Pre-Conference Workshops

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Outline

Part I: An Introduction to GAMS

Part II: Stochastic programming in GAMS

Part III: The GAMS Object-Oriented API's

Part IV: Code embedding in GAMS
Stochastic Programming - Introduction

Stochastic programs are mathematical programs that involve uncertain data.

Motivation:
Real world problems frequently include some uncertain parameters. Often these uncertain parameters follow a probability distribution that is known or can be estimated.

Goal:
Find some policy that is feasible for all (or almost all) the possible data instances and that maximizes the expectation of some function of the decision variables and the random variables.

Example:
In a two-stage stochastic programming problem with recourse the decision maker has to make a decision now and then minimize the expected costs of the consequences of that decision.
Simple Example: Newsboy (NB) Problem

• Data:
  • A newsboy faces a certain demand for newspapers
    \( d = 63 \)
  • He can buy newspapers for fixed costs per unit
    \( c = 30 \)
  • He can sell newspapers for a fixed price
    \( v = 60 \)
  • For leftovers he has to pay holding costs per unit
    \( h = 10 \)
  • He has to satisfy his customers demand or has to pay a penalty
    \( p = 5 \)

• Decisions:
  • How many newspapers should he buy: \( X \) 63
  • How many newspapers should he sell: \( S \) 63

• Derived Outcomes:
  • How many newspapers need to be disposed: \( I \) 0
  • How many customers are lost: \( L \) 0
**Simple NB Problem – GAMS Formulation**

**Variable** Z Profit;
**Positive Variables**
  X Units bought
  I Inventory
  L Lost sales
  S Units sold;

**Equations** Row1, Row2, Profit;

* demand = UnitsSold + LostSales
Row1.. d =e= S + L;

* Inventory = UnitsBought - UnitsSold
Row2.. I =e= X - S;

* Profit, to be maximized;
Profit.. Z =e= v*S - c*X - h*I - p*L;

**Model** nb / all /;

**Solve** nb max z use lp;

→ nbsimple.gms
NB Problem – Add Uncertainty

• Uncertain demand $d$

• Decisions to make:
  • How much newspaper should he buy “here and now” (without knowing the outcome of the uncertain demand)?
    → First-stage decision

  • How many newspapers are sold?
  • How many customers are lost after the outcome becomes known?
  • How many unsold newspapers go to the inventory?
    → Second-stage or recourse decisions
  • Recourse decisions can be seen as
    • penalties for bad first-stage decisions
    • variables to keep the problem feasible
Stochastic NB Problem – GAMS Extension

Idea:

Use deterministic model formulation plus some annotation to define uncertainty.

Parameter | Probability | Value of $d$
--- | --- | ---
randvar $d$ discrete

0.7 | 45
0.2 | 40
0.1 | 50

stage 2 I L S $d$

stage 2 Row1 Row2

Make demand $d$ uncertain

Define (non-default) stage 2 variables and equations
file emp / '%emp.info%' /; put emp '* problem %gams.i%'/;

$onput
randvar d discrete 0.7 45
  0.2 40
  0.1 50
stage 2 I L S d
stage 2 Row1 Row2
$offput
putclose emp;

Syntax to write an EMP info file on the fly, e.g. [...]\225a\empinfo.dat

EMP, what? → Excursus
The EMP Framework

EMP stands for Extended Mathematical Programming
Dictionary with output-handling information

- The expected value of the solution can be accessed via the regular .L (and .M) fields.
- Additional information can be stored in a parameter by scenario, e.g.:
  - **level**: Levels of variables or equations
  - **randvar**: Realization of a random variable
  - **opt**: Probability of each scenario

- This needs to be stored in a separate dictionary:

```plaintext
Set scen  Scenarios / s1*s3 /;
Parameter
  s_d(scen) Demand realization by scenario
  s_x(scen) Units bought by scenario
  s_s(scen) Units sold by scenario
  s_o(scen,*) scenario probability / #scen.prob 0 /;

Set dict / scen .scenario.''
  d .randvar .s_d
  s .level .s_s
  x .level .s_x
  '' .opt .s_o /;

solve nb max z use emp scenario dict;
```
3 parts of a GAMS EMP stochastic model

1. The deterministic core model
2. EMP annotations in EMP info file
3. The dictionary with output-handling information
Extensions to the Simple NB Problem

- Multiple stages:

\[
\text{stage stageNo rv | equ | var } \{\text{rv | equ | var}\}
\]

- **StageNo** defines the stage number
- The default **StageNo** for the objective variable and objective equation is the highest stage mentioned
- The default **StageNo** for all the other random variables, equations and variables not mentioned is 1

- Several probability distributions for random variables:
  - Discrete distributions:
    \[
    \text{randVar rv discrete prob val } \{\text{prob val}\}
    \]
  - Continuous distributions: normal, binomial, exponential, ...
    \[
    \text{randVar rv distr par } \{\text{par}\}
    \]
    \[
    \text{sample rv } \{\text{rv}\} \text{ sampleSize}
    \]

- Joint Random variables:
Independent vs. Joint Random Variables

Demand
- Prob: 0.2
  - d: 40
- Prob: 0.7
  - d: 45
- Prob: 0.1
  - d: 50

Price
- Prob: 0.2
  - p: 55
- Prob: 0.7
  - p: 60
- Prob: 0.1
  - p: 65

Demand / Price
- Prob: 0.04
  - d: 40 / p: 55
- Prob: 0.14
  - d: 40 / p: 60
- Prob: 0.02
  - d: 40 / p: 65
- Prob: 0.14
  - d: 45 / p: 55
- Prob: 0.49
  - d: 45 / p: 60
- Prob: 0.07
  - d: 45 / p: 65
- Prob: 0.02
  - d: 50 / p: 55
- Prob: 0.02
  - d: 50 / p: 60
- Prob: 0.07
  - d: 50 / p: 65
- Prob: 0.01
  - d: 50 / p: 65

Demand / Price
- Prob: 0.2
  - d: 40 / p: 55
- Prob: 0.7
  - d: 45 / p: 60
- Prob: 0.1
  - d: 50 / p: 65
Extensions to the Simple NB Problem

- Multiple stages:
  
  ```gams
  stage stageNo rv | equ | var {rv | equ | var}
  ```
  
  - `StageNo` defines the stage number
  - The default `StageNo` for the objective variable and objective equation is the highest stage mentioned
  - The default `StageNo` for all the other random variables, equations and variables not mentioned is 1

- Several probability distributions for random variables:
  
  - Discrete distributions:
    ```gams
    randVar rv discrete prob val {prob val}
    ```
  
  - Continuous distributions: normal, binomial, exponential, ...
    ```gams
    randVar rv distr par {par}
    sample rv {rv} sampleSize
    ```

- Joint Random variables:
  ```gams
  jRandVar rv rv {rv} prob val val {val}
  {prob val val {val}}
  ```
OBJ.. Z =e= X1 + X2;
E1.. om1*X1 + X2 =g= 7;
E2.. om2*X1 + 3*X2 =g= 12;
Model sc / all /;
solve sc min z use lp;

chance E1 0.6
chance E2 0.6
Chance Constraints with EMP

3 out of 4 must be true

\[ 0.75 \geq 0.6 \]

- \( 1 \times X_1 + X_2 = g = 7; \)
- \( 2 \times X_1 + X_2 = g = 7; \)
- \( 3 \times X_1 + X_2 = g = 7; \)
- \( 4 \times X_1 + X_2 = g = 7; \)

2 out of 3 must be true

\[ 0.66 \geq 0.6 \]

- \( 1 \times X_1 + 3 \times X_2 = g = 12; \)
- \( 2 \times X_1 + 3 \times X_2 = g = 12; \)
- \( 3 \times X_1 + 3 \times X_2 = g = 12; \)
Chance Constraints [\textit{chance}]

- Defines individual or joint chance constraints (CC):

\[
\text{chance equ \{equ\} [holds] minRatio [weight|varName]}
\]

- Individual CC: A single constraint \textit{equ} has to hold for a certain ratio \((0 \leq \text{minRatio} \leq 1)\) of the possible outcomes

- Joint CC: A set of constraints \textit{equ} has to hold for a certain ratio \((0 \leq \text{minRatio} \leq 1)\) of the possible outcomes

- If \textit{weight} is defined, the violation of a CC gets penalized in the objective \((\text{weight violationRatio})\)

- If \textit{varName} is defined the violation get multiplied by this existing variable

\(\rightarrow\) simplechance.gms
SP in GAMS - Summary & Outlook

• The Extended Mathematical Programming (EMP) framework can be used to replace parameters in the model by random variables

• Support for Multi-stage recourse problems and chance constraint models

• Easy to add uncertainty to existing deterministic models, to either use specialized algorithms (e.g. solvers Lindo, DECIS) or create Deterministic Equivalent (free solver DE)

• Besides the expected value, EMP also supports optimization of other risk measures (e.g. VaR)

• GAMS/Scenred2 interfaces GAMS with the well-known scenario reduction software Scenred (https://www.gams.com/latest/docs/T_SCENRED2.html)

• More information: https://www.gams.com/latest/docs/UG_EMP_SP.html
Thank You!
Meet us at the GAMS booth!
Extended Example: Newsboy (NB) Problem

- **Data:**
  - A newsboy faces a certain demand for newspapers \( d = 63 \)
  - He can buy newspapers for fixed costs per unit \( c = 30 \)
  - He can sell newspapers for a fixed price \( v = 60 \)
  - For leftovers he has to pay holding costs per unit \( h = 10 \)
  - He has to satisfy his customers demand or has to pay a penalty \( p = 5 \)
  - He can return units for a refund (stage 3) \( r = 9 \)

- **Stage 1: Decisions:**
  - How many newspapers should he buy: \( X \)

- **Stage 2: Decisions & Derived Outcomes**
  - How many newspapers should he sell: \( S \)
  - How many newspapers go to his inventory: \( I \)
  - How many customers are lost: \( L \)

- **Stage 3: Decisions & Derived Outcomes**
  - How many units returned for refund: \( Y \)
  - How many units kept for holding cost \( h \) again: \( E \)
Stages \([\text{stage}]\)

- Defines the stage of random variables \((\text{rv})\), equations \((\text{equ})\) and variables \((\text{var})\): 

  \[
  \text{stage} \text{ stageNo} \text{ rv} \mid \text{equ} \mid \text{var} \{\text{rv} \mid \text{equ} \mid \text{var}\}
  \]

- \text{StageNo} \text{ defines the stage number}

- The default \text{StageNo} for the objective variable and objective equation is the highest stage mentioned

- The default \text{StageNo} for all the other random variables, equations and variables not mentioned is 1
Random Variables

Discrete Distribution

Normal Distribution

Poisson Distribution

Exponential Distribution
Random Variables (RV) \([\text{randVar}]\)

Defines both discrete and parametric random variables:

\[\text{randVar rv discrete prob val \{prob val\}}\]

The distribution of discrete random variables is defined by pairs of the probability \(\text{prob}\) of an outcome and the corresponding realization \(\text{val}\).

\[\text{randVar rv distr par \{par\}}\]

The name of the parametric distribution is defined by \(\text{distr, par}\) defines a parameter of the distribution.

For parametric distributions a sample can be created.
Joint RVs \([\text{jRandVar}]\)

- Defines discrete random variables and their joint distribution:

\[
\text{jRandVar \(rv \ rv \ \{rv\} \ prob \ val \ val \ \{val\}} \\
\{prob \ val \ val \ \{val\}\}
\]

- At least two discrete random variables \(rv\) are defined and the outcome of those is coupled

- The probability of the outcomes is defined by \(prob\) and the corresponding realization for each random variable by \(val\)

\[\text{nbdiscjoint.gms}\]
Correlation between RVs [correlation]

- Defines a correlation between a pair of random variables:

  \[ \text{correlation } \text{rv} \ \text{rv} \ \text{val} \]

- \text{rv} is a random variable which needs to be specified using the \text{randvar} keyword and \text{val} defines the desired correlation (-1 \leq \text{val} \leq 1).

\[ \Rightarrow \text{nbcontjoint.gms} \]