Stochastic Optimization: Solvers and Tools

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Stochastic Optimization in the Energy Industry
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GAMS Development / GAMS Software

• Roots: Research project World Bank 1976
• Pioneer in Algebraic Modeling Systems used for economic modeling
• Went commercial in 1987
• Offices in Washington, D.C and Cologne

• Professional software tool provider
• Operating in a segmented niche market
• Broad academic & commercial user base and network
GAMS at a Glance


Design Principles:
- Balanced mix of declarative and procedural elements
- Open architecture and interfaces to other systems
- Different layers with separation of:
  - model and data
  - model and solution methods
  - model and operating system
  - model and interface
AML and Stochastic Programming (SP)

• Algebraic Modeling Languages/Systems good way to represent optimization problems
  – Algebra is a universal language
  – Hassle free use of optimization solvers
  – Simple connection to data sources (DB, Spreadsheets, …) and analytic engines (GIS, Charting, …)

• Large number of (deterministic) models in production
  – Opportunity for seamless introduction of new technology like Global Optimization, Stochastic Programming, …
  – AML potential framework for SP
Stochastic Programming Claims and ‘Facts’

• Lots of application areas (Finance, Energy, Telecommunication)
• Mature field (Dantzig ’55)
• Variety of SP problem classes with specialized solution algorithms (e.g. Bender’s Decomposition)

• Compared to deterministic mathematical programming (MP) small fraction
  • Only 0.2% of NEOS submission to SP solvers
• No/few commercially supported solvers for SP
• Various frustrations with industrial SP projects
Some Stochastic Programming Classes

SP problems

Distribution Problems
- Wait and See
- Expected Value

Recourse Problems
- Distribution-based
- Scenario-based

Chance Constrained Problems

Stochastic Measures: EVPI and VSS

Source: G. Mitra
Chance Constraints

\[
Z_{CCP} = \min cx
\]
\[
s.t. \quad A_0 x = b_0
\]
\[
P\{A_i x \geq h_i\} \geq \beta_i \quad i = 1..I, \quad \beta_i \in [0,1]
\]

• $\beta$ is the reliability level
Dynamic Representation

Expected Value

<table>
<thead>
<tr>
<th>Two-Stage SP</th>
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| Wait and See |

| Multi-Stage SP |

Source: G. Mitra
Two-Stage SLP with recourse

\[
\min \quad z = cx + E f^\omega y^\omega \\
\text{s.t.} \quad Ax - B^\omega x + D^\omega y^\omega = b \\
\quad y^\omega \geq 0, \quad \omega \in \Omega
\]

- \( x \) first-stage decision variables
- \( y^w \) second-stage decision variables
- \( c, f^w \) objective coefficients
- \( A, b \) first-stage coefficients
- \( B^w, D^w, d^w \) second-stage random parameters
Multi-Stage SLP with recourse

\[
Z_{HN} = \min_{x_1} \left\{ c_1 x_1 + E_{\xi_2} \left[ \min_{x_2} c_2 x_2 + E_{\xi_3|\xi_2} \left[ \min_{x_3} c_3 x_3 + \ldots + E_{\xi_T|\xi_{T-1}|\ldots|\xi_2} \min_{x_T} c_T x_T \right]\right]\right\}
\]

subject to:

\[
A_{11} x_1 + A_{21} x_2 + \ldots + A_{T1} x_T = b_1
\]

\[
A_{21} x_1 + A_{22} x_2 + A_{31} x_1 + A_{32} x_2 + A_{33} x_3 + \ldots + A_{TT} x_T = b_2
\]

\[
\vdots
\]

\[
A_{T1} x_1 + A_{T2} x_2 + \ldots + A_{TT} x_T = b_T
\]

\[
\ell_t \leq x_t \leq u_t,
\]
Discrete distribution (Two-stage SLP)

\[
\begin{align*}
\min \ z &= s/t \\
&= cx + p^1 f y^1 + p^2 f y^2 + \cdots + p^W f y^W \\
&\quad - B^1 x + Dy^1 \\
&\quad - B^2 x + Dy^2 \\
&\quad \vdots \\
&\quad - B^W x + Dy^W = b \\
x, \quad y^1, \quad y^2, \quad \ldots, \quad y^W &\geq 0, \\
\end{align*}
\]
Deterministic Equivalent (DE)

- Discrete Distribution/Scenarios
- Standard (Mixed Integer Nonlinear) Linear Program with block structure (opportunity for special purpose algorithms)
- Non-anticipativity (or locking) constraints
  - Explicit by new constraints (Split-variable representation maintains block structure)
  - Implicit through variable substitution
- Size equivalent to number of possible realizations (i.e. exponential in the number of random variables)
- Size of LP can quickly explode
Personalized Software Overview

• SP Solvers
  – Standard LP Solvers
  – SP solvers based on decomposition
• Modeling Tools
  – SMPS (core, time, stochastic)
  – Extension to AML
  – APIs
• Others
  – Scenario (tree) management
  – Visualization of results
Stochastic Programming Solvers

• LP solver
  – Interior point methods seem to be better than simplex
• All other ready to use solver (e.g. NEOS) for $n$-stage SP with recourse:
  – DECIS, Infanger (2-stage)
  – FortSP, OptiRisk
  – OSL/SE, IBM (discontinued)
  – Academic codes: MSLiP (Gassmann), bnbs (Altenstedt), ddsip (Schultz et. al) (2-stage MIP), …
• SLP-IOR (Mayer/Kall) started in 1992
  – Model management system for SLP
  – Rich solver library (21) for 18 SP model types
DECIS

- Solves two-stage stochastic linear programs with recourse
- Two-stage decomposition (Benders)
- Stores only one instance of the problem and generates scenario sub-problems as needed
- LP engine for sub-problems CPLEX or MINOS
- Solution Strategies
  - Universe problem (all scenarios)
  - Sampling: Crude Monte Carlo/Importance sampling
SP Modeling Tools

- Stochastic MPS
  - make it easy to convert existing deterministic LP into SLP
  - Add information about dynamic and stochastic structure.
- Core file (deterministic problem)
- Time file (map core file dynamic structure into stages)
- Stoch file (information about the random variables)
- SMPS format is extremely flexible exceeding the capability of any existing stochastic programming solver
- Difficult for a human to manage
SP Modeling Tools cont.

• Algebraic Modeling Languages
  – Similar to SMPS:
    • Supply time/stage map
    • Supply stochastic information
  – Integrated in language: AIMMS, GAMS, Mosel
  – Extensions: SAMPL, SMPL, StAMPL

• API
  – OSL/SE (discontinued)
  – COIN-OR Stochastic Modeling Interface (SMI)
    [https://projects.coin-or.org/Smi](https://projects.coin-or.org/Smi)

• Gear towards generating and solving DE
MPL

INDEX I = 1..7;

DECISION
    h[I,T];
    b[I,T];
    d[I,T];
    c[T];

DATA
    C0 = 50000;    ! Initial amount of cash
    U = 10;       ! Coupon
    G = 0.005;    ! Transaction cost rate
    V = 0.15;    ! Diversification limit

MODEL
    obj = max SUM(T=4: c[T]);

SUBJECT TO
    BAL[T,I] WHERE (T<4) AND (I<7):

    BAL[T,I] WHERE (T=4) AND (I<7):
        H[I,T] = H[I+1,T-1];

....

AMPL

set I := 1 .. 7;

var h{i in I, t in T} >=0;
var b{i in I, t in T} >=0;
var d{i in I, t in T} >=0;
var c{t in T} >=0;

param C0 := 50000;  # Initial amount of cash
param U := 10;      # Coupon
param G := 0.005;   # Transaction cost rate
param V := 0.15;    # Diversification limit

maximize obj: sum {t in T: t=4} c[t];

subject to bal{t in T: t< 4,i in I: i <7}:
    h[i,t] = h[i+1, t-1] + b[i,t] - d[i,t];

subject to bal {t in T: t<4,i in I: i = 7}:
    h[i,t] = b[i,t];

subject to bal{t in T: t=4,i in I: i <7}:
    h[i,t] = h[i+1,t-1];
TIME
  T := 1..4;

SCENARIO
  Scen:= 1..8;

PROBABILITIES
  Prob[Scen]:= FILE("ScenProb.pro");

RANDOM DATA
  PR[T,I,Scen] := DATABASE("tbl_Prices","Price");

STAGES:
  AGGREGATION:
    1: 1..1;
    2: 2..2;
    3: 3..3;
    4: 4..4;

TREE
  BINARY;

SAMPL

set T := 4;

scenario set Scen:= 1..8;

probability param Prob{Scen};

random param PR{T, I, Scen};

stages theStages :=

  1  {t IN T: t=1}
  2  {t IN T: t=2}
  3  {t IN T: t=3}
  4  {t IN T: t=4};

tree theTree:= twostage(2);
Other Tools and Frameworks

- Scenario (tree) management
  - Generation
    - From random variables to scenarios
    - SAMPL, SMPL, AIMMS, Mosel tools for tree generation
  - Reduction
    - ScenRed, (Römisch et. al.)
- Framework
  - SLP-IOR
  - SPInE (Stochastic Programming Integrated Environment), OptiRisk
    - Special purpose scenario generators
    - Connection to SAMPL and SMPL
    - FortSP SP solver
ScenRed (Römisch et. al.)

- Find good approximation of original scenario tree of significant smaller size.
Observations

• Skills for SP Application
  – Application domain expert
  – Modeling expert
  – Statistic/Stochastic expert (scenario generation)
  – Algorithmic expert

• Focus has shifted from solving SP to represent/model SP

• Little progress in specialized solver technology

• GUPOR: *Large Scale Stochastic Programming: Still an unsolved problem in OR* (Birge, 2006)
GAMS Approach to SP

• Emphasis on performance:
  – Support 64bit OS to overcome memory issues
  – Efficient out-of-core solver runs
  – Reduce SP problem size (GAMS/ScenRed)
  – Make professional special purpose SP solvers available
  • GAMS/DECIS (GAMS/OSL-SE)
    – Utilize emerging grid computing technology
    – High performance data exchange
• Minor modifications to GAMS language
  – Emphasize functionality over beauty
Wait-and-see Model

- Independent problems → Solve in parallel
- GAMS Grid Facility
  - Supports SMP and various grid engines

A pool of connected computers managed and available as a common computing resource

- Effective sharing of CPU power
- Massive parallel task execution
- Scheduler handles management tasks
- E.g. Condor, Sun N6 Grid Engine, Globus
- Can be rented or owned in common
Examples

- Too few (lots of repeats)
- Too simple
- Difficult to reproduce
- Important steps in example missing
  - Creating stochastic data/scenarios
  - Analyzing results (vast amount of data)

- Power Expansion Problem (GAMS/DECIS, AIMMS)
  - Simple, small, demand scenarios given
  - But at least reproducible
Power Expansion Problem

- Determine capacity expansion for different power plants (coal, hydro, nuclear) based on uncertain demand
- Cost structure: capital cost + operating cost
- Demand given as load block:
  - Additional constraint: No nuclear in peak.
  - Recourse: Purchase electricity
Conclusion

• Stochastic Programming still challenging and developing field
  – GUPOR: *Uncertainty: An OR Frontier* (Greenberg, 2006)
• Lack of solution technology for \( n \)-stage SLP limits the dissemination of SP
• Reduce the skill level for SP applications
• Collection of comprehensive & reproducible examples could help to *spread the word*
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