A Student-centric Class and Exam Scheduling System at West Point

Monique Guignard
University of Pennsylvania
The Wharton School

Michael Bussieck
Alex Meeraus
GAMS Development Corp.

Siqun Wang
Singapore Management University

Fred O'Brien
University Apps Inc

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Change in Focus

Computation – Past

- **Algorithm limits application**
- Problem representation is low priority
- Large costly projects
- Long development times
- Centralized expert groups
- High computational cost, mainframes
- **Users left out**

Model – Present

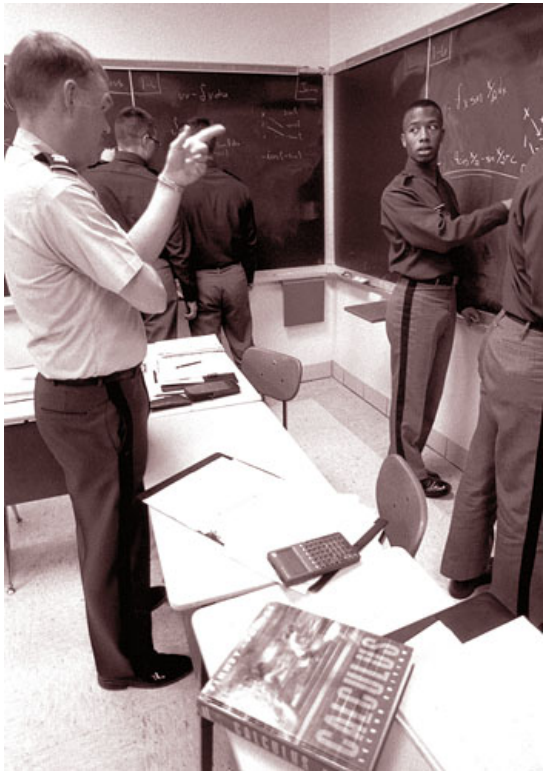
- **Modeling skill limits applications**
- Algebraic model representation
- Smaller projects
- Rapid development
- Decentralized modeling teams
- Low computational cost, workstations
- Machine independence
- **Users involved**

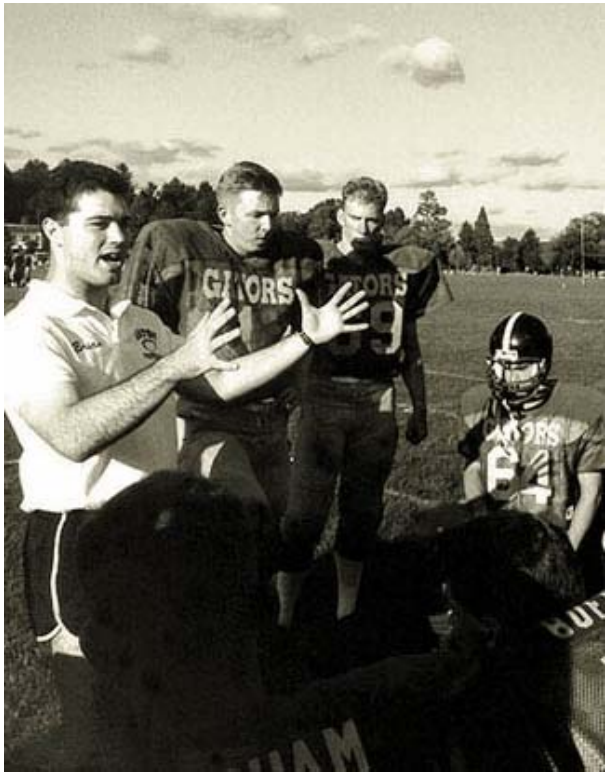
Application – Future

- **Domain expertise limits application**
- Off-the-shelf graphical user interfaces
- Links to other types of models
- Models embedded in business applications
- Internet/web
- **Users hardly aware of model**

Scheduling US Military Academy West Point

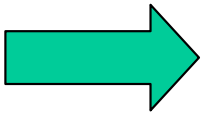
" … each student's daily activities are a carefully regimented balance of academic, military, and physical requirements."





USMA is Different

- Technically
  - Day1/day2 schedule
  - Special rules (e.g. < 30% athletes in class)
  - Sufficient number of rooms, teachers, …
- Scheduling around the cadets needs
  - No conflicting activities
  - Individual schedule of activities is compliant to vast catalogue of *business rules*



Software evaluation did not find an “off the shelf” product that could handle USMA requirements

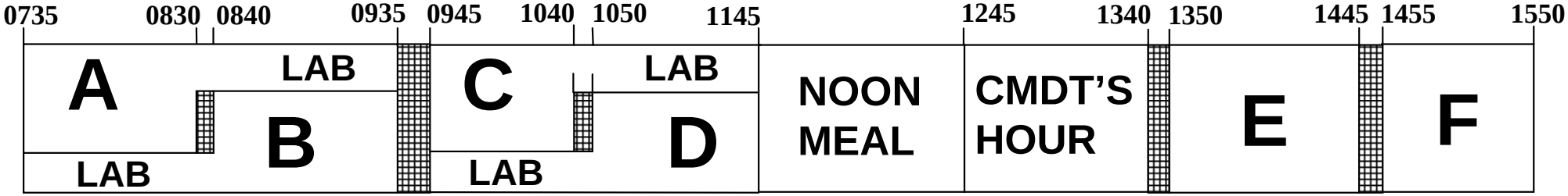
2 Day Schedule

OFFICE OF THE DEAN
UNITED STATES MILITARY ACADEMY

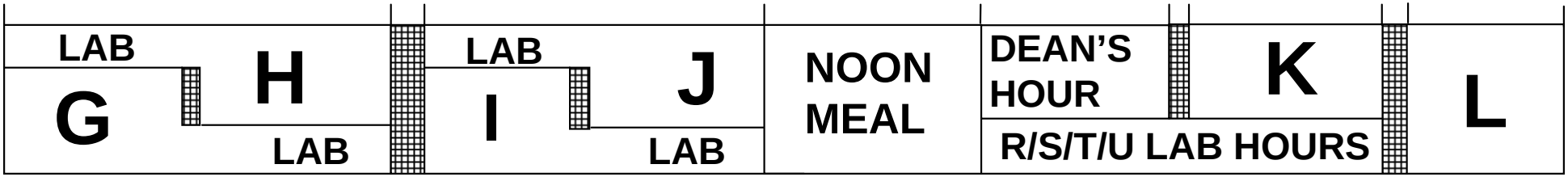
AUGUST 1999
(EDITION OF AUG 98 IS OBSOLETE)

ATTENDANCE PERIODS

1-DAY CLASS PERIODS



2-DAY CLASS PERIODS



Academic Scheduling

- Course scheduling
  - For a given set of *course offerings* find *good* schedules for all cadets.
- Term End Exam (TEE) scheduling
  - Scheduling preparation
  - Find *good* schedules for exam courses **and** cadets.

Course Scheduling

- Given course hours & capacity

MA481,AB,36	MA481,CD,18	MA481,EF,18
PE300,C,180	PE300,J,60	MA371,F,18

- Given cadet's course registration

```
043671XXX,MA481
043671XXX,PE300
```

- Objective: Find a *good* assignment of cadet's course requests to course hours

```
043671XXX,MA481,CD
043671XXX,PE300,J
```

Problems with a Model

- There is no solution subject to *all* constraints/rules for real data
- Infeasibilities
  - Individual Cadet Infeasibilities
  - System Infeasibility (e.g. Capacity)
- *Goal Programming:*
  - Relax constraints/rules by penalizing violations
  - How to Select penalties for constraint violations
  - Penalty depend on individual Cadet

An Optimization Model

$$\min \sum_{ro}(p1_{ro} * \pi 1_{ro} + p2_{ro} * \pi 2_{ro}) + \sum_{c}(p3_c * \pi 3_c + p4_c * \pi 4_c)$$

$$\sum_{o} x_{c,ro} = 1 \quad \text{(for all 8TAP entries)}$$

$$\sum_{r} x_{c,ro} \leq 1 + \pi 3_c \quad \text{(for all cadets } c \text{ for all time slots } o\text{)}$$

$$-\sigma - \pi 4_c \leq \sum_{ro \text{ on day-1}} x_{c,ro} - \sum_{ro \text{ on day-2}} x_{c,ro} \leq \sigma + \pi 4_c \quad \text{(for all cadets } c\text{)}$$

$$x_{c,ro} = 0 \quad \text{(for all } c, ro \text{ where } c \text{ has activity at } o\text{)}$$

$$\sum_{c} x_{c,ro} \leq cap_{ro} + \pi 1_{ro} \quad \text{(for all course hours } ro\text{)}$$

$$\sum_{c \text{ freshman\&athlete}} x_{c,ro} - 0.6 \sum_{c} x_{c,ro} \leq \pi 2_{ro} \quad \text{(for all course hours } ro\text{)}$$

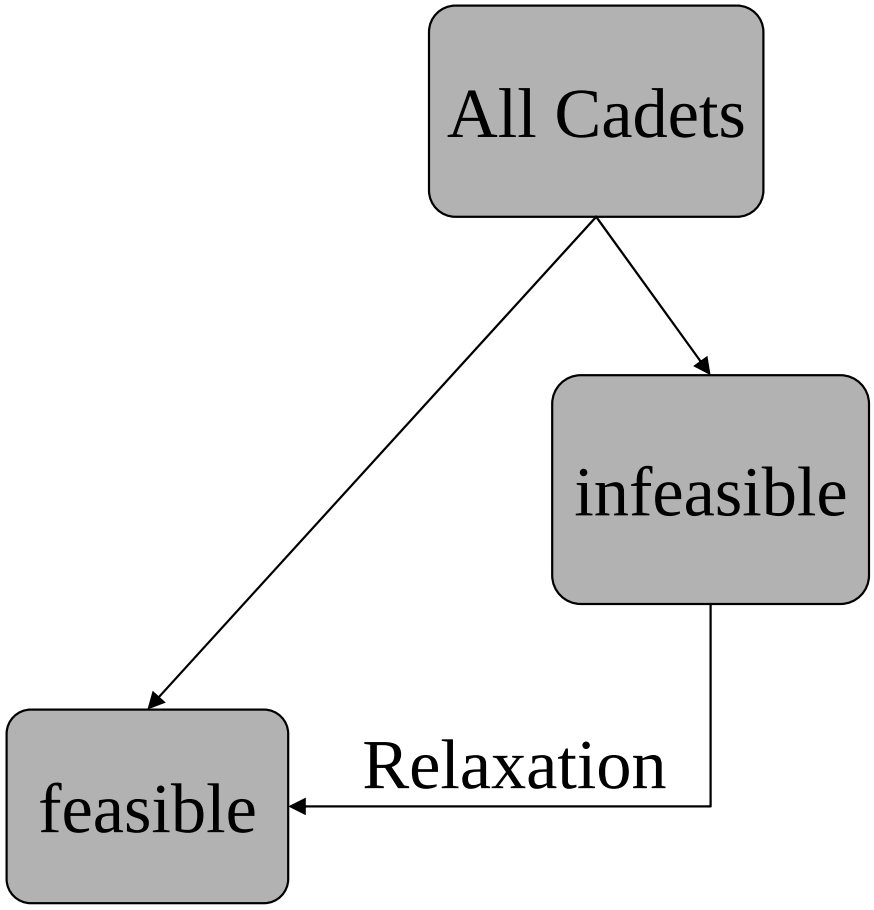
- 60,000 Variables, 500,000 Non-Zeros
- 24 hours CPLEX 6.6 and no integer solution

Decomposition

- Pre-Scheduling
  - Filter cadets with no feasible schedule
  - Overcome infeasibility by relaxation/data changes
- Scheduling
  - All individual constraints/rules are hard constraints
  - Find assignment that does not exceed capacity (or penalize overloads)

Pre-Scheduling

- One cadet at a time
  - Check feasibility
  - If infeasible produce several infeasible schedules ranked by severeness of infeasibility
    - Hour Conflict
    - Day – Day Balance
    - Last Hour Free
  - Human Intervenes
- Thousands of small MIPs



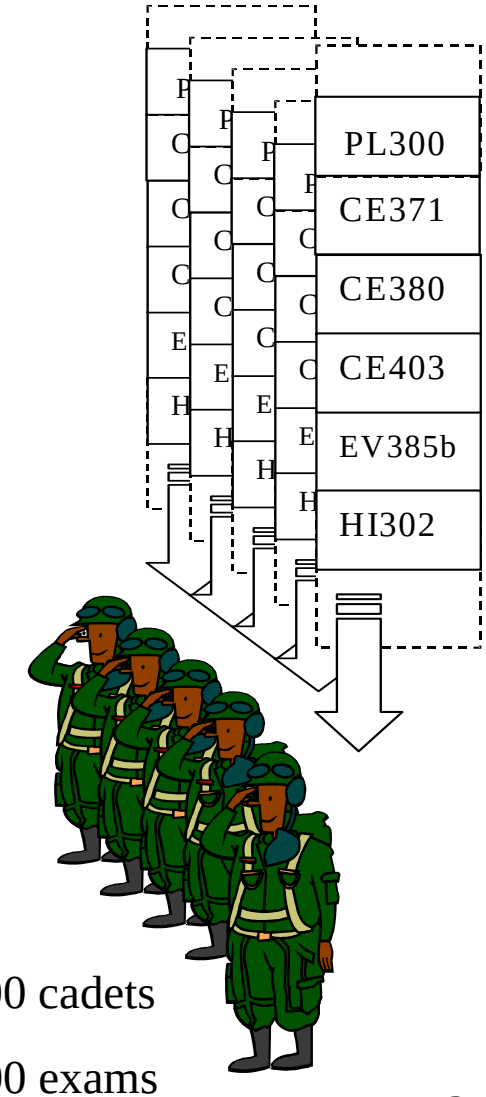
Results

- AY 2000/2 parallel tested
- AY 2001/1 deployed

	Legacy System + human deconflicter	New System
Individual Relaxations	**203/304/116**	**58/25/4**
Capacity Overloads	**12/54**	**9/21**
Number of Schedulers	**3**	**1**
Time to produce Schedule	**4 Weeks**	**1 Day**

Term End Exam Scheduling

	1	2		6
morning period	CE371 CH384 CS383 HI366	CH101 CS408 EE301 EN302		EV203 PH203 PL300 LR204
afternoon period	CE404 LG484 LS362 MS350	LF382 SE388 SS388 …		CE403 CS380 SS201 …



Term End Exam Courses (≈250 courses)

≈ 4000 cadets

≈20000 exams

Overcoming Conflicts

- Schedule with conflicts

1	2		6
CE371 CH384 CS383 HI366	CH101 CS408 EE301 EN302		EV203 PH203 PL300 LR204
CE404 LG484 LS362 MS350	LF382 SE388 SS388 CE403		CE403 CS380 SS201 …

Cadet's 8TAP:

PL300
CE372
CE403
CS380
EV180
HI302

- Makeup/ahead for an exam course:
  - An additional exam offering for a small group of cadets who can not go to the primary exam offering
- Resolve conflicts by adding makeup/ahead

TEE Scheduling

- Given exam courses
  - **MA481 CE371 CH100 ...**
- Given exam periods
  - **p1, p2, p3, ... p12**
- Given cadet's exam course '*requests*'
  - **043671571,CE403**
  - **043671571,CE380**
- Find an assignment of exam course sessions (primaries, makeups) to periods and cadet's requests to exam courses sessions.
  - **CE403,prim,p12 CE403,mkup,p4 CE380,prim,p4 ...**
  - **043671571,CE403,p4 043671571,CE380,p4 ...**
- Objective: Minimize the total number of makeups

An Optimization Model

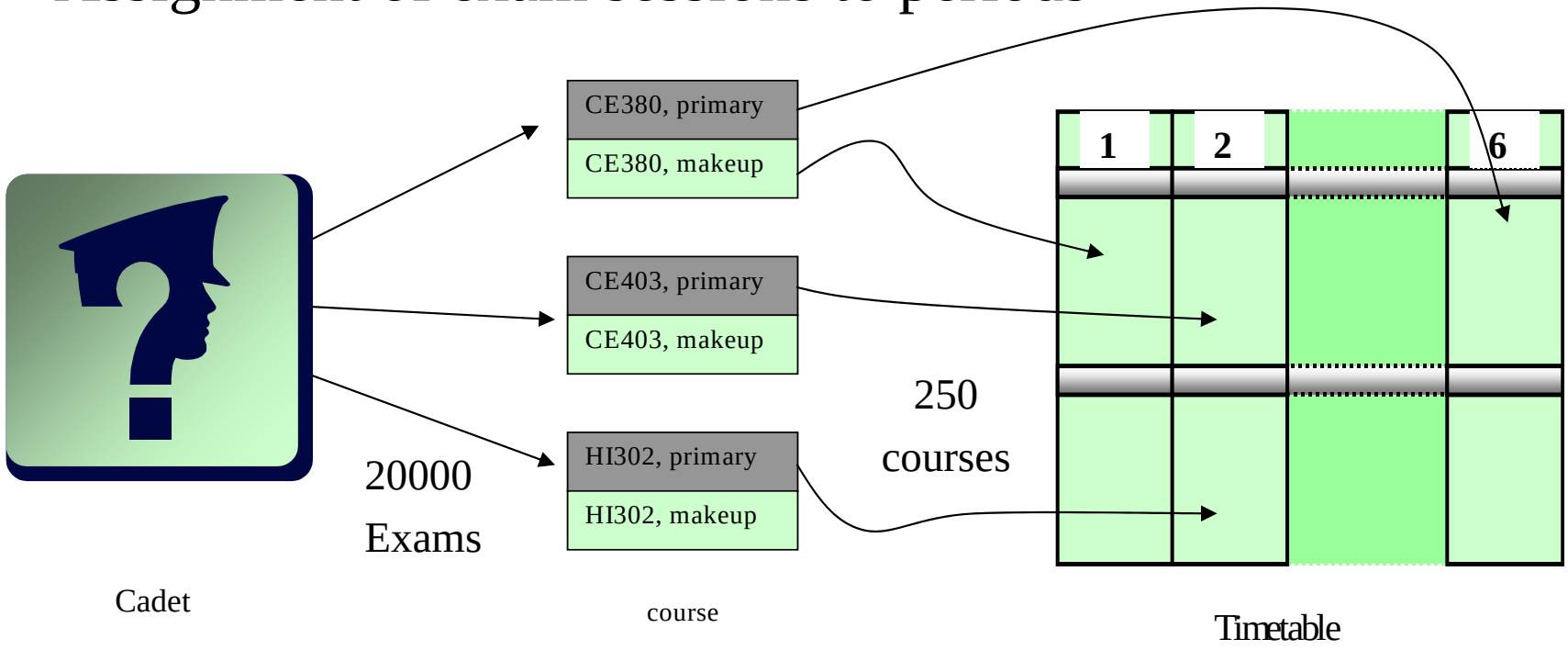
- Variables
  - `x(c,r,p)` urse/period **250.000**
  - `y(r,s,p)` d **|S|*3.000**
  - `z(r,p)` prim **3.000**
- Constraints
  - `Conflict` **50.000**
  - `Assign` **20.000**
  - `PrimEnroll` $\sum_c x(c,r,p) \geq 0.75$ $p) = 1$ **3.250**
  - `Consecutive` **36.000**
  - `Exams per day` **6.000**
  - `Inclusive` **T|*12**
  - `Exclusive` **|*12**
  - `Fixed,Prohibit,No makeup,Finishe` ble fixing
  - Coupling of `x` and `y` $\sum_c x(c,r,p) \leq enroll(r) \cdot y(r,s,p)$ **|S|*3.000**

Solution Approach

- Heuristic based on a collection of medium sized optimization models produces conflict free schedules and automatically relaxes constraints.
- Improvement module starts with a good/ mediocre solution and a set of relaxed constraints and tries to
  - Improve number of makeups
  - Reinforce relaxed constraints

Solution Improvement

- Decompose the problem
  - Assignment of cadet request to exam course session
  - Assignment of exam sessions to periods



- Given a feasible schedule – iterate until no progress

Feasibility Study

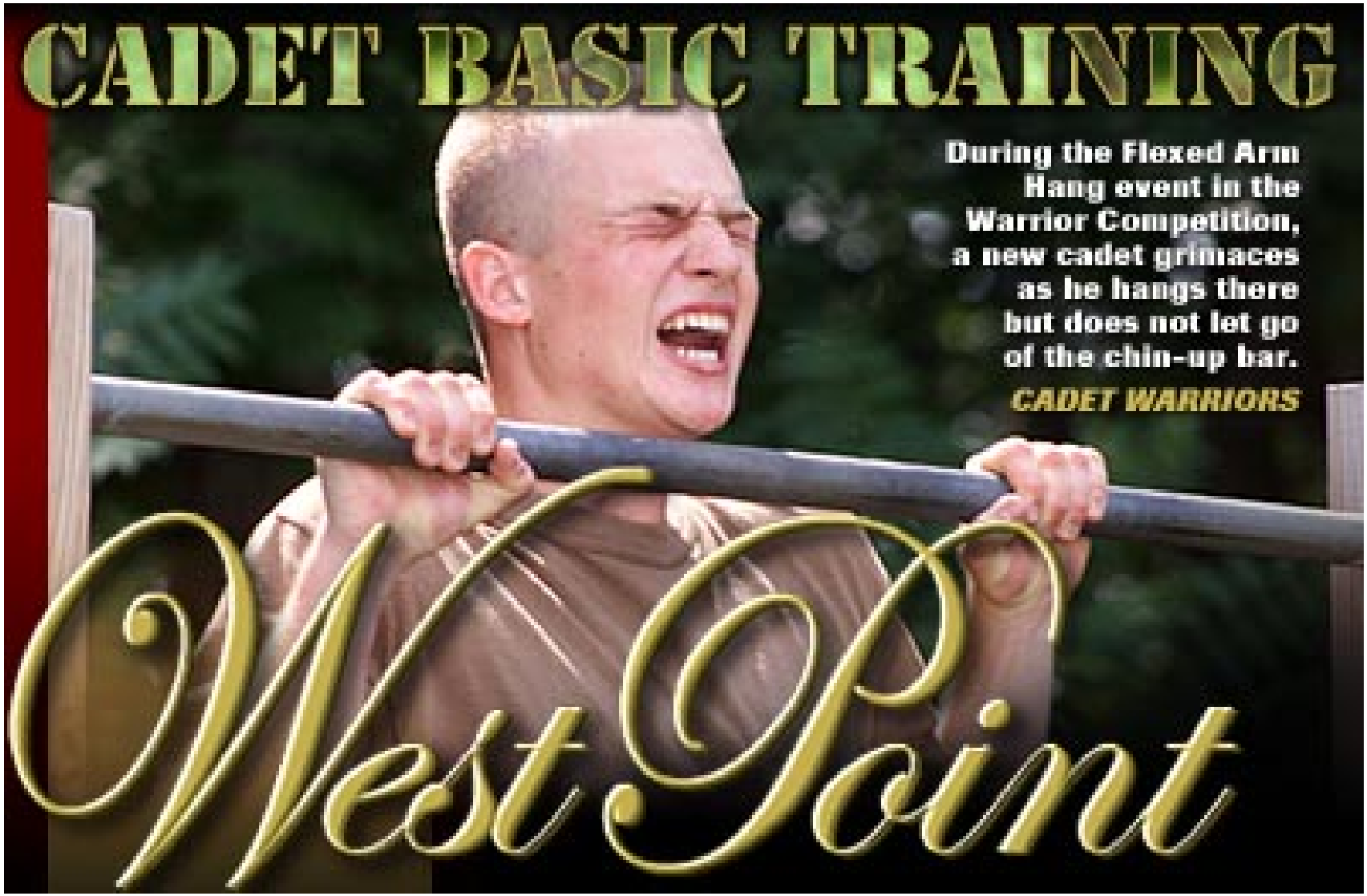
- TEE last application of legacy system
  - Mainframe, Cobol, ~1980
  - Maintenance + on-site personnel: $500,000/year
- By March 2001: decision for renewal
- TEE Schedule for AY2001/2 (End of May 2001)
  - Chuck + Legacy system
    - Partial schedule, approx. 90 makeups (4 Weeks)
  - Chuck + GAMS TEE scheduler
    - Complete schedule, no conflicts, 60 makeups (10 minutes)
    - The improver module produced schedule with 40 makeups

More Computational Results

- Three data sets 01/2,02/1(early),02/1
- Constraint violations ‘OK’

Year	Courses	Periods	Requests	Makeups
01/2	226	12	18937	38
02/1 early	213	12	18512	49
02/1	252	11	21175	61

Before


CADET BASIC TRAINING
During the Flexed Arm Hang event in the Warrior Competition, a new cadet grimaces as he hangs there but does not let go of the chin-up bar.
CADET WARRIORS
West Point

After

Conclusions

- Two Student-Centric Scheduling Problems
  - Course Scheduling
  - TEE scheduling
- Math. Programming Approaches
- Successful Applications
- Running at USMA without model changes for several years (changes in hardware, interface, newer solver versions, …)