colorado at Boulder
<u><b>MPSWRITE</b></u>	MPS file generator that comes with any GAMS System
<u><b>OQNLP</b></u>	Multi-start method for global optimization from Optimal Methods Inc.
<u><b>OSL</b></u>	High performance LP/MIP solver from IBM
<u><b>OSLSE</b></u>	OSL Stochastic Extension for solving stochastic models
<u><b>PATH</b></u>	Large scale MCP solver from University of Wisconsin at Madison
<u><b>SBB</b></u>	Branch-and-Bound algorithm from ARKI for solving MINLP models
<u><b>SNOPT</b></u>	Large scale SQP based NLP solver from Stanford University
<u><b>XA</b></u>	Large scale LP/MIP system from Sunset Software
<u><b>XPRESS</b></u>	High performance LP/MIP solver from Dash



# GAMS Global Solvers

Available solvers:

- **BARON** Branch-and-reduce solver system  
by The Optimization Firm
- **LGO** Global/nonlinear optimization solver suite  
by PCS
- **OQNLP** OptQuest/NLP solver system  
by OptTek Systems and Optimal Methods

Key differences between the global solver options:

- underlying algorithms
- external solver needs
- optimality guarantees provided
- model forms they can handle
- model sizes they can handle

# The Relevance of GO

Optimization is often based on highly nonlinear descriptive models; several important and general model-classes are:

“Black box” systems design and operations  
(experimental design, response surface methods,  
confidential models,...)

Decision-making under uncertainty

Dynamic optimization models

- Nonlinear models frequently possess multiple optima: hence, their solution requires a suitable global scope search approach
- The objective of global optimization is to find the “absolutely best” solution, in the possible presence of a multitude of local sub-optima

# Continuous GO Model

$$\min f(x)$$

$$f: R^n \rightarrow R^1$$

$$g(x) \leq 0$$

$$g: R^n \rightarrow R^m$$

$$x_l \leq x \leq x_u$$

$x, x_l, x_u, (x_l < x_u)$  are real  $n$ -vectors

“Minimal” analytical assumptions:

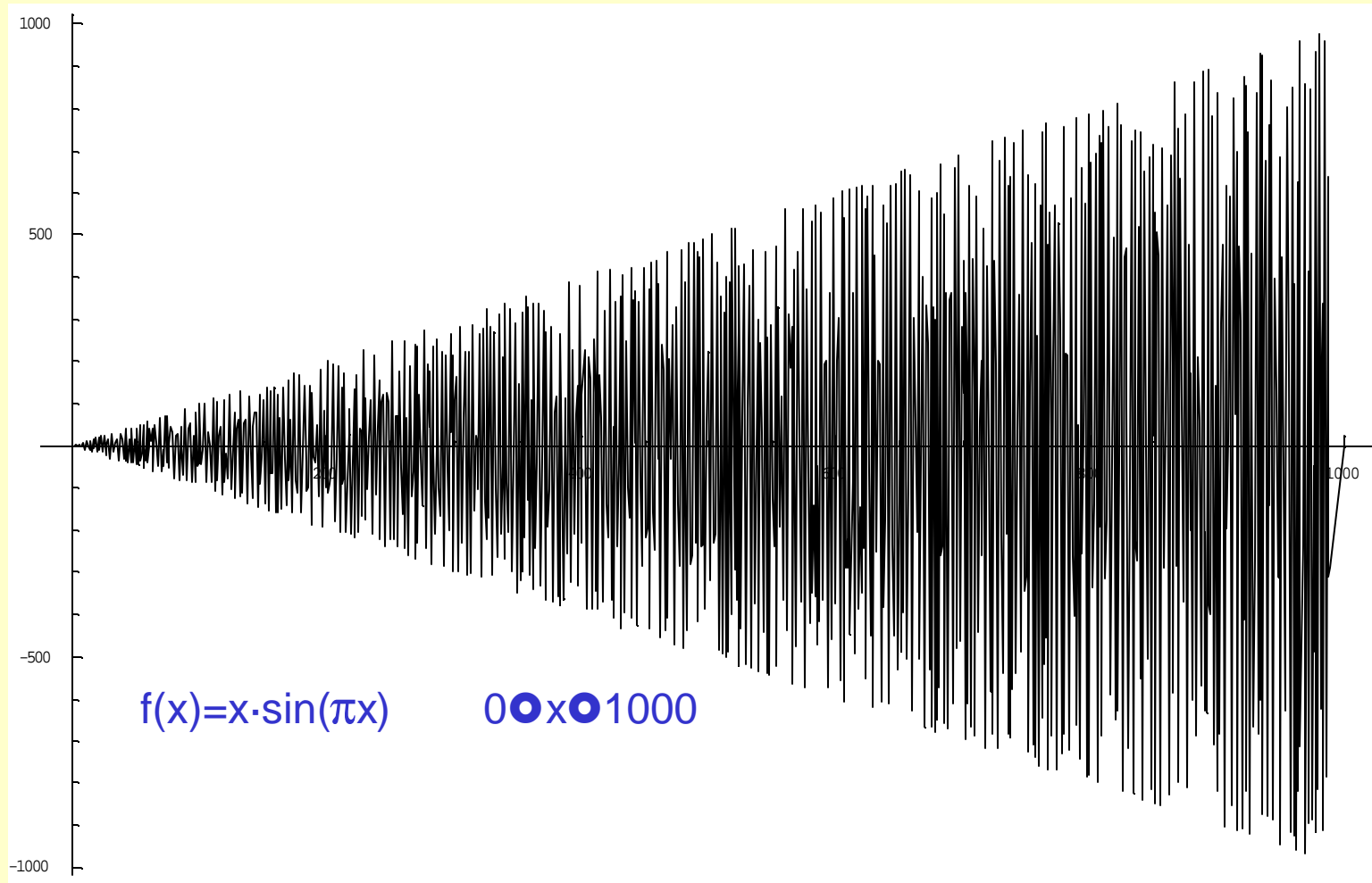
- $x_l, x_u$  finite
- feasible set  $D = \{x_l \leq x \leq x_u : g(x) \leq 0\}$  non-empty
- $f, g$  continuous

These guarantee the existence of global solution set  $X^*$

Typically, we assume that  $X^*$  is finite (at most, countable)

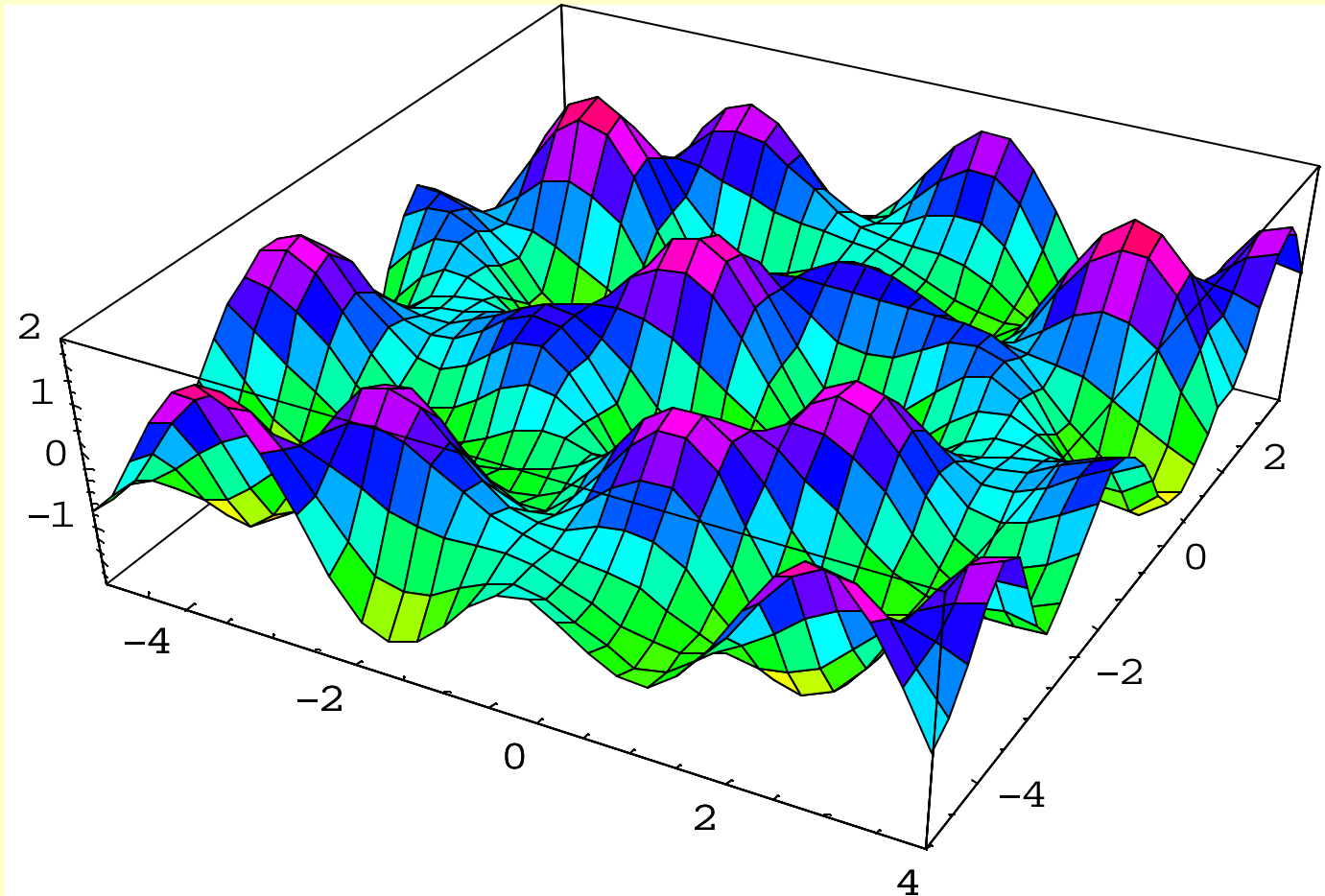
CGO covers a very general class of models, including all combinatorial and mixed integer (MIP) problems

# Illustrative GO Model, $n=1$



GO models can be arbitrarily difficult to solve,  
even in (very) low dimensions...

# Illustrative GO Model, $n=2$



Numerical difficulty may increase exponentially,  
as model size ( $n, m$ ) grows

# Solving GO Models

- Practical objective: solution estimates, on the basis of finite sequences of search points and corresponding function values  $\{x_1, \dots, x_k\}$ ,  $\{f_1, \dots, f_k\}$ ,  $\{g_1, \dots, g_k\}$  (Note: higher order information, as a rule, has no general value in the context of global search)
- If the functions  $f$  and  $g$  are Lipschitz-continuous, then - based on the sample sequence - deterministic lower/upper bounds can be derived; hence, a rigorous convergence theory can be developed  $(|h(x_1) - h(x_2)| \leq L \|x_1 - x_2\| \text{ for } x_1, x_2 \in D; L = L(D, h) > 0)$
- If the model functions are “only” continuous, then stochastic convergence is the best one can aim for: indeed, it can be properly established
- Implementation issues and challenges: “only a few details”...

# LGO Solver Suite: General Scope

- The LGO (Lipschitz Global Optimizer) solver suite has been designed and developed with GO models in mind that do not - or may not - have an easily identifiable and “exploitable” special structure
- This scope specifically includes “black box” models, and models with computable (perhaps non-analytical) functions: examples will be shown and listed later on
- Note that more structured GO models - e.g. with convex, concave or indefinite QP objectives over convex domains - also belong to the scope of LGO (as a rule, with a relatively small performance “hit”, due to its “broad-minded” methods...)



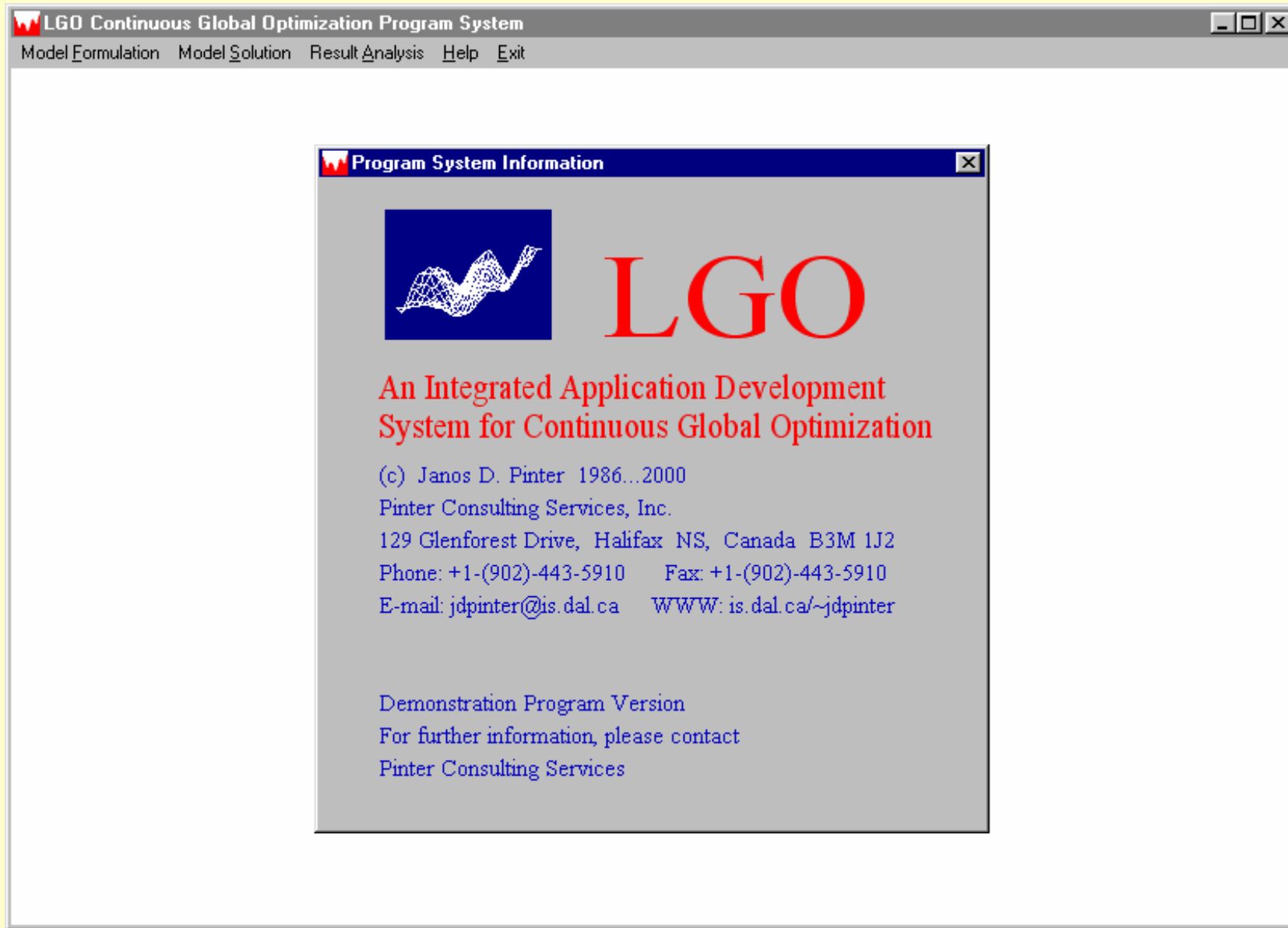
# LGO Solver Suite: Components

- LGO integrates several global search algorithms
  - adaptive partition and search (branch-and-bound)
  - adaptive random search
  - adaptive multi-start
  - and “traditional” local optimization methods
    - exact penalty method, box-constrained local search
    - generalized reduced gradient method
- This approach flexibly supports the search for global or local solutions in optimization models defined by continuous functions, w/o further necessary analytical conditions

# LGO Solver Suite:

## Characteristics and Implementations

- No external solver needs; combinations with local solvers are possible, and easy to add
- Optimality guarantees provided: exact and stochastic (deterministic, if model Lipschitz constants are known or can be properly over-estimated during optimization)
- Model forms handled: arbitrary continuous model functions
- Model sizes: in principle, arbitrary (depending on processor and RAM), but: “curse of dimensionality”... typical current delivery versions: thousand(s) of variables and constraints
- Customized versions of LGO implemented for several major modeling platforms; used in advanced scientific, engineering, and economic applications for over a decade



LGO modeling and solver system (+ IDE): MS Windows version

Fortran - Lahey Computer Systems, Inc. - Microsoft Internet Explorer

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**Computational Global Optimization  
in Nonlinear Systems — An Interactive Tutorial**  
By János D. Pintér, Ph.D., D.Sc.

# ORMS

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SOFTWARE REVIEW



## LGO

*Versatile tool for global  
optimization*

By Harold P. Benson and Erjiang Sun

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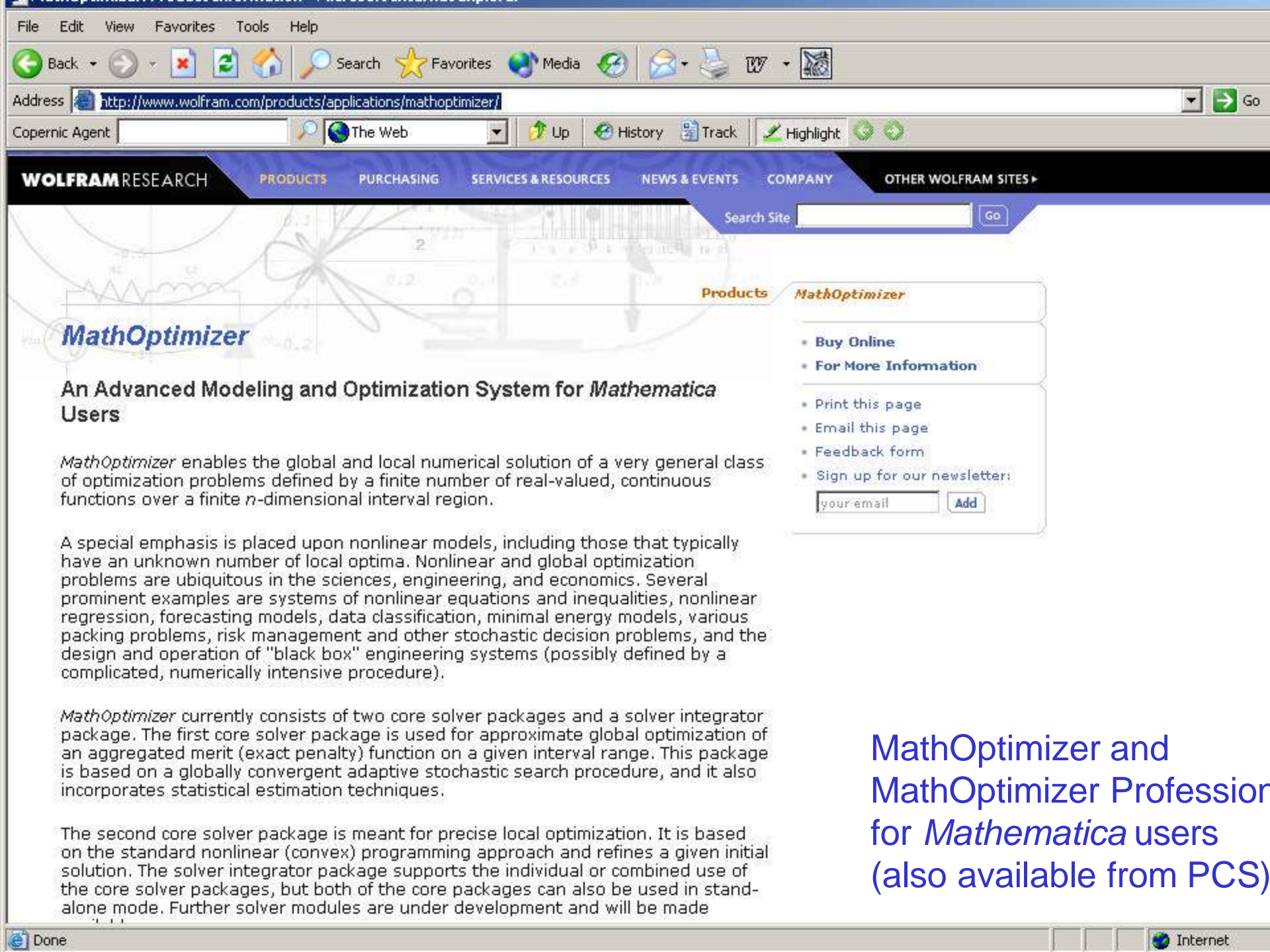
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### An Advanced Modeling and Optimization System for *Mathematica* Users

*MathOptimizer* enables the global and local numerical solution of a very general class of optimization problems defined by a finite number of real-valued, continuous functions over a finite  $n$ -dimensional interval region.

A special emphasis is placed upon nonlinear models, including those that typically have an unknown number of local optima. Nonlinear and global optimization problems are ubiquitous in the sciences, engineering, and economics. Several prominent examples are systems of nonlinear equations and inequalities, nonlinear regression, forecasting models, data classification, minimal energy models, various packing problems, risk management and other stochastic decision problems, and the design and operation of "black box" engineering systems (possibly defined by a complicated, numerically intensive procedure).

*MathOptimizer* currently consists of two core solver packages and a solver integrator package. The first core solver package is used for approximate global optimization of an aggregated merit (exact penalty) function on a given interval range. This package is based on a globally convergent adaptive stochastic search procedure, and it also incorporates statistical estimation techniques.

The second core solver package is meant for precise local optimization. It is based on the standard nonlinear (convex) programming approach and refines a given initial solution. The solver integrator package supports the individual or combined use of the core solver packages, but both of the core packages can also be used in stand-alone mode. Further solver modules are under development and will be made

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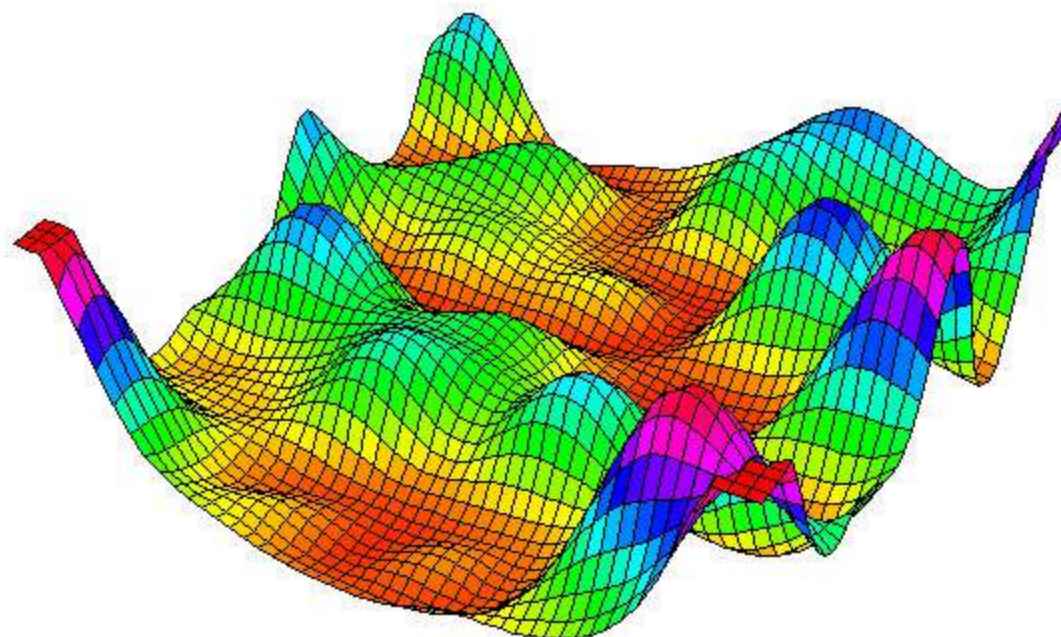
MathOptimizer and  
MathOptimizer Profession  
for *Mathematica* users  
(also available from PCS)



# MathOptimizer Professional

An Advanced Modeling and Optimization System  
for *Mathematica* Users with an External Solver Link

## User Guide



# *MathOptimizer and MathOptimizer Professional* Software review in Scientific Computing World (July - August 2003)

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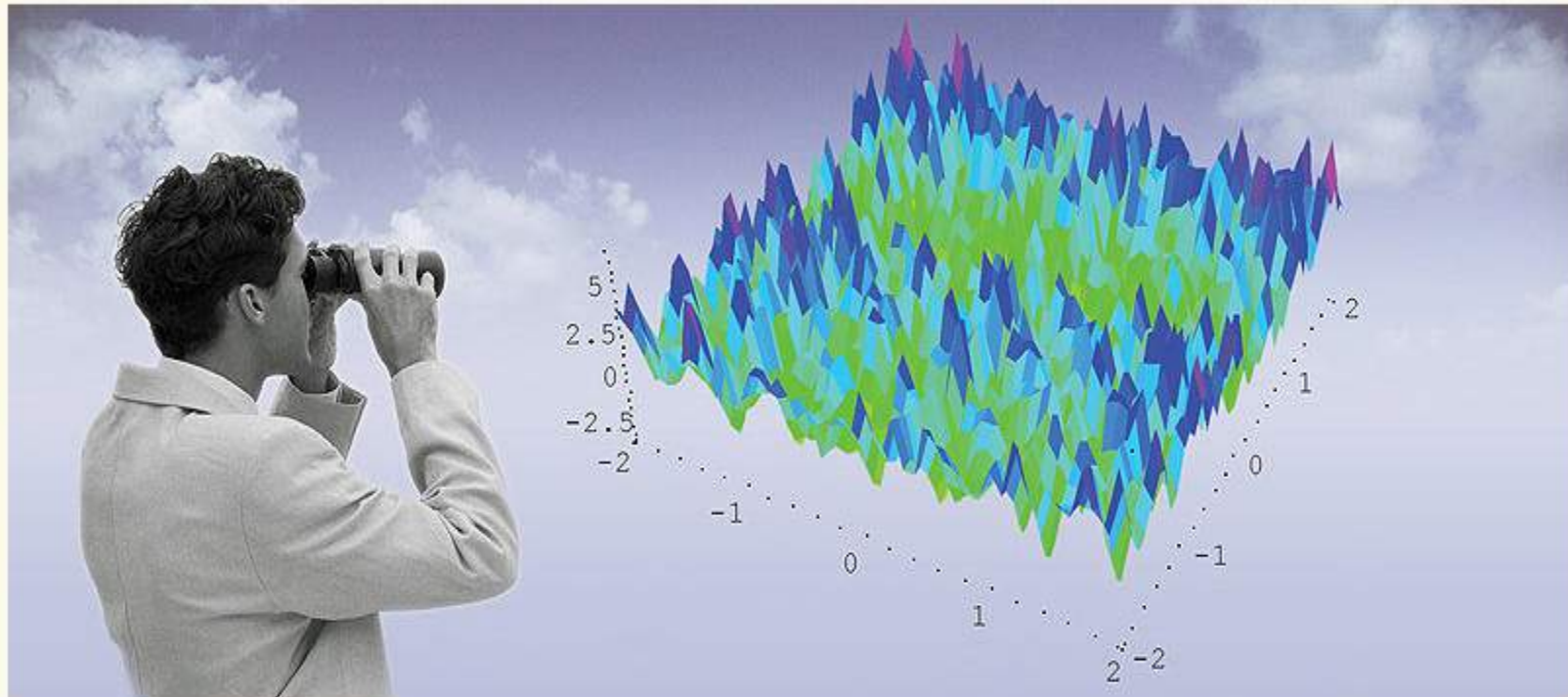
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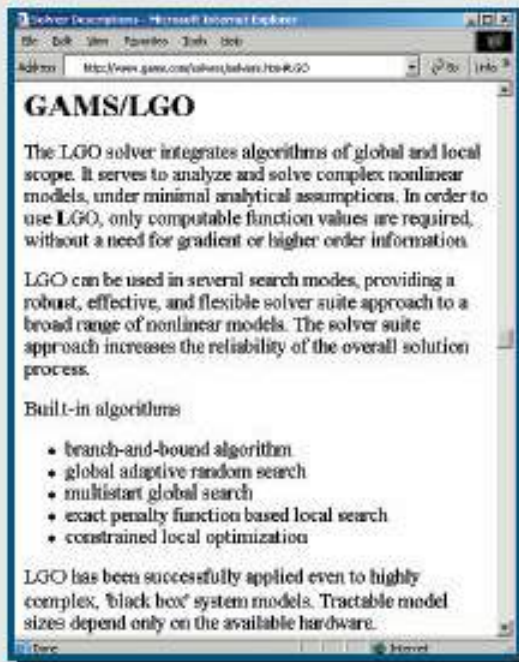
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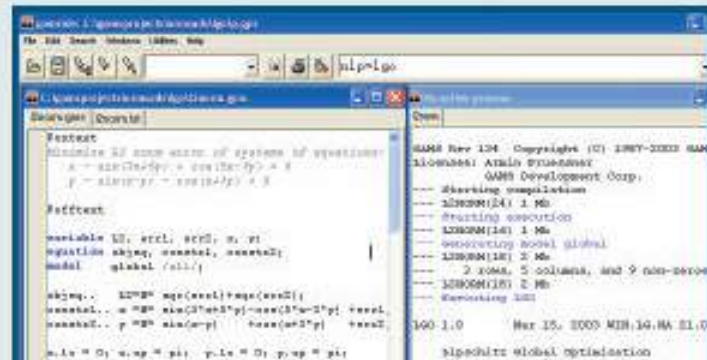


High performance and reliability of solvers come as a result of technological and theoretical developments in solution technology and modeling systems.

LGO is an integrated solver suite for handling nonlinear optimization models. The LGO solution approach is based on the seamless combination of globally convergent and traditional local search strategies.

**LGO has been successfully applied in many application areas:**

- Advanced Engineering Design





# GAMS/LGO Solver Operations

## GAMS Model

Model description/preprocessing      GAMS/LGO

Solver options      Solution report

—

—

## LGO Solver Suite

Global search methods      for multiextremal models

Local search methods      for convex nonlinear models

—

—

Optional calls to other GAMS solvers  
and to external program systems



# GAMS/LGO

The LGO solver integrates algorithms of global and local scope. It serves to analyze and solve complex nonlinear models, under minimal analytical assumptions. In order to use LGO, only computable function values are required, without a need for gradient or higher order information.

LGO can be used in several search modes, providing a robust, effective, and flexible solver suite approach to a broad range of nonlinear models. The solver suite approach increases the reliability of the overall solution process.

## LGO Solver Components

- branch-and-bound based global search
- global adaptive random search
- multistart global search
- exact penalty function based local search
- constrained local optimization

LGO has been successfully applied even to highly complex, 'black box' system models. Tractable model sizes depend only on available hardware.

## Illustrative Application Areas

- advanced engineering design
- econometrics and finance
- medical research and biotechnology
- chemical and process industries
- scientific modeling



# GAMS/LGO

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## 1 Introduction

### 1.1 The LGO Solver Suite

# GAMS/LGO

## General Options

Option	Description	Default
Opmode	Specifies the search mode used. 0 Local search from the given nominal solution without a preceding global search (LS) 1 Global branch-and-bound search and local search (BB+LS) 2 Global adaptive random search and local search (GARS+LS) 3 Global multistart random search and local search (MS+LS)	3
Tlimit	Time limit in seconds. This is equivalent to the GAMS option reslim. If specified, this overrides the GAMS reslim option.	1000
Log_time	Iteration log time interval in seconds. Log output occurs every log_time seconds.	0.5
Log_iter	Iteration log time interval. Log output occurs every log_iter iterations.	1
Log_err	Iteration log error output. Error reported (if applicable) every log_err iterations.	10
Debug	Debug option. Prints out complete LGO status report to listing file. <b>0:</b> No <b>1:</b> yes	0
CallConopt	Number of seconds given for cleanup phase using CONOPT. CONOPT terminates after at most CallConopt seconds. The cleanup phase determines duals for final solution point.	10
Help	Prints out all available GAMS/LGO solver options in the log and listing files.	



# GAMS/LGO

## Limits and Tolerances

Option	Description	Default
G_maxfct	Maximum number of merit function evaluations before termination of global search phase (BB, GARS, or MS). The difficulty of global optimization models varies greatly: for difficult models, g_maxfct can be increased to 1.0E+6, or even larger values, as needed.	1000 *(n+m)
Max_nosuc	Maximum number of merit function evaluations in global search phase (BB, GARS, or MS) where no improvement is made. Algorithm terminates upon reaching this limit.	200 *(n+m)
Penmult	Constraint penalty multiplier. Global merit function is defined as objective + the violated constraints weighted by penmult.	1
Acc_tr	Global search termination criterion parameter (acceptability threshold). The global search phase (BB,GARS, or MS) ends, if an overall merit function value found in the global search phase is less than acc_tr.	-1.0E10
Fct_trg	Partial stopping criterion in second local search phase.	-1.0E10
Fi_tol	First local search merit function improvement.	1.0E-6
Con_tol	Maximal constraint violation in local search.	1.0E-6
Kt_tol	Kuhn-Tucker local optimality condition tolerance.	1.0E-6
Irngs	Random number seed.	0
Var_lo	Smallest lower bound	-1.0E10
Var_up	Largest upper bound	1.0E10
Bad_obj	Default value for objective function given evaluation errors.	1.0E10

# An Illustrative Test Model (N.L. Trefethen)

## A Hundred-dollar, Hundred-digit Challenge

Each October, a few new graduate students arrive in Oxford to begin research for a doctorate in numerical analysis. In their first term, working in pairs, they take an informal course called the “Problem Solving Squad.” Each week for six weeks, I give them a problem, stated in a sentence or two, whose answer is a single real number. Their mission is to compute that number to as many digits of precision as they can.

Ten of these problems appear below. I would like to offer them as a challenge to the SIAM community. Can you solve them?

I will give \$100 to the individual or team that delivers to me the most accurate set of numerical answers to these problems before May 20, 2002. With your solutions, send in a few sentences or programs or plots so I can tell how you got them. Scoring will be simple: You get a point for each correct digit, up to ten for each problem, so the maximum score is 100 points.

Fine print? You are free to get ideas and advice from friends and literature far and wide, but any team that enters the contest should have no more than half a dozen core members. Contestants must assure me that they have received no help from students at Oxford or anyone else who has already seen these problems.

Hint: They’re hard! If anyone gets 50 digits in total, I will be impressed. The ten magic numbers will be published in the July/August issue of *SIAM News*, together with the names of winners and strong runners-up.—Nick Trefethen, Oxford University.

### The Hundred-dollar, Hundred-digit Challenge Problems

1. What is  $\lim_{\epsilon \rightarrow 0} \int_{\epsilon}^1 x^{-1} \cos(x^{-1} \log x) dx$  ?
2. A photon moving at speed 1 in the  $x$ - $y$  plane starts at  $t = 0$  at  $(x,y) = (0.5, 0.1)$  heading due east. Around every integer lattice point  $(i,j)$  in the plane, a circular mirror of radius  $1/3$  has been erected. How far from the origin is the photon at  $t = 10$ ?
3. The infinite matrix  $A$  with entries  $a_{11} = 1$ ,  $a_{12} = 1/2$ ,  $a_{21} = 1/3$ ,  $a_{13} = 1/4$ ,  $a_{22} = 1/5$ ,  $a_{31} = 1/6$ , etc., is a bounded operator on  $\ell^2$ . What is  $\|A\|$  ?
4. What is the global minimum of the function  
$$\exp(\sin(50x)) + \sin(60e^y) + \sin(70 \sin(x)) + \sin(\sin(80y)) - \sin(10(x+y)) + \frac{1}{4}(x^2 + y^2) ?$$





abel.gms | abel.lst | abel.log

\$Title Linear Quadratic Control Problem (ABEL,SEQ=64)

\$Ontext

*Linear Quadratic Riccati Equations are solved as a General  
Nonlinear Programming Problem instead of the usual Matrix  
Recursion.*

*Kendrick, D, Caution and Probing in a Macroeconomic Model. Journal of  
Economic Dynamics and Control 4, 2 (1982).*

\$Offtext

```
Sets      n      states / consumpt, invest /
          m      controls / gov-expend, money /
          k      horizon / 1964-i, 1964-ii, 1964-iii, 1964-iv
                               1965-i, 1965-ii, 1965-iii, 1965-iv /
          ku(k)   control horizon
          ki(k)   initial period
          kt(k)   terminal period ;
```

**Alias** (n,np), (m,mp) ;

ku(k) = **yes**\$(ord(k) lt card(k));

ki(k) = **yes**\$(ord(k) eq 1);

kt(k) = **not** ku(k);

**Display** k, ki, kt, ku;

**Table** a(n,np) state vector matrix

	consumpt	invest
consumpt	.914	-.016
invest	.097	.424

About

# GAMS

Integrated Development Environment

GAMS IDE	2.0.23.10
Module	GAMS Rev 135
Lic date	Jun 2, 2003
Build	VIS 21.1 135

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3: 50

Insert

# Trefethen's Problem 4 (continued)

**\$title** Trefethen's HDHD 2002 Challenge, Problem 4

**\$ontext**

Trefethen's GO model: a 2-var, 0-cons problem, with many local optima

Source: SIAM News, Jan - Feb 2002 issue

Solution:  $x^* \sim (-0.0244030796, 0.2106124272)$ ;  $f(x^*) \sim -3.306868647$

**\$offtext**

**variables** f, x1, x2;

**equations** objf;

objf.. f =e= 0.25\*(x1\*x1+x2\*x2) + exp(sin(50.\*x1)) + sin(60.\*exp(x2))  
+ sin(70.\*sin(x1)) + sin(sin(80.\*x2)) - sin(10.\*(x1+x2));

**model** trefethen4 / all / ;

x1.lo = -3;

x1.l = -2;

x1.up = 3;

x2.lo = -3;

x2.l = 2;

x2.up = 3;

**solve** trefethen4 **using** nlp **minimizing** f; // nlp=lgo option is set

# Trefethen's Problem 4 (continued)

- Solution found by LGO:

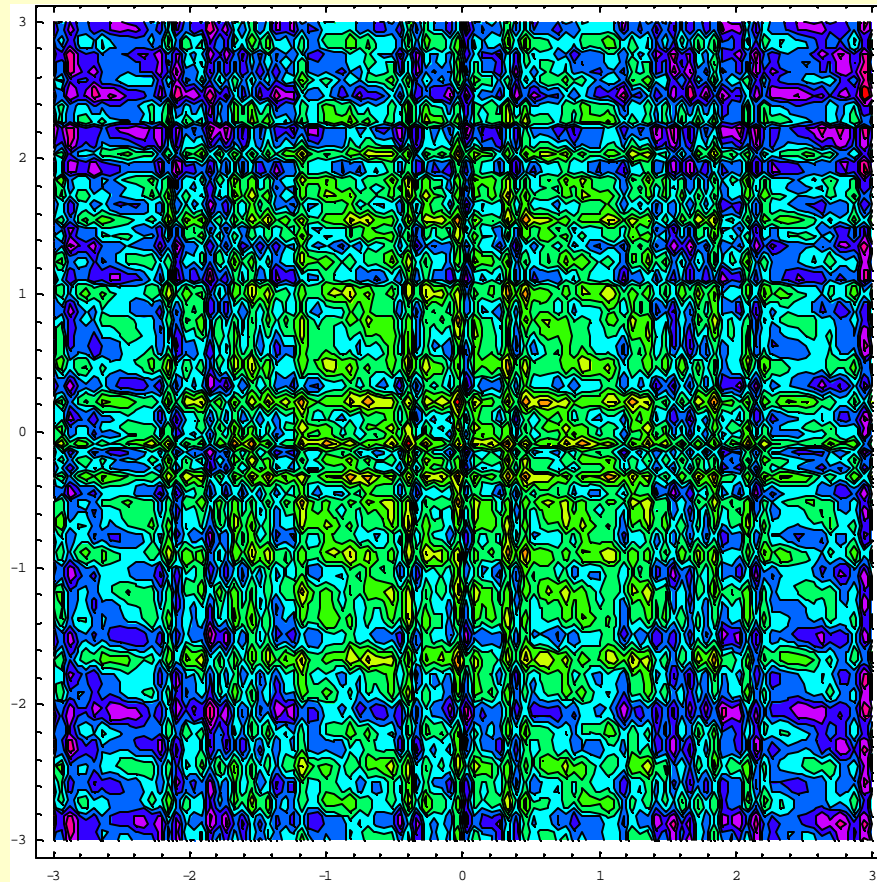
$x^* \sim (-0.0244030796, 0.2106124272);$

$f(x^*) \sim -3.306868647$

- This solution is identical to at least 10 decimal digits to the “true” solution reported by SIAM News (July-August 2002)
- LGO runtime  $\ll$  1 second (P4 desktop PC)

# Trefethen's Problem 4

## Contour Plot of Objective Function



Thousands of  
local optima;  
a single global  
solution...

$$\frac{1}{4} \|x\|^2 + y^2 - \tilde{a}^T x - \sin(\tilde{a}^T x - y) - \tilde{a}^T y - \sin(\tilde{a}^T y)$$

# Advanced GO Applications

## An Illustrative List

The list below is primarily based on actual (PCS client) case studies and projects, and a few articles / reports

- Chemotherapy and radiotherapy design
- Computational physics and chemistry
- Chemical data and process analysis
- Data classification (clustering) and visualization
- Differential equations (numerical solution)
- Engineering design
- Experimental design
- Environmental engineering
- Financial model development

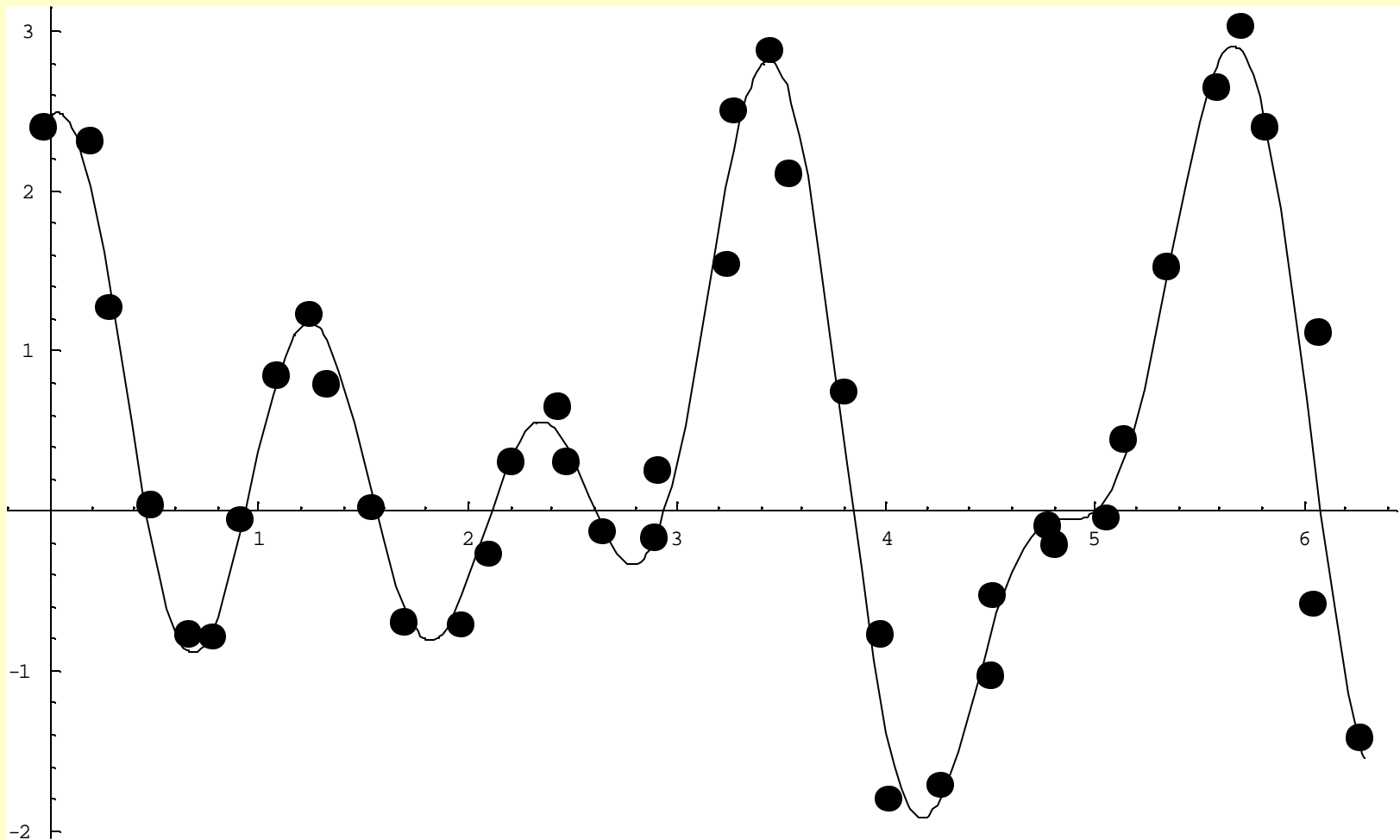


# Application Examples (Continued)

- Laser design
- Model fitting to experimental data (in various calibration or forecasting contexts)
- Packing and loading configuration design
- Resource / population (fish stock,...) management
- Robotics (grasp) design
- Staff scheduling
- Systems of (nonlinear) equations and inequalities
- Vehicle routing and scheduling
- Waste-water engineering system design  
and some other areas

Several advanced applications are highlighted on the next slides

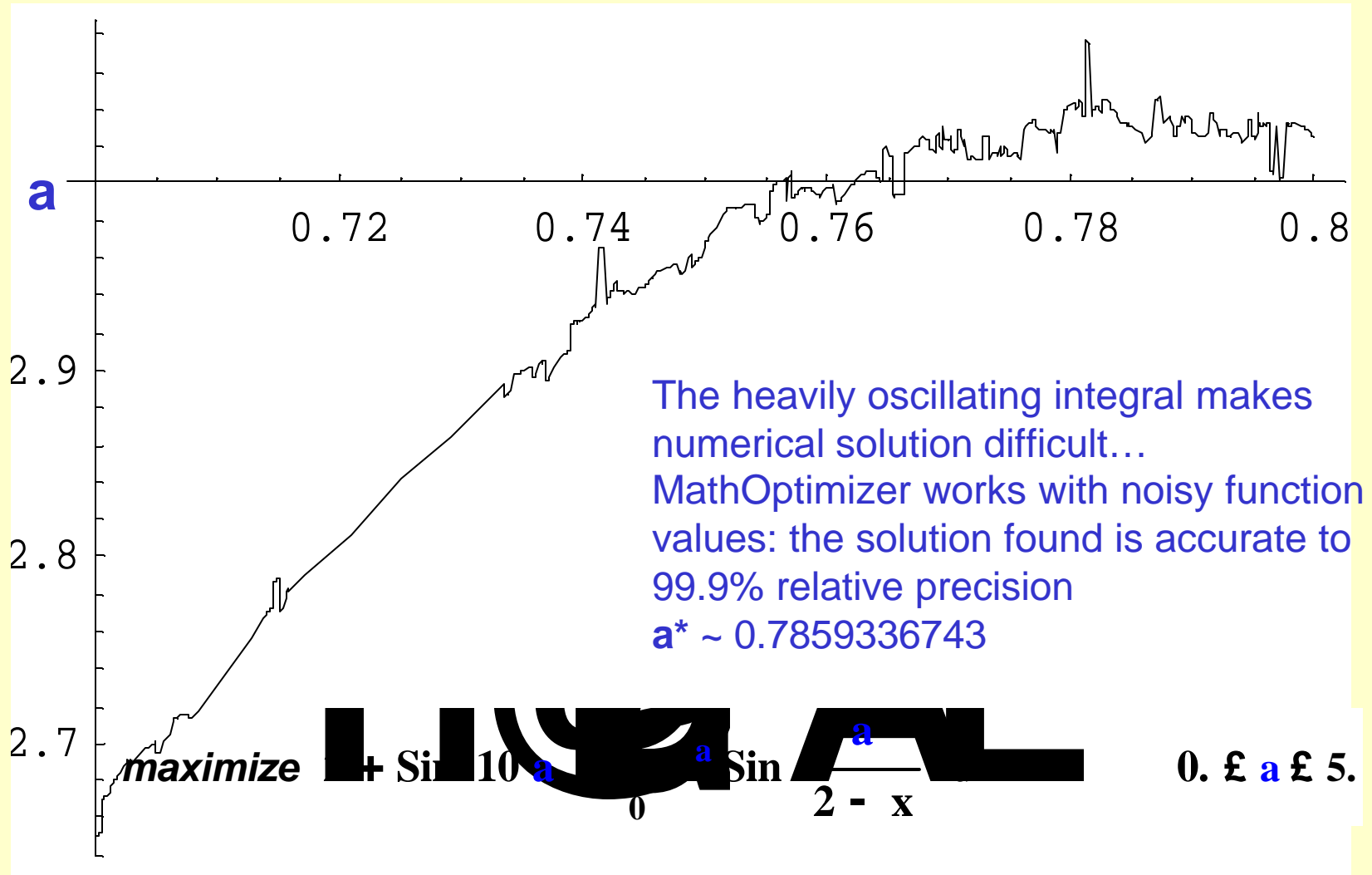
# Calibration of Nonlinear Model in Presence of Noise



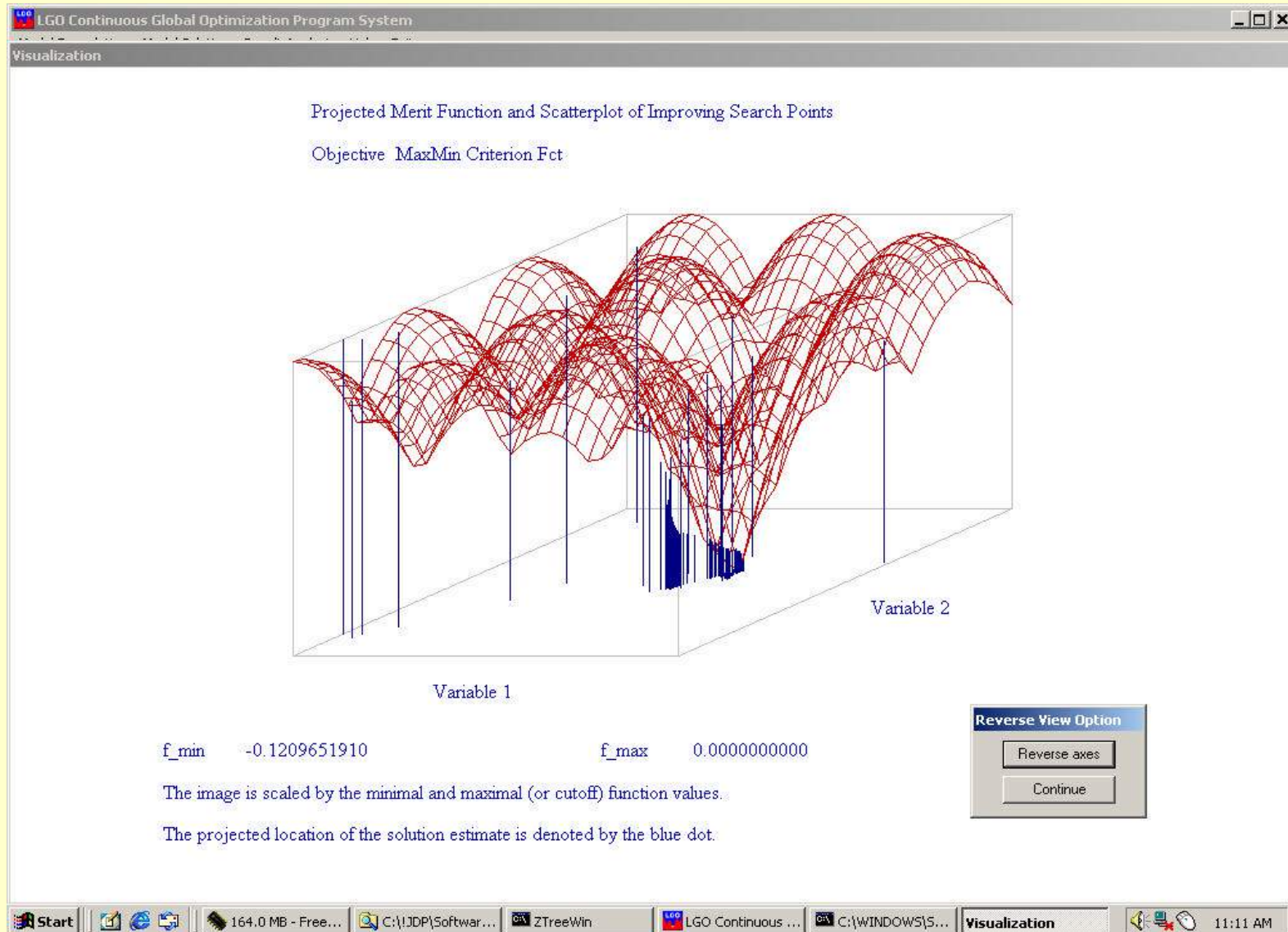
$$a + \sin[b \cdot (\pi \cdot t) + c] + \cos[d \cdot (3\pi \cdot t) + e] + \sin[f \cdot (5\pi \cdot t) + g] + \xi$$

Globally optimized model fit (found by MathOptimizer)

# HDHD Challenge, Problem 9: Parametric Integration



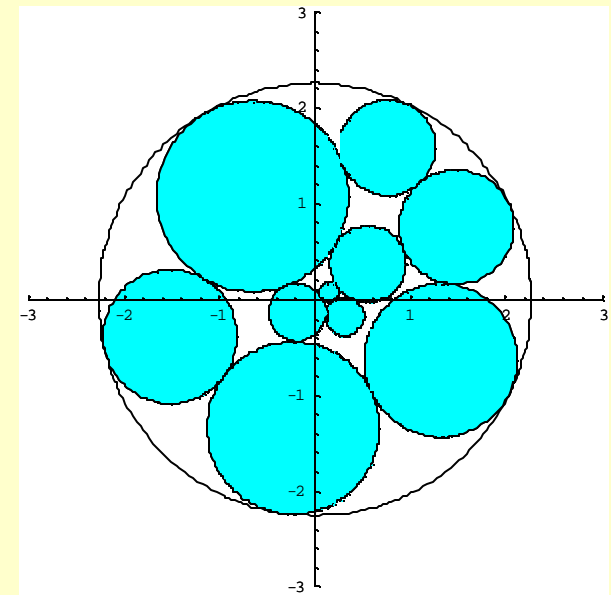
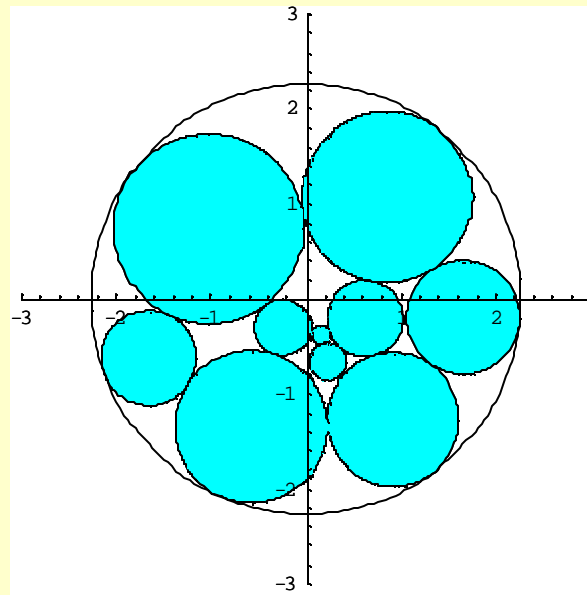
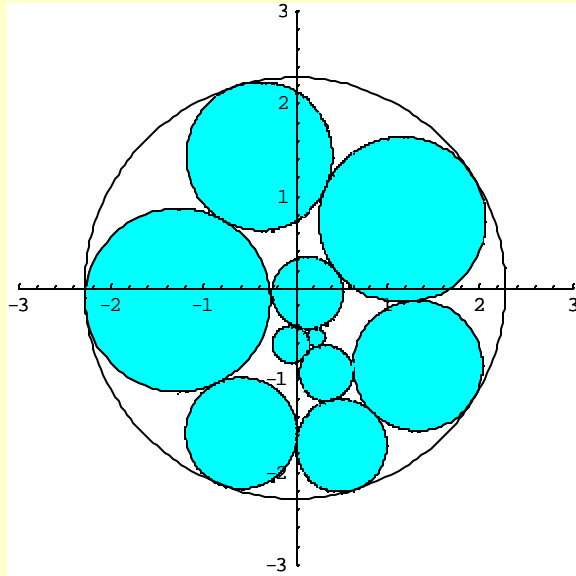
# MaxiMin Point Arrangement Problem (U. of Tilburg, NL)



LGO IDE: model visualization ( $m=13$ ,  $d=2$ )

# Non-Uniform Circle Packings

On the pictures shown below, 10 circles - with radii 0.1, 0.2,..., 1.0 - are arranged; equal consideration is given to the “tightness” of the configuration and the radius of the embedding circle (emb\_c\_rad)



Three different configurations found (MOP runtimes ~ 2 to 5 min)

emb\_c\_rad=2.28345

emb\_c\_rad=2.26559

emb\_c\_rad=2.26107

# Laser Equipment Design (NRC, Canada)

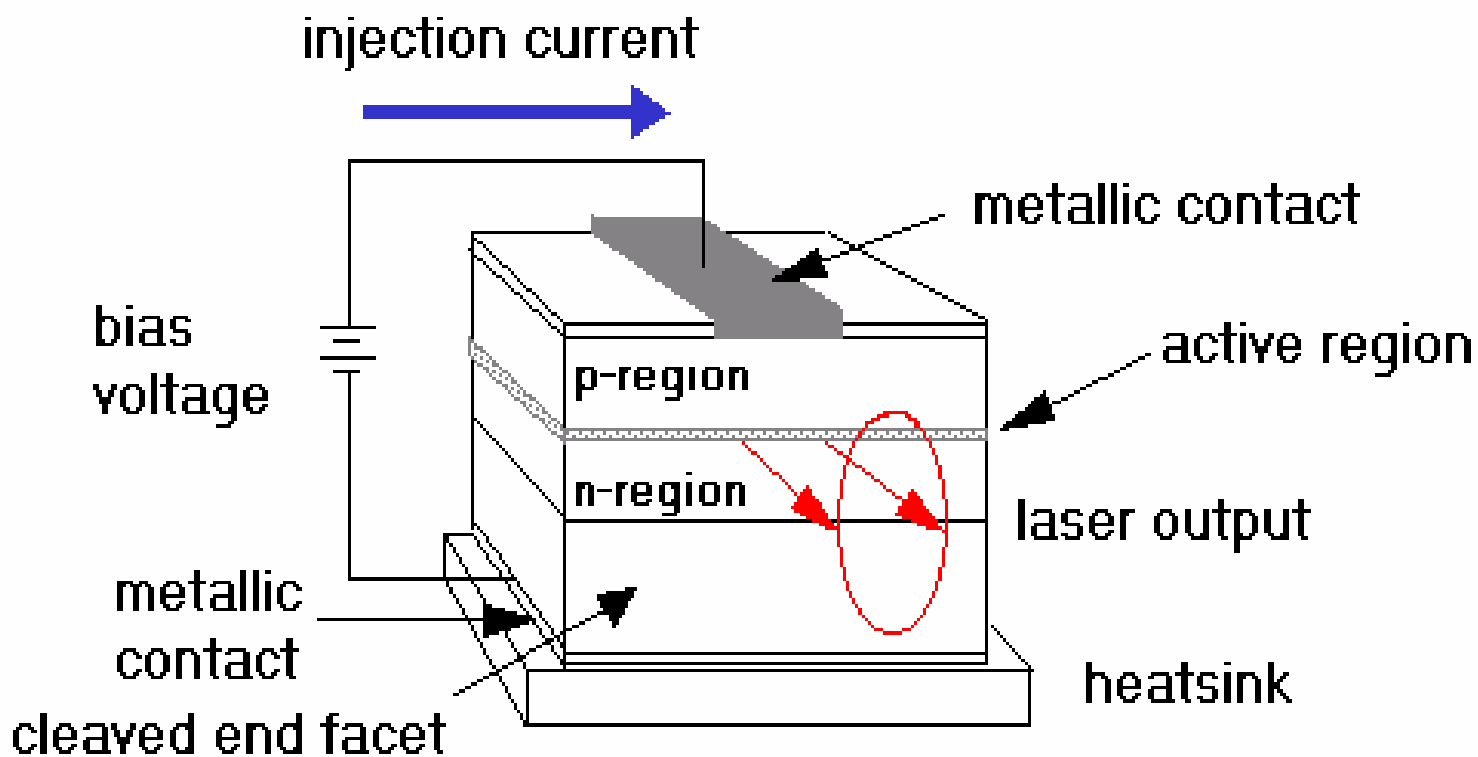
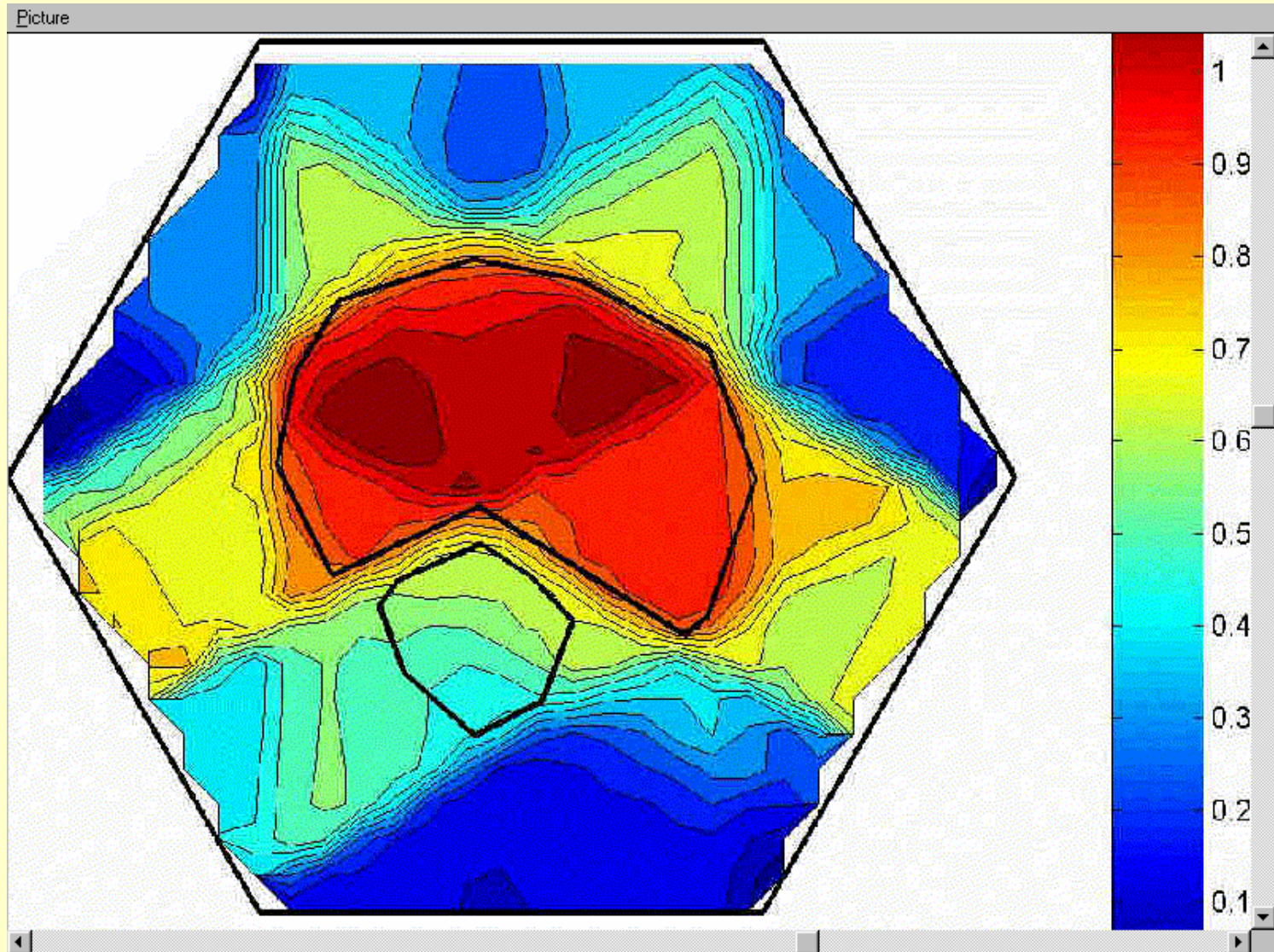


Fig. 1  
Semiconductor Laser Diode  
(simplified diagram)

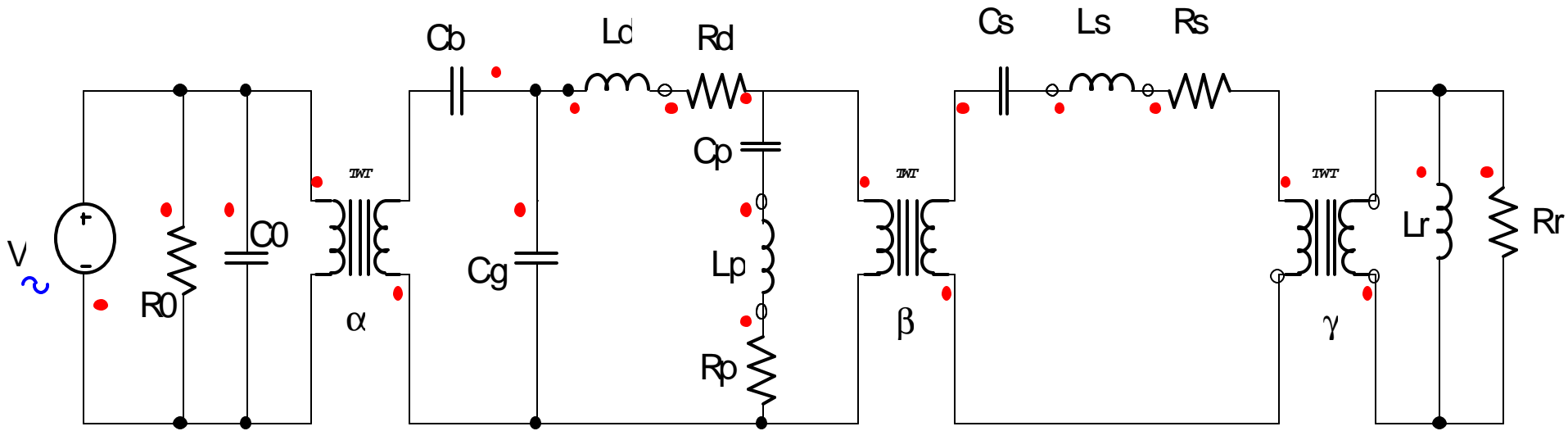


# Intensity Modulated Radiotherapy (U. of Kuopio, FI)



Globally optimized dose distribution

# Sonar Transducer Design: Numerical Model (DRDC)

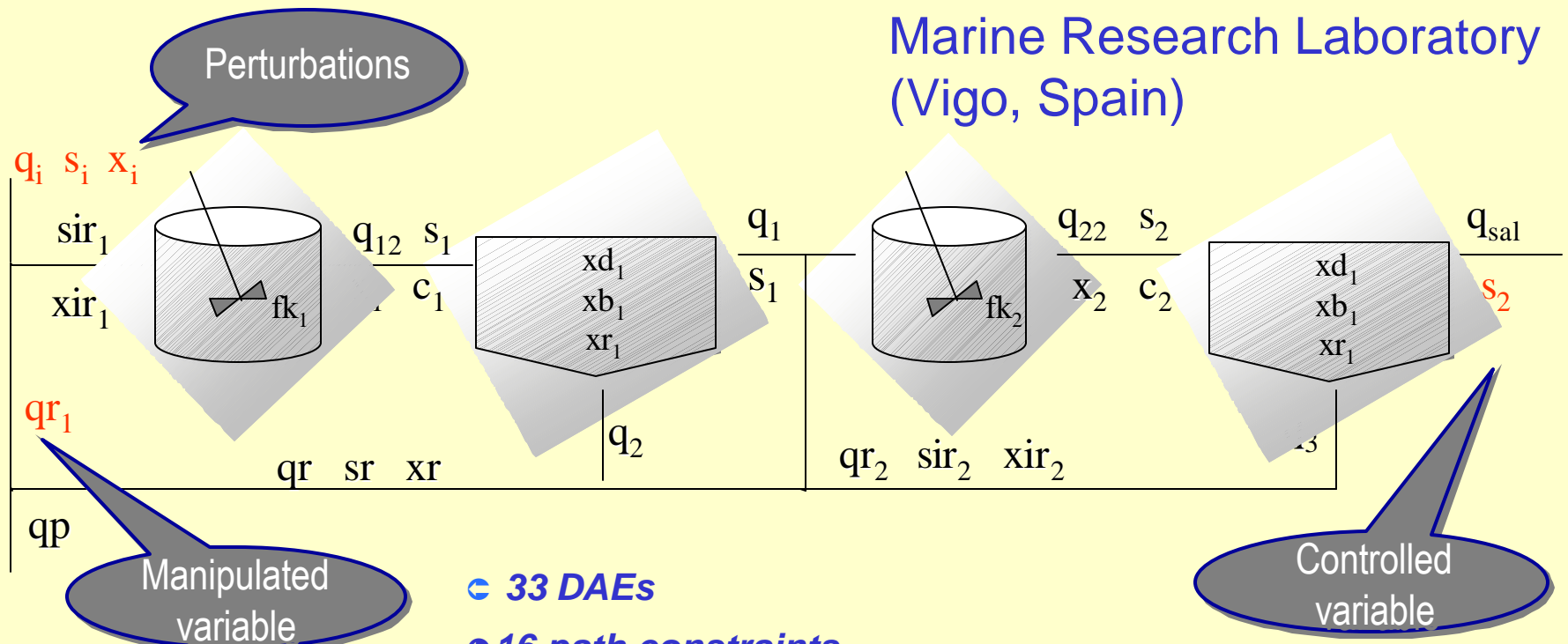


This electric circuit simulates a piezoelectric sonar projector


The optimization problem consists of finding a set of circuit design parameters such that the sonar projector gives a broad efficiency vs. frequency

This model has been solved using MathOptimizer: the results have been applied to the actual design of sonar equipment

# Integrated Design and Operation of Waste-Water Plant



- 33 DAEs
- 16 path constraints
- 30 bounds on states
- 44 states, 8 design variables

 **Objective:** to find the design of the units, the operating conditions and the parameters of the controller which minimize a weighted sum ( $C$ ) of economic terms ( $f_{econ}$ ) and a controllability measure ( $ISE$ )

This model has been analyzed and solved using an LGO DLL version

# Conclusions

- Brief Review:  
GAMS, GO, LGO  
LGO solver modes  
Illustrative applications
- Benefits of Global Optimization:  
Globally sought (hence, often improved) solutions  
Added modeling freedom: more realistic models, new possibilities and markets
- Models welcome; additional documentation available  
Workshops, consulting services offered

# Illustrative References

- Over one hundred GO books
- Kluwer AP Non-Convex Opt. And its Appls. series: 60+ volumes (2003)
- *Handbook of GO*, Vols. 1-2
- *J. of Global Optimization*
- *J. of Heuristics*
- Many thousands of research articles and reports
- ...

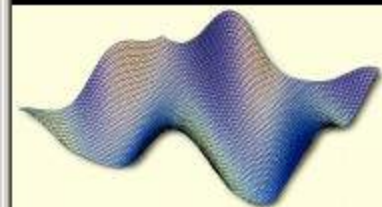
# Illustrative References (cont'd)

## Some recent work by JDP & colleagues

- Chapter 15 in *Handbook of Global Optimization*, Vol. 2
- Radiotherapy: Tervo et al; *Annals of Opns. Res.* (2003)
- Model calibration: JDP; *Opt. Methods & Software* (2003)
- Laser design: Isenor, JDP & Cada, *Opt. & Engrg.* (2003)
- MathOptimizer / MathOptimizer Pro applications:  
Kampas and JDP; *The Mathematica Journal* (to appear)
- *Global Optimization: Selected Case Studies*  
JDP, ed.; Kluwer AP (forthcoming)
- Nonlinear optimization in modeling environments (in ed. Vol.)
- Numerical GO tests: Khompatraporn, JDP & Zabinsky;  
*J. of Glob. Opt.* (to appear)
- GAMS/LGO: JDP and Pruessner (work in progress)
- MathOptimizer and ModelMaker: JDP and Purcell (w.i.p.)

...





# Nonlinear Systems Modeling and Optimization Software Development and Applications

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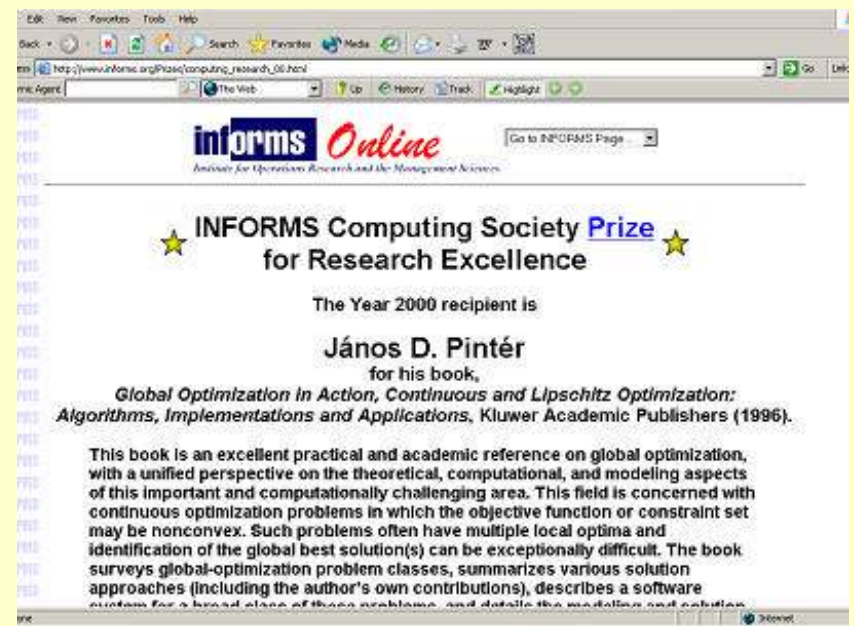
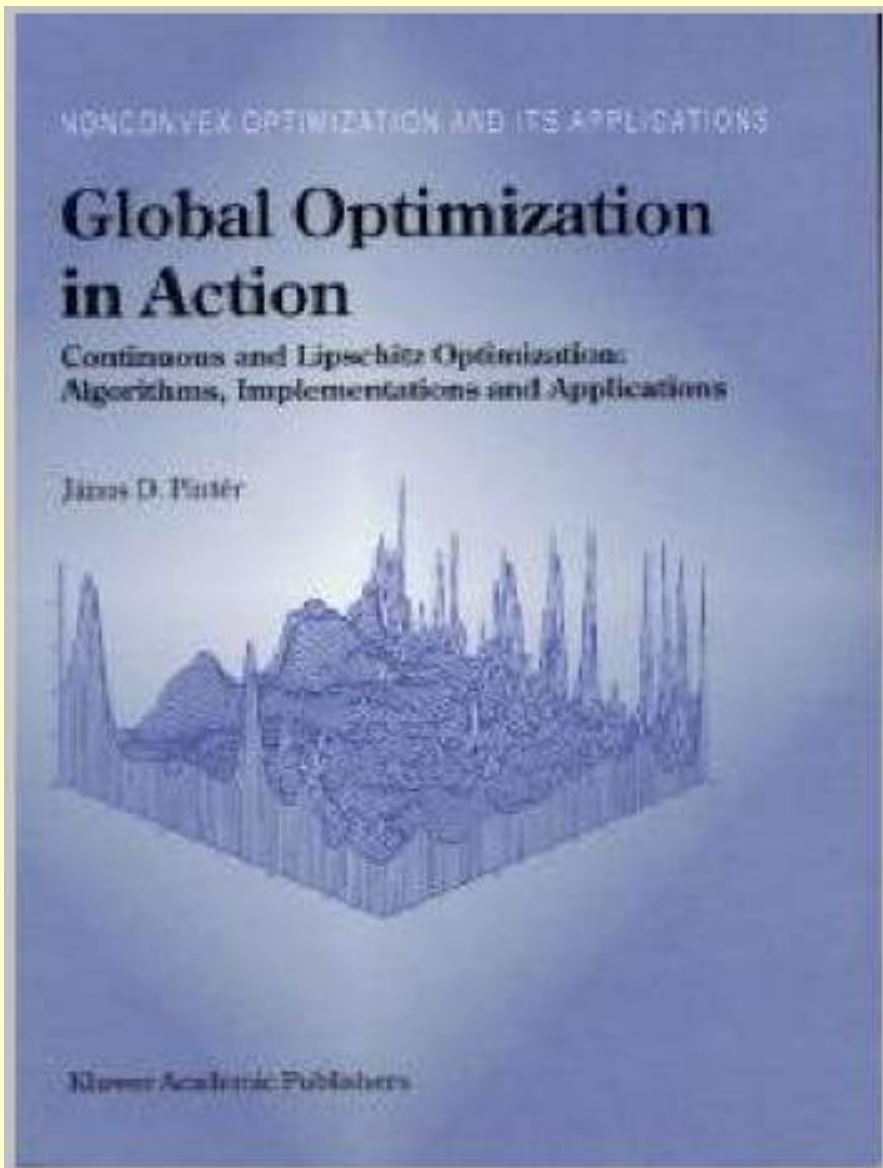
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Research monograph: theoretical background of LGO solvers; applications

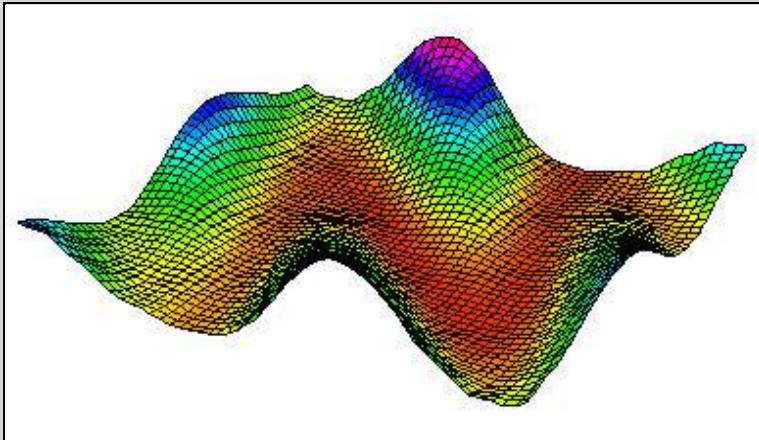


“Easygoing” e-book; GO review, includes demos of the LGO IDE



# Global Optimization

*Scientific and Engineering Applications  
with Mathematica® Examples*



János D. Pintér

ELSEVIER

SCIENCE

Joint work with  
Frank J. Kampas  
(forthcoming, 2004)

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# Thanks for your attention!

