Agricultural Impact Analysis using GAMS
Including Firm Level Risk

Bruce A. McCarl

Specialist in Applied Optimization
Professor of Agricultural Economics, Texas A&M
Principal, McCarl and Associates

mccarl@tamu.edu
mccarl@bihs.net
agrinet.tamu.edu/mccarl

409-693-5694
409-845-1706

Including Firm Level Risk

Agricultural producers face pervasive uncertainty.

Weather induces large changes in yields, working rates and resources available. General market conditions across the sector cause prices and input costs to be uncertain.

When incorporating risk into the program models there are three big issues

1. What is the nature of risk? -- What parameters of the model are uncertain? and How do we describe their distribution?

2. When during the model time horizon are risk outcomes revealed? Are there times when the model should reflect that the producer has received information about uncertain events and will make adaptive decisions?

3. How do we model the farmers behavioral reaction to risk? Is expected profit maximization not to the proper objective but rather some degree of aversion to the variation caused by risk?

Including Firm Level Risk

Our risk treatment will be somewhat specialized because of time constraints (see newbook.pdf chapter 14 and probab.pdf for more extensive treatment)

The treatment will be limited in several principal ways

1. Will specialize in risk in yields and prices. Will not discuss how to form such probability distributions except through a few casual remarks (see probab.pdf for more extensive treatment)

2. We will only cover the expected value variance and MOTAD formulations of model objective function alterations to risk (these are the two main ones used in the literature -- see newbook.pdf chapter 14 for more extensive treatment)

3. We will specialize along lines of discrete stochastic programming or stochastic programming with recourse and will not produce the most compact models that could be used to handle cases without recourse.

Including Firm Level Risk
Why model risk

Why not just solve for all values of risky parameters

Curses of dimensionality and certainty

Dimensionality Number of possible plans
(3 possible values for 5 parameters $3^5 = 243$)

Certainty Each plan would be certain of data so we would have
243 different things we could do –
What would we do?

General Risk Modeling Aim

Generate a plan which is Robust in the face of the Uncertainty

Not best performer necessarily in any setting, but a good performer across many or most of the uncertainty spectrum

Including Firm Level Risk
Risk entry into a programming model

Maximize	CX		
Subject to	AX	#	b
X	$	0	

Objective function returns - C

- variability in prices
- variability in production quantities
- variability in costs
- variability in market sales

Resource usages - A

- variability in raw input quality
- variability in working conditions
- variability in intermediate product yields
- variability in product requirements

Resource endowments - b

- variability in demand firm faces
- variability in resources available
- variability in working conditions

Including Firm Level Risk
Forms of assumed reaction to risk

Non Recourse or non adaptive decision making

- Decisions made now consequence felt later
- No additional decisions made between now and when consequences felt
- Example – Buy stock now make no decisions for one year

Recourse or adaptive decision making

- Decisions made now consequences arise over time

- Later time during model additional decisions made.
  - In this later decision period
    - Decision maker knows what happened between first decision and now.
    - Decisions can revise prior actions ie the decisions can adjust in the face of events -- a phenomena called recourse

- Example – Buy stock now, review decisions quarterly possibly selling and buying other stocks

Including Firm Level Risk
Decision Maker reaction to risk

Expected Value Maximization

$$\text{Maximize} \quad \overline{C} X$$
$$\text{Subject to} \quad \overline{\overline{A}} X \leq \overline{\overline{b}}$$
$$X \geq 0$$

Conservative - Fat or thin coefficients

$$\text{Maximize} \quad C_c X$$
$$\text{Subject to} \quad A_c X \; \# \; b_c$$
$$X \; \$ \; 0$$
$$\text{where} \quad C_c \; ' \; \overline{\overline{C}} \; \& \; \text{Risk Discount}$$
$$A_c \; ' \; \overline{\overline{A}} \; \% \; \text{Risk Discount}$$
$$b_c \; ' \; \overline{\overline{b}} \; \& \; \text{Risk Discount}$$

E- V

Maximize E(income) - RAP * Variance(income)

Expected utility

Maximize Sum(p,Probability(p)*U[Wealth(p)])
S.T. Wealth(p)=InitWealth + Income(p) for all p
Income(p)=C(p)*X for all p

Safety First based

Maximize Sum(p,Probability(p)*Income(p))
S.T. Income(p)=C(p)*X for all p
Income(p)$safety for all p

Including Firm Level Risk
First Risk Model

Markowitz mean-variance portfolio choice formulation

Given Problem

max sum(invest, moneyinvest(invest)*avgreturn(invest))
s.t sum(invest,moneyinvest(invest)*price(invest))# funds

Markowitz observed not all money is invested in the highest valued stock

Inconsistent with LP formulation
Why? Not a basic solution

Markowitz posed the hypothesis that average returns and the variance of returns were important

Including Firm Level Risk
E-V Model

Commonly Used Formulation

$$\text{Max } E - \phi\sigma^2 = E - RAP * Var$$

$$\text{Max} \quad -\sum_j c_j X_j + \sum_k p_k GI_k - \phi \sum_k p_k (GI_k - AVGI)^2 \qquad (1)$$

$$\text{s.t.} \quad \sum_j X_j \le Acres \qquad (2)$$

$$-\sum_j y_{ikj} X_j + S_{ik} = 0 \quad \text{for all } i, k \qquad (3)$$

$$-\sum_i sp_{ik} S_{ik} + GI_k = 0 \quad \text{for all } k \qquad (4)$$

$$-\sum_k p_k GI_k + AVGI = 0 \qquad (5)$$

$$X_j, \quad S_{ik}, \quad GI_k, \quad AVGI \ge 0 \quad \text{for all } j, k \qquad (6)$$

Including Firm Level Risk
E-V Model

$$
\begin{array}{lllllllr}
\text{Max} & -\sum_j c_j X_j & & + \sum_k p_k GI_k - \Phi \sum_k p_k (GI_k - AVGI)^2 & & & & (1) \\
\text{s.t.} & \sum_j X_j & & & & & \le \text{Acres} & (2) \\
 & -\sum_j y_{ikj} X_j & + S_{ik} & & & & = 0 \text{ for all } i, k & (3) \\
 & & -\sum_i sp_{ik} S_{ik} & + GI_k & & & = 0 \text{ for all } k & (4) \\
 & & & -\sum_k p_k GI_k & + AVGI & & = 0 & (5) \\
 & X_j, & S_{ik}, & GI_k, & AVGI & & \ge 0 \text{ for all } j, k & (6)
\end{array}
$$

where

- j identifies the production possibilities;
- k identifies the states nature;
- i identifies the commodities sold;
- X_j is acres of production possibility j grown;
- c_j is the nonstochastic cost of growing X_j;
- y_{ijk} is the uncertain yield of commodity i realized under state of nature k when the growing X_j;
- S_{ik} is the total sales of commodity i under state of nature k;
- sp_{ik} is the sale price for commodity i under state of nature k;
- GI_k gross income from sales under state of nature k;
- p_k is probability of state of nature k;
- AVGI is average gross income; and
- Φ is a risk aversion parameter

Including Firm Level Risk
E-V Model

$$
\begin{array}{llr}
\text{Max} & -\sum_j c_j X_j + \sum_k p_k GI_k - \Phi \sum_k p_k (GI_k - AVGI)^2 & (1) \\
\text{s.t.} & \sum_j X_j \le Acres & (2) \\
 & -\sum_j y_{ikj} X_j + S_{ik} = 0 \text{ for all } i, k & (3) \\
 & -\sum_i sp_{ik} S_{ik} + GI_k = 0 \text{ for all } k & (4) \\
 & -\sum_k p_k GI_k + AVGI = 0 & (5) \\
 & X_j,\ S_{ik},\ GI_k,\ AVGI \ge 0 \text{ for all } j, k & (6)
\end{array}
$$

(1) Depicts maximization of gross income from sales minus cost of production minus the risk aversion parameter times the variance of gross income

(2) Limits acreage available

(3) Adds production into sales variables by commodity for each state of nature. Yield is stochastic here

(4) Sums sales times price into gross income by state of nature variable. Sales vary by state of nature due to the stochastic yields in (3) and price is stochastic

(5) Taking probabilities into account computes average gross income.

(6) Requires the variables to be nonnegative.

Including Firm Level Risk
E-V Model optimality conditions

$$\text{Max} \quad -\sum_j c_j X_j + \sum_k p_k GI_k - \Phi \sum_k p_k (GI_k - AVGI)^2 \qquad (1)$$

$$\text{s.t.} \quad \sum_j X_j \le \text{Acres} \qquad (2)$$

$$-\sum_j y_{ikj} X_j + S_{ik} = 0 \text{ for all } i, k \qquad (3)$$

$$-\sum_i sp_{ik} S_{ik} + GI_k = 0 \text{ for all } k \qquad (4)$$

$$-\sum_k p_k GI_k + AVGI = 0 \qquad (5)$$

$$X_j, \; S_{ik}, \; GI_k, \; AVGI \ge 0 \text{ for all } j, k \qquad (6)$$

Assume average income, income by state of nature and sales by commodity and state of nature are nonzero.

Under these assumptions Kuhn Tucker conditions become equality constraints and we can assert that the Lagrangian multipliers for rows 3,4,5 are given by

$$\lambda_5 = 2\Phi \sum_k p_k (GI_k - AVGI)$$

$$\lambda_{4k} = p_k * (1 + \lambda_5)$$

$$\lambda_{3ik} = \lambda_{4k} * sp_{ik}$$

and the conditions for x can be expressed as

$$\lambda_2 = \sum_i \sum_k \lambda_{3ik} * y_{ikj} - c_j$$

$$\text{MVP of Land} = \sum_{\text{commodity } i} \sum_{\text{SON } k} \text{MVP of Commodity}_{ik} * \text{Yield}_{ikj} - \text{production cost}_j$$

Including Firm Level Risk
E-V Model optimality conditions

Assuming Risk Neutrality

$$8_5 \quad ' \quad N(\mathbf{j}_k \; p_k \; (\; GI_k \; \& \; AVGI \;) \quad ' \quad 0$$

$$8_{4k} \quad ' \quad p_k \; (\; (1 \; \% \; 8_5 \;) \quad ' \quad p_k$$

$$8_{3ik} \quad ' \quad 8_{4k} \; (\; sp_{ik} \quad ' \quad p_k (sp_{ik}$$

The KT conditions for x can be expressed as

$$8_2 \quad ' \quad \mathbf{'}_i \; \mathbf{'}_k \; 8_{3ik} \; (\; y_{ikj} \; \& \; c_j$$

$$8_2 \quad ' \quad \mathbf{'}_i \; \mathbf{'}_k \; p_k (sp_{ik} \; (\; y_{ikj} \; \& \; c_j$$

$$\text{MVP of Land} \quad ' \quad \mathbf{'}_{\text{commodity } i} \; \text{Average Marg Revenue}_i \; \& \; \text{production cost}_j$$

Thus the decision rule employed in the risk neutral model is that the firm produces to the point at which its resource values are equated with the average marginal revenue product and less the marginal cost. The only differences from a nonstochastic model is the average process in calculating revenue.

When we move away from his neutrality then we also factor in a cost of bearing risk through 8_5

Including Firm Level Risk
E-STD Model GAMS (farmev.gms)

Max E - N F = E - RAP * STD

Max	$-\sum_j c_j X_j + \sum_k p_k GI_k - M \sqrt{\sum_k p_k (GI_k - AVGI)^2}$	(1)
s.t.	$\sum_j X_j \le Acres$	(2)
	$-\sum_j y_{ikj} X_j + S_{ik} = 0$ for all i, k	(3)
	$-\sum_i sp_{ik} S_{ik} + GI_k = 0$ for all k	(4)
	$-\sum_k p_k GI_k + AVGI = 0$	(5)
	$X_j, \; S_{ik}, \; GI_k, \; AVGI \ge 0$ for all j, k	(6)

```
POSITIVE VARIABLES      acres(crop)                 acres by crop        (x)
                  sales(stateofnat,crop)      sales by crop        (S)
                   grossinc(stateofnat)        gross income         (GI)
                   avggrosinc                  average gross income (AVGI);
VARIABLES   OBJ                     MAXIMand;
EQUATIONS   OBJJ                    OBJECTIVE FUNCTION   (1)
            acresAV                         acres AVAILABLE      (2)
            commodbal(stateofnat,crop) commodity balance    (3)
            RETURNDEF(stateofnat)      RETURNS DEFINITION   (4)
            AVRET                      AVERAGE RETURNS      (5);
 OBJJ..    OBJ =E=   -sum(crop,cost(crop)*acres(crop))
                     +sum(stateofnat,probab(stateofnat)*grossin
                     c(stateofnat))
            -rap*sqrt(sum(stateofnat,probab(stateofnat)
               *sqr(grossinc(stateofnat)-avggrosinc)));
acresAV..    SUM(crop, acres(crop)) =L= acresavail ;
commodbal(stateofnat,crop)..-
acres(crop)*yield(stateofnat,crop)+sales(stateofnat,crop)=e=0
RETURNDEF(stateofnat).. -SUM(crop, prices(stateofnat,crop)*
sales(stateofnat,crop))
             + grossinc(stateofnat) =E= 0 ;
AVRET..        -
SUM(stateofnat,probab(stateofnat)*grossinc(stateofnat)) +
avggrosinc =E= 0
 MODEL farmEV /ALL/ ; SOLVE farmEV USING NLP MAXIMIZING OBJ ;
```

Including Firm Level Risk
E-STD Model GAMS

$$\text{Max} \quad -\sum_j c_j X_j + \sum_k p_k GI_k - M\sqrt{\sum_k p_k (GI_k - AVGI)^2} \qquad (1)$$

$$\text{s.t.} \quad \sum_j X_j \le \text{Acres} \qquad (2)$$

$$-\sum_j y_{ikj} X_j + S_{ik} = 0 \text{ for all } i, k \qquad (3)$$

$$-\sum_i sp_{ik} S_{ik} + GI_k = 0 \text{ for all } k \qquad (4)$$

$$-\sum_k p_k GI_k + AVGI = 0 \qquad (5)$$

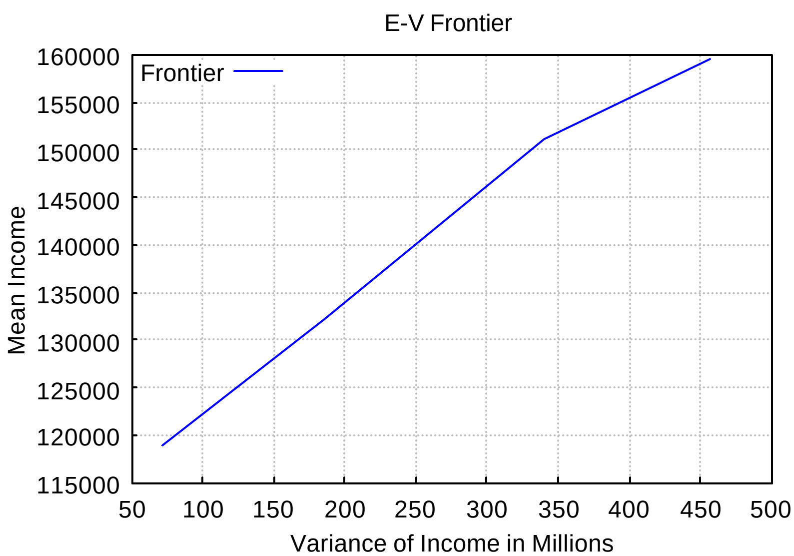
$$X_j, \quad S_{ik}, \quad GI_k, \quad AVGI \ge 0 \text{ for all } j, k \qquad (6)$$

```
           |                             A       R
           |                             V       H
           |                     G G G G G       S     P     N
           |                     R R R R G             O     E     R
           |                     O O O O R       C     S     G     O
           | A A S S S S S S S S S S S S O       O     I A   A A   W
           | C C A A A A A A A A S S S S S       E     T I   T I   C
           | R R L L L L L L L L I I I I I O     F     I J     J   N
           | E E E E E E E E E E N N N N N B     F     V ,   V ,   T
           | S S S S S S S S S S C C C C C J     S     E S   E S   S
           |
           | 1 2 1 2 3 4 5 6 7 8 1 2 3 4 1 1
            --------------------------------
     OBJJ 1| F F                 2 A 1 1 $ C   = 0       4     4     8
  ACRESAV 1| C C                               < F       2     0     2
COMMODBAL 1| 6   C                             < 0       1     1     2
COMMODBAL 2|   5   C                           < 0       1     1     2
COMMODBAL 3| 6       C                         < 0       1     1     2
COMMODBAL 4|   5       C                       < 0       1     1     2
COMMODBAL 5| 6           C                     < 0       1     1     2
COMMODBAL 6|   5           C                   < 0       1     1     2
COMMODBAL 7| 6               C                 < 0       1     1     2
COMMODBAL 8|   5               C               < 0       1     1     2
RETURNDEF 1|     4 4             C             < 0       1     2     3
RETURNDEF 2|         4 4           C           < 0       1     2     3
RETURNDEF 3|             4 4         C         < 0       1     2     3
RETURNDEF 4|                 4 4       C       < 0       1     2     3
    AVRET 1|                     1 1 1 1 C     = 0       1     4     5
```

Including Firm Level Risk
E-STD Model GAMS Solution (farmev.gms)

```
SET RAPS   RISK AVERSION PARAMETERS /R0*R3/
PARAMETER RISKAVER(RAPS) RISK AVERSION COEFICIENT BY RISK AVERSION PARAMETER
    /R0   0.00000,  R1   0.5000,  R2    1.00000,  R3   1.50000  /
PARAMETER OUTPUT(*,RAPS) RESULTS FROM MODEL RUNS WITH VARYING RAP
OPTION SOLPRINT = OFF;
LOOP (RAPS,RAP=RISKAVER(RAPS);
            SOLVE farmev USING NLP MAXIMIZING OBJ ;
            std =  sqrt(sum(stateofnat,probab(stateofnat)
                    *sqr(grossinc.l(stateofnat)-avggrosinc.l)));
            OUTPUT("OBJ",RAPS)=OBJ.L;
            OUTPUT("RAPx100",RAPS)=RAP*100;
            OUTPUT(crop,RAPS)=acres.L(crop);
            OUTPUT("MEAN",RAPS)=avggrosinc.L;
            OUTPUT("STD",RAPS)=std;
            OUTPUT("SHADPRICE",RAPS)=acresAV.M;
            OUTPUT("IDLE",RAPS)=acresavail-acresAV.L );
 DISPLAY OUTPUT;
```

	R0	R1	R2	R3
corn	500	381	116	61
soybeans		119	384	439
OBJ	54616	44017	36553	29904
RAPX100		50	100	150
MEAN	159616	151086	132159	128170
STD	21359	18443	13628	13089
SHADPRICE	109	88	73	60



Including Firm Level Risk
Dissecting the GAMS formulation
Graphing

```
parameter graphit (*,raps,*);
graphit("Frontier",raps,"Mean")=OUTPUT("MEAN",RAPS);
graphit("frontier",raps,"Var")=OUTPUT("std",RAPS)**2;
*$include gnu_opt.gms
* titles
$setglobal gp_title  "E-V Frontier "
$setglobal gp_xlabel "Variance of Income"
$setglobal gp_ylabel "Mean Income"

$batinclude gnupltxy graphit mean var fig1 windows
```

This is done using a GNUPLOT interface originally developed by Rutherford but modified here as documented on the Web page agrinet.tamu.edu/mccarl

Including Firm Level Risk
Modeling Support from GAMSCHK Nonlinear Models

```
## EQU  OBJJ
  VAR                              Aij          Xj        Aij*Xj
 ACRES(corn)                      210.00     0.00000E+00 0.00000E+00
 ACRES(soybeans)                  150.00      500.00      75000.
 GROSSINC(year93)            ***-0.89528     0.10432E+06 -93396.
 GROSSINC(year94)            *** 0.19325E-07 0.12457E+06 0.24073E-02
 GROSSINC(year95)            ***-0.19877E-07 0.12457E+06-0.24761E-02
 GROSSINC(year96)            ***-0.10472     0.12220E+06 -12796.
 AVGGROSINC                  ***-0.38605E-14 0.11891E+06-0.45907E-09
 OBJ                             1.0000      31192.      31192.
    =E=                                                    =E=
 RHS COEFF                                               0.00000E+00

##  GROSSINC(year96)  SOLUTION VALUE                       122200.
  EQN                               Aij          Ui        Aij*Ui
 OBJJ                        ***-0.10472      1.0000     -0.10472
 RETURNDEF(year96)                1.0000     0.10472     0.10472
 AVRET                           -0.25000    0.00000E+00 0.00000E+00
 TRUE REDUCED COST                                       0.00000E+00

 ##  AVGGROSINC       SOLUTION VALUE               118914.
  EQN                              Aij         Ui        Aij*Ui
 OBJJ                    ***-0.38605E-14  1.0000    -0.38605E-14
 AVRET                            1.0000     0.00000E+00 0.00000E+00
 REDUCED COST EXCLUDING BOUNDS                        -0.38605E-14
 Accounting Error -NLP/MIP?                            0.38605E-14
 TRUE REDUCED COST                                     0.00000
```

Starting point and accuracy is an issue

Including Firm Level Risk
Linear Alternative Motad Model

$$\text{Max} \quad -\sum_j c_j X_j + \sum_k p_k GI_k - M\sqrt{\frac{\pi N}{2(N-1)}}\sum_k p_k \left(Dev_k^+ + Dev_k^-\right) \qquad (1)$$

$$\text{s.t.} \quad \sum_j X_j \le \#\,Acres \qquad (2)$$

$$\sum_j y_{ikj} X_j - S_{ik} \ge 0 \quad \text{for all } i, k \qquad (3)$$

$$\sum_i sp_{ik} S_{ik} - GI_k = 0 \quad \text{for all } k \qquad (4)$$

$$\sum_k p_k GI_k - AVGI = 0 \qquad (5)$$

$$GI_k - AVGI - Dev_k^+ + Dev_k^- = 0 \quad \text{for all } k \qquad (6)$$

$$X_j \ge 0, \quad S_{ik} \ge 0, \quad GI_k \gtrless 0, \quad AVGI \gtrless 0, \quad Dev_k^+ \ge 0, \quad Dev_k^- \ge 0 \quad \text{for all } i, j, k \qquad (7)$$

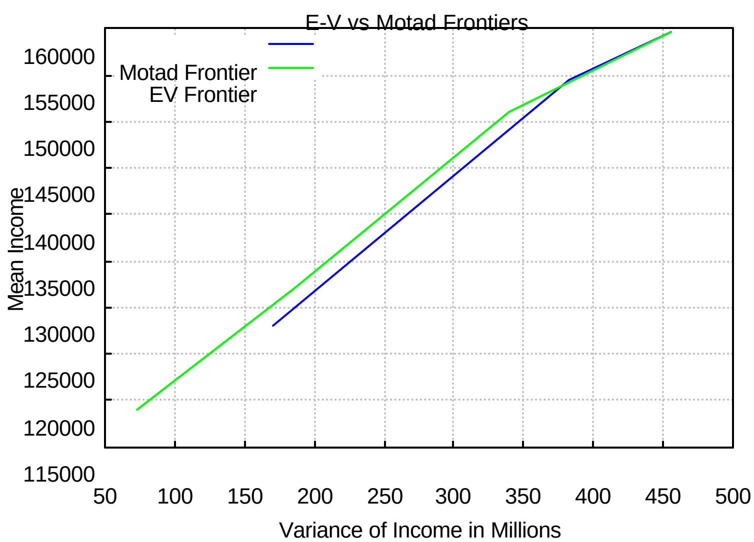
A linear alternative

Square root term converts to estimate of standard error

Including Firm Level Risk
Linear Alternative Motad Model (farmotad.gms)

$$\begin{array}{llr}
\text{Max} & -\sum_j c_j X_j + \sum_k p_k GI_k - \Phi \sqrt{\frac{\pi N}{2(n-1)}} \sum_k p_k \left(Dev_k^{+} + Dev_k^{-} \right) & (1) \\
\text{s.t.} & \sum_j X_j \leq \text{Acres} & (2) \\
 & -\sum_j y_{ikj} X_j + S_{ik} = 0 \quad \text{for all } i, k & (3) \\
 & -\sum_i sp_{ik} S_{ik} + GI_k = 0 \quad \text{for all } k & (4) \\
 & -\sum_k p_k GI_k + AVGI = 0 & (5) \\
 & GI_k - AVGI - Dev_k^{+} + Dev_k^{-} = 0 \quad \text{for all } k & (6) \\
 & X_j, \; S_{ik}, \; GI_k, \; AVGI, \; Dev_k^{+}, \; Dev_k^{-} \geq 0 \quad \text{for all } j, k & (7)
\end{array}$$

	R0	R1	R2	R3
corn	500	429	58	58
soybeans		71	442	442
OBJ	54616	42434	34222	26578
RAPX100		50	100	150
MEAN	159616	154502	127992	127992
STD	21359	19574	13070	13070
SHADPRICE	109	85	68	53



Including Firm Level Risk
Finding a Risk Aversion Parameter

$$\begin{array}{llll} \text{Max} & cX\&Q\ F^2(X) & & \\ \text{s.t.} & AX & \# & b \\ & X & \$ & 0 \end{array} \quad \text{versus} \quad \begin{array}{llll} \text{Max} & cX\&>\ F(X) & & \\ \text{s.t.} & AX & \# & b \\ & X & \$ & 0 \end{array}$$

$$c\ \&\ 2\ Q\ F(X)\frac{MF(X)}{MX}\&8A\ '\ 0 \qquad c\ \&\ >\ \frac{MF(X)}{MX}\&8\ A\ '\ 0$$

$$Q\ '\ \frac{>}{2\ F(X)}$$

Given

$$0\ \#\ >\ \#\ 5$$

$$0\ \#\ Q\ \#\ \frac{5}{2F(x)}$$

Including Firm Level Risk
Now what about recourse (simpspr.gms)

What is recourse?
  Decision now – planting (Y)
  Decision later – harvesting (X)

Two states of nature can occur

	Price	Yield	Harvest Capacity	Probability
Son 1	2	1.1	2	0.3
Son 2	5	0.9	3	0.7

$$
\begin{array}{lllllll}
\text{Max} & \&3\ Y & \% & .3(2X_1) & \% & .7(5X_2) & \\
\text{s.t.} & \&1.1Y & \% & X_1 & & & \#\ 0 \\
 & & & X_1 & & & \#\ 2 \\
 & \&0.9Y & & & \% & X_2 & \#\ 0 \\
 & & & & & X_2 & \#\ 3 \\
 & Y, & & X_1, & & X_2 & \$\ 0
\end{array}
$$

Solution   obj=1.7   Y=3.33 X_1=2 X_2=3

How many X's ? 2 one for each event
  X_1=2 X_2=3

Where is uncertainty ? expected value

Including Firm Level Risk with Recourse (resourse.gms) – add cattle feeding

```
           |       C C C C C C C C                                 A         R
           |       A A A A A A A A                                 V         H
           |       T T T T T T T T                         G G G G G         S
           |       T T T T T T T T                         R R R R G
           |     C L L L L L L L L                         O O O O R         C
           | A A A E E E E E E E E                         S S S S O         O
           | C C T F F F F F F F F S A L E S - S A L E S - S S S S S         E
           | R R T E E E E E E E E                         I I I I I O       F
           | E E L E E E E E E E E                         N N N N N B       F
           | S S E D D D D D D D D                         C C C C C J       S
           | 1 2 1 1 2 3 4 5 6 7 8 1 2 3 4 5 6 7 8 9 0 1 2 1 2 3 4 1 1
            ----------------------------------------------------------
     OBJJ 1|                                               2 A 1 A $ C   = 0
  ACRESAV 1| C C A                                                       < F
     COWS 1|     C 3 3                                                   = 0
     COWS 2|     C     3 3                                               = 0
     COWS 3|     C         3 3                                           = 0
     COWS 4|     C             3 3                                       = 0
    C     1| F     5 5             3                                     = 0
    O     2|   E   4 5               3                                   = 0
    M     3|       E E                 3                                 = 0
    M     4| F         5 5               3                               = 0
    O     5|   E       4 5                 3                             = 0
    D     6|           E E                   3                           = 0
    B     7| F             5 5                 3                         = 0
    A     8|   E           4 5                   3                       = 0
    L     9|               E E                     3                     = 0
    -    10| F                 5 5                   3                   = 0
    C    11|   E               4 5                     3                 = 0
    O    12|                   E E                       3               = 0
RETURNDEF 1| 6 6 6 6 5             D D E                   3             = 0
RETURNDEF 2| 6 6 6     6 5               D D E               3           = 0
RETURNDEF 3| 6 6 6         6 5                 D D E           3         = 0
RETURNDEF 4| 6 6 6             6 5                   D D E       3       = 0
    AVRET 1|                                               A A A A 3     = 0
```

Including Firm Level Risk – add cattle feeding without Recourse (norecrse.gms)

```
           |                                         A       R
           |                                         V       H
           |                                 G G G G G       S     P     N
           |                                 R R R R G             O     E     R
           |     C C                         O O O O R       C     S     G     O
           | A A A A                         S S S S O       O     I A   A A   W
           | C C T T S A L E S - S A L E S - S S S S S       E     T I   T I   C
           | R R T T                         I I I I I O     F     I J     J   N
           | E E L L                         N N N N N B     F     V ,   V ,   T
           | S S E E                         C C C C C J     S     E S   E S   S
           |                           1 1 1
           | 1 2 1 2 1 2 3 4 5 6 7 8 9 0 1 2 1 2 3 4 1 1
            --------------------------------------------
     OBJJ 1| F F F F                         2 A 1 A $ C   = 0       8     2   1 0
    C     1| F   5 5 3                                     = 0       1     3     4
    O     2|   E 4 5   3                                   = 0       1     3     4
    M     3|     E E     3                                 = 0       2     1     3
    M     4| F   5 5       3                               = 0       1     3     4
    O     5|   E 4 5         3                             = 0       1     3     4
    D     6|     E E           3                           = 0       2     1     3
    B     7| F   5 5             3                         = 0       1     3     4
    A     8|   E 4 5               3                       = 0       1     3     4
    L     9|     E E                 3                     = 0       2     1     3
    -    10| F   5 5                   3                   = 0       1     3     4
    C    11|   E 4 5                     3                 = 0       1     3     4
    O    12|     E E                       3               = 0       2     1     3
RETURNDEF 1|         D D E                   3             = 0       3     1     4
RETURNDEF 2|               D D E               3           = 0       3     1     4
RETURNDEF 3|                     D D E           3         = 0       3     1     4
RETURNDEF 4|                           D D E       3       = 0       3     1     4
    AVRET 1|                                 A A A A 3     = 0       4     1     5
  ACRESAV 1| C C A A                                       < F       4     0     4
```

Firm Level Risk Solutions with and without Recourse

Without recourse			R0	R1	R2	R3
1	.OBJ	.Overall	54616	44341	39026	34409
2	.RAPx100	.Overall		50	100	150
3	.Income	.Mean	159616	187275	193837	195072
4	.Income	.Year93	126250	164246	178673	181388
4	.Income	.Year94	157070	188296	200163	202397
4	.Income	.Year95	183060	196910	193152	192444
4	.Income	.Year96	172085	199650	203359	204057
5	.Income	.STD	21359	13940	9502	9063
6	.Land	.SHADPRICE	109	89	78	69
8	.Corn	.acres	500	365	199	168
8	.Soybeans	.acres		116	273	302
8	.cattle	.Ration2		93	139	148
sell	.Corn	.Year93	50500	31327	11804	8129
sell	.Corn	.Year94	69500	45211	19380	14518
sell	.Corn	.Year95	56500	35711	14196	10146
sell	.Corn	.Year96	63500	40827	16988	12500
sell	.Soybeans	.Year93		2388	6810	7643
sell	.Soybeans	.Year94		3409	9211	10303
sell	.Soybeans	.Year95		2701	7547	8459
sell	.Soybeans	.Year96		2968	8174	9154
sell	.Beef	.Year93		930	1389	1476
sell	.Beef	.Year94		930	1389	1476
sell	.Beef	.Year95		930	1389	1476
sell	.Beef	.Year96		930	1389	1476

With Recourse			R0	R1	R2	R3
1	.OBJ	.Overall	54616	44341	39030	35614
2	.RAPx100	.Overall		50	100	150
3	.Income	.Mean	54616	51311	46701	43692
4	.Income	.Year93	21250	28282	34461	35923
4	.Income	.Year94	52070	52331	51806	47664
4	.Income	.Year95	78060	60945	46143	41548
4	.Income	.Year96	67085	63685	54394	49632
5	.Income	.STD	21359	13940	7671	5385
6	.Land	.SHADPRICE	109	89	78	71
8	.Corn	.acres	500	365	269	144
8	.Soybeans	.acres		116	194	321
8	.cattle	.number		93	182	171
cattle	.Year93	.Ration2		93	182	171
cattle	.Year94	.Ration1				171
cattle	.Year94	.Ration2		93	182	
cattle	.Year95	.Ration2		93	182	171
cattle	.Year96	.Ration1			182	171
cattle	.Year96	.Ration2		93		
sell	.Corn	.Year93	50500	31327	16312	4333
sell	.Corn	.Year94	69500	45211	26547	11531
sell	.Corn	.Year95	56500	35711	19544	6066
sell	.Corn	.Year96	63500	40827	25130	9798
sell	.Soybeans	.Year93		2388	3614	7913
sell	.Soybeans	.Year94		3409	5324	11596
sell	.Soybeans	.Year95		2701	4138	8781
sell	.Soybeans	.Year96		2968	5493	10374
sell	.Beef	.Year93		930	1815	1709
sell	.Beef	.Year94		930	1815	1709
sell	.Beef	.Year95		930	1815	1709
sell	.Beef	.Year96		930	1815	1709

Including Firm Level Risk

compare with and without Recourse (recourse.gms)

```
            |       T T T T T T T T                         G G G G G       S
            |       T T T T T T T T                         R R R R G
            |     C L L L L L L L L                         O O O O R       C
            | A A A E E E E E E E E                         S S S S O       O
            | C C T F F F F F F F F S A L E S - S A L E S - S S S S S       E
            | R R T E E E E E E E E                         I I I I I O     F
            | E E L E E E E E E E E                         N N N N N B     F
            | S S E D D D D D D D D                         C C C C C J     S
            |                                         1 1 1
            | 1 2 1 1 2 3 4 5 6 7 8 1 2 3 4 5 6 7 8 9 0 1 2 1 2 3 4 1 1
     OBJJ 1|                                               2 A 1 A $ C   = 0
  ACRESAV 1| C C A                                                       < F
     COWS 1|     C 3 3                                                   = 0
     COWS 2|     C     3 3                                               = 0
     COWS 3|     C         3 3                                           = 0
     COWS 4|     C             3 3                                       = 0
    C     1| F     5 5             3                                     = 0
    O     2|   E   4 5               3                                   = 0
    M     3|       E E                 3                                 = 0
    M     4| F         5 5               3                               = 0
    O     5|   E       4 5                 3                             = 0
    D     6|           E E                   3                           = 0
    B     7| F             5 5                 3                         = 0
    A     8|   E           4 5                   3                       = 0
    L     9|               E E                     3                     = 0
    -    10| F                 5 5                   3                   = 0
    C    11|   E               4 5                     3                 = 0
    O    12|                   E E                       3               = 0
RETURNDEF 1| 6 6 6 6 5             D D E                   3             = 0
RETURNDEF 2| 6 6 6     6 5               D D E               3           = 0
RETURNDEF 3| 6 6 6         6 5                 D D E           3         = 0
RETURNDEF 4| 6 6 6             6 5                   D D E       3       = 0
    AVRET 1|                                               A A A A 3     = 0
            |                                 R R R R G
            |     C C                         O O O O R       C
            | A A A A                         S S S S O       O
            | C C T T S A L E S - S A L E S - S S S S S       E
            | R R T T                         I I I I I O     F
            | E E L L                         N N N N N B     F
            | S S E E                         C C C C C J     S
            |                           1 1 1
            | 1 2 1 2 1 2 3 4 5 6 7 8 9 0 1 2 1 2 3 4 1 1
     OBJJ 1| F F F F                         2 A 1 A $ C   = 0
    C     1| F   5 5 3                                     = 0
    O     2|   E 4 5   3                                   = 0
    M     3|     E E     3                                 = 0
    M     4| F   5 5       3                               = 0
    O     5|   E 4 5         3                             = 0
    D     6|     E E           3                           = 0
    B     7| F   5 5             3                         = 0
    A     8|   E 4 5               3                       = 0
    L     9|     E E                 3                     = 0
    -    10| F   5 5                   3                   = 0
    C    11|   E 4 5                     3                 = 0
    O    12|     E E                       3               = 0
RETURNDEF 1|         D D E                   3             = 0
RETURNDEF 2|               D D E               3           = 0
RETURNDEF 3|                     D D E           3         = 0
RETURNDEF 4|                           D D E       3       = 0
    AVRET 1|                                 A A A A 3     = 0
  ACRESAV 1| C C A A                                       < F
```

Forming Probability Distributions

Probability distributions state the relative frequency of occurrence of a set of mutually exclusive events.

Finding Probability Distributions Based on Objective Data

Desirable Characteristics

1) each of the states of nature must be mutually exclusive;

2) probability of occurrence of each of the states of nature must be an unbiased measure of the current probability of that state of nature occurring;

3) the sum of the probabilities across the states of nature must equal one

Second property is the most troubling in when using objective, historical data. trends, events

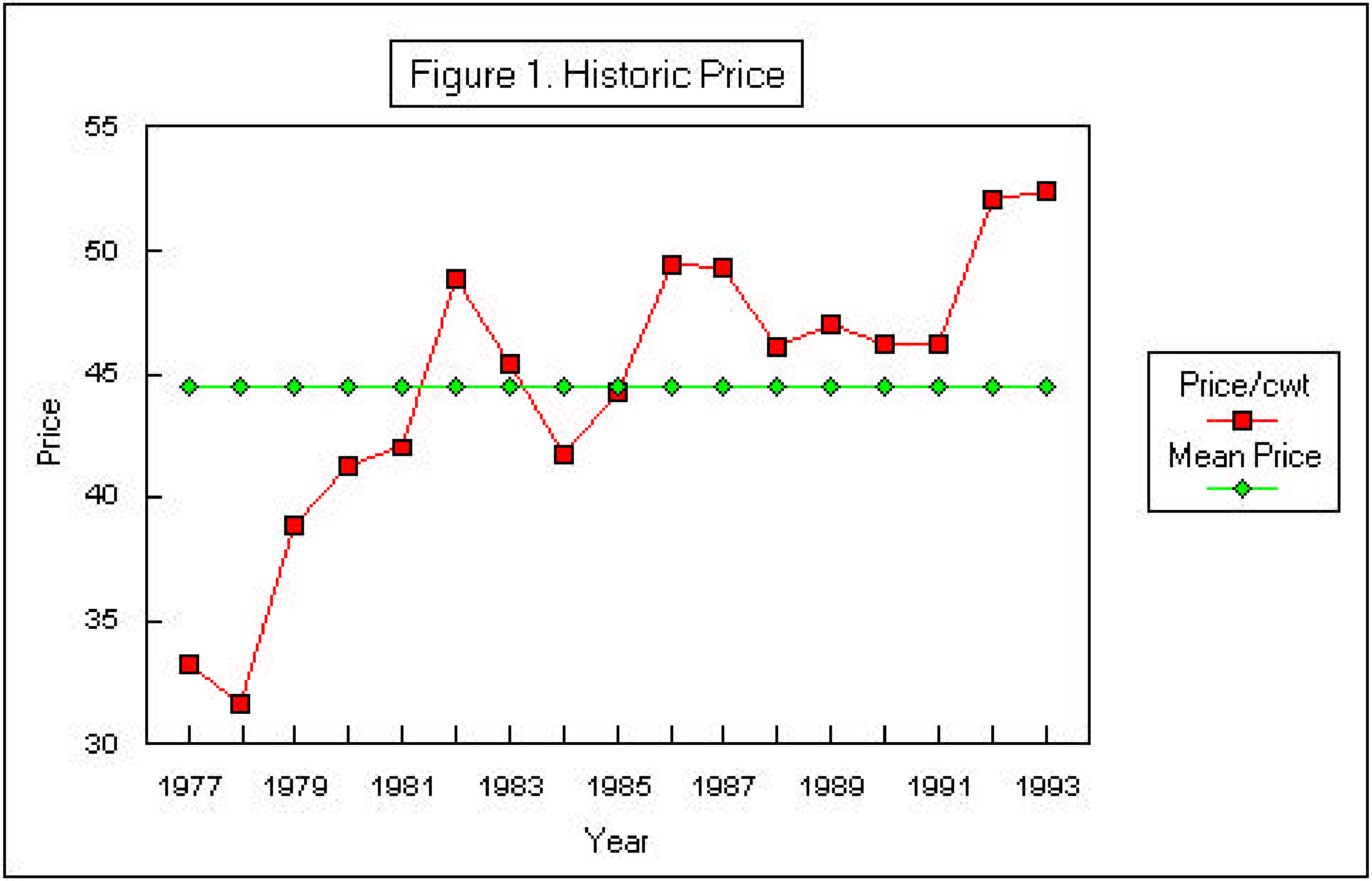


Figure 1. Historic Price
55
50
45
40
35
30
Price
1977
1979
1981
1983
1985
1987
1989
1991
1993
Year
Price/cwt
Mean Price

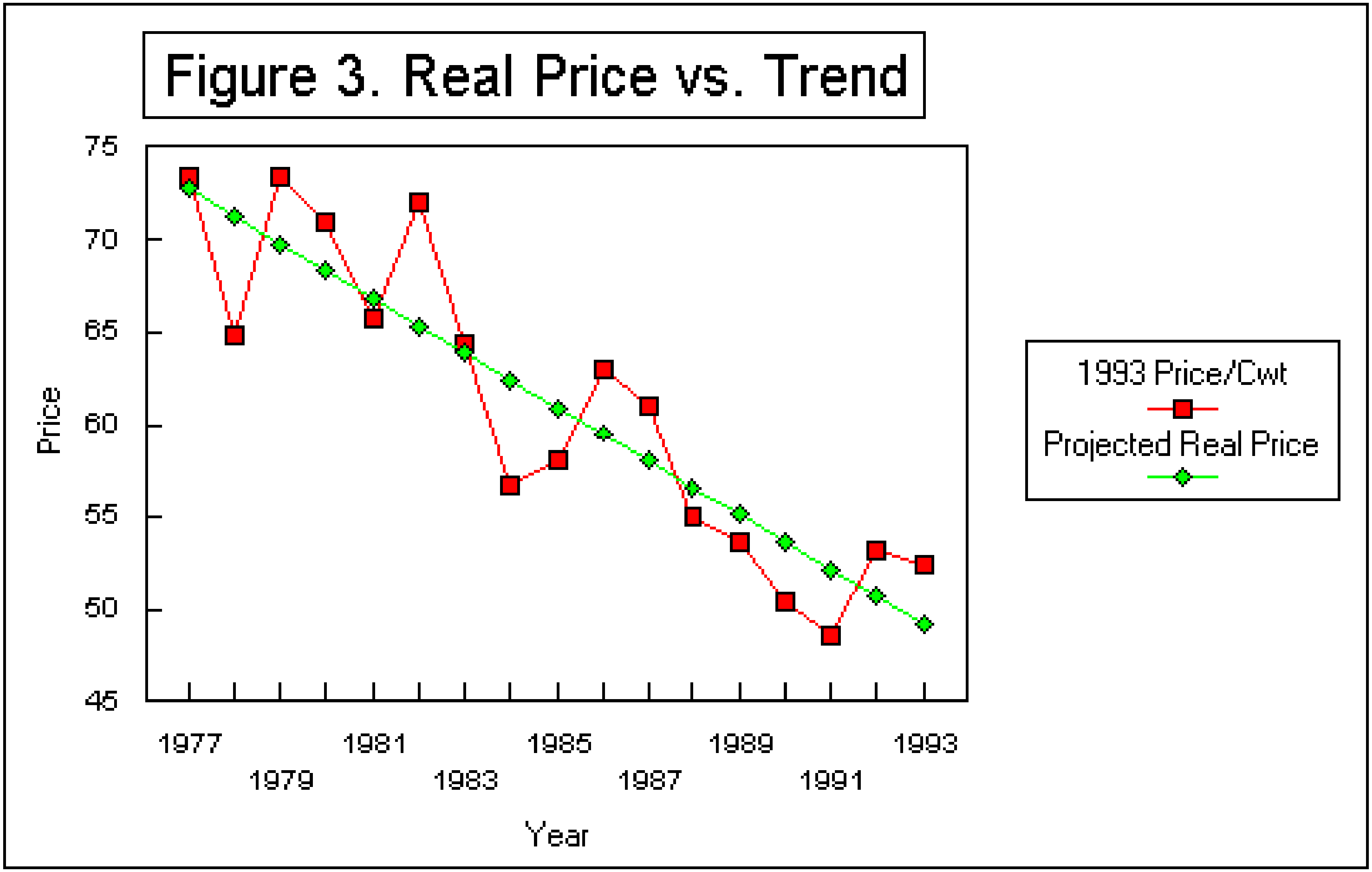


Figure 3. Real Price vs. Trend
Price
75
70
65
60
55
50
45
1977
1979
1981
1983
1985
1987
1989
1991
1993
Year
1993 Price/Cwt
Projected Real Price

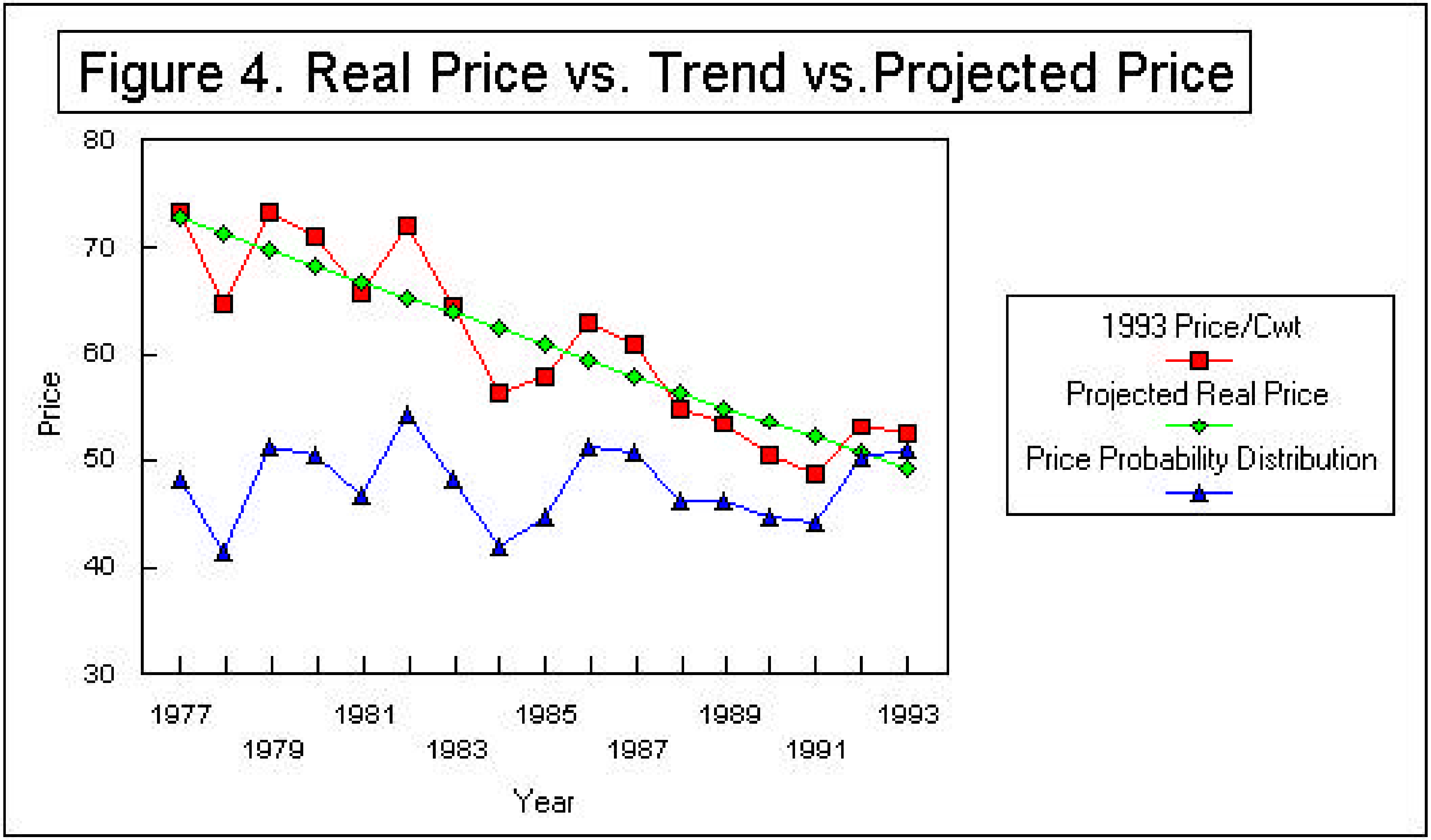


Figure 4. Real Price vs. Trend vs.Projected Price
80
70
60
50
40
30
Price
1977
1979
1981
1983
1985
1987
1989
1991
1993
Year
1993 Price/Cwt
Projected Real Price
Price Probability Distribution

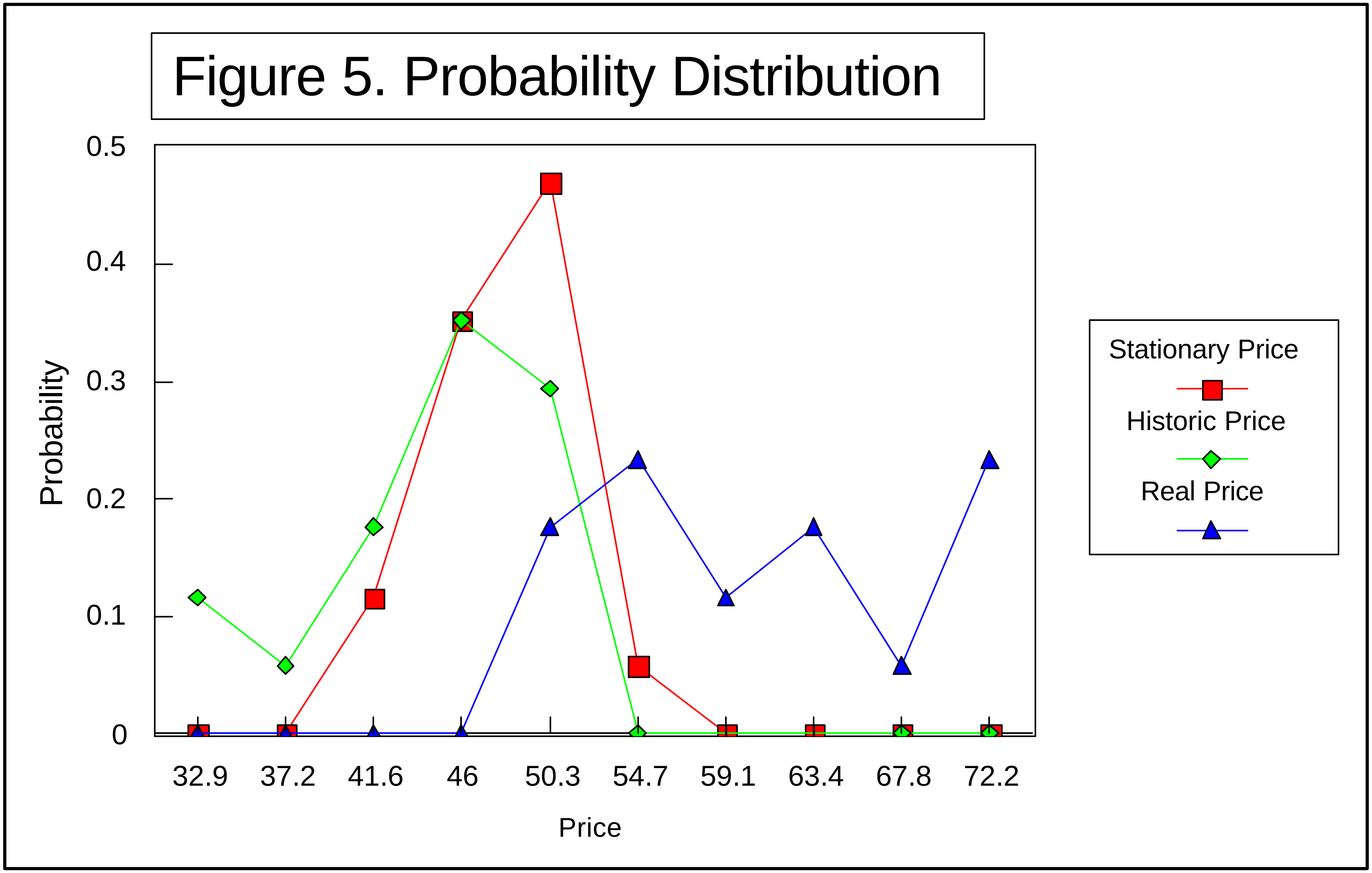


Figure 5. Probability Distribution
0.5
0.4
0.3
0.2
0.1
0
Probability
32.9
37.2
41.6
46
50.3
54.7
59.1
63.4
67.8
72.2
Price
Stationary Price
Historic Price
Real Price

General Lessons Learned on Objective Probabilities

Use objective data – trends and other systematic effects can bias

Use a procedure like regression to develop values expected

One may find residual terms are hetereoskedastic